Q_6 as the Flavor Symmetry in the non-minimal SUSY $SU(5)$ Model

Juan Carlos Gómez Izquierdo, Myriam Mondragón and F. González Canales.

IF-UNAM

XVIII International Symposium on Particles, String and Cosmology PASCOS 2012

June 8, 2012

Motivation

NEUTRINOS

Experimental Data:

$$\theta_{12} \approx 34^\circ$$
$$\theta_{23} \approx 46^\circ$$

$$\theta_{13} \approx \begin{cases} 6.5^\circ\ (7.2^\circ) \\ 9.6^\circ\ (11^\circ)\ T2K \\ 5.7^\circ\ (8^\circ)\ MINOS \\ 9.8^\circ\ RENO \end{cases}$$

Guido Altarelli, *et, al.* Arxiv:1205.4670

Theory: Tri-Bimaximal scenario

$\theta_{12} = 35^\circ;\ \theta_{23} = 45^\circ;\ \theta_{13} = 0^\circ$

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},\ A_4.$$

$$M_\nu = U^* diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$$

$$M_\nu = \begin{pmatrix} a & b & b \\ b & a+c & b-c \\ b & b-c & a+c \end{pmatrix}$$

Bi-maximal scenario:

$\theta_{12} = 45^\circ;\ \theta_{23} = 45^\circ;\ \theta_{13} = 0^\circ$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix},\ S_4.$$

Quarks

$$V_{CKM} = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix},$$

Particle Data Group

1. Nearest Neighbor Interaction (NNI)

$$M_f = \begin{pmatrix} 0 & a_f & 0 \\ \pm a_f & 0 & b_f \\ 0 & c_f & d_f \end{pmatrix}; \; f = u, d, \ell, \nu_\ell.$$

H. Fritzsch, *et, al*. Phys. Lett., B697:357, 2011.

2. Mass matrices with two textures

$$M_f = \begin{pmatrix} 0 & a_f & 0 \\ a_f & b_f & c_f \\ 0 & c_f & d_f \end{pmatrix}; f = u, d, \ell, \nu_\ell$$

H. Fritzsch and Z. Z. Xing. Prog. Part. Nucl. Phys., 45:1-81, 2000.

Flavor Symmetries play an important role to understand mixings

Hajime Ishimori,et al. Prog. Theor. Phys. Suppl., 183:1-163,2010

Q_6

- 12 elements

$$Q_6 = \{1, A, A^2, A^3, A^4, A^5, B, AB, A^2B, A^3B, A^4B, A^5B\};$$

$$A = \begin{pmatrix} \cos(2\pi/6) & \sin(2\pi/6) \\ -\sin(2\pi/6) & \cos(2\pi/6) \end{pmatrix}, \qquad B = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

- 6 irr-representation: $1_{+,0}$, $1_{+,2}$, $1_{-,1}$, $1_{-,3}$, 2_1 and 2_2.
- 2_1 and 2_2 allow to construct mass matrices in NNI.

Q_6 works out very well for fermions in supersymmetric models.

K. S. Babu and Jisuke Kubo. Phys. Rev., D71:056006, 2005;

Yuji Kajiyama, et, al. Nucl. Phys., B743:74-103;

K. S. Babu, et, al. Phys. Rev., D83:095008

SUSY $SU(5) \times U(1) \times Q_6$ Model

How Q_6 looks like within GUT scenario?

Matter content in the SUSY $SU(5) \times U(1) \times Q_6$ model

	$SU(5)$	Q_6	$U(1)$
(H_1^d, H_2^d)	$\bar{5}$	2_1	$-x$
H_3^d	$\bar{5}$	$1_{+,2}$	$-x$
(H_1^u, H_2^u)	5	2_1	x
H_3^u	5	$1_{+,2}$	x
(F_1, F_2)	$\bar{5}$	2_2	$3x/2$
F_3	$\bar{5}$	$1_{-,3}$	$3x/2$
(T_1, T_2)	10	2_2	$-x/2$
T_3	10	$1_{-,3}$	$-x/2$
(N_1^c, N_2^c)	1	2_2	$-5x/2$
N_3^c	1	$1_{-,1}$	$-5x/2$
ϕ	1	$1_{+,2}$	$5x$
$\bar{\phi}$	1	$1_{+,2}$	$-5x$
$H_{\bar{45}}$	$\bar{45}$	$1_{+,2}$	$-x$
H_{45}	45	$1_{+,2}$	x
Φ	24	$1_{+,0}$	0

We assume that R-parity is conserved.

Explicitly, we have

$$
\begin{aligned}
F_{ia} &= \begin{pmatrix} d^c_{i,1} \\ d^c_{i,2} \\ d^c_{i,3} \\ \ell^-_i \\ -\nu_{\ell_i} \end{pmatrix}_L, \; T^{ab}_j = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_{j,3} & u^c_{j,2} & -u^1_j & -d^1_j \\ -u^c_{j,3} & 0 & u^c_{j,1} & -u^2_j & -d^2_j \\ u^c_{j,2} & -u^c_{j,1} & 0 & -u^3_j & -d^3_j \\ u^1_j & u^2_j & u^3_j & 0 & -\ell^+_j \\ d^1_j & d^2_j & d^3_j & \ell^+_j & 0 \end{pmatrix}_L \\
H^{ua} &= \begin{pmatrix} \mathrm{H}^u \\ \mathbf{H}^u \end{pmatrix}, \; H^d_b = \begin{pmatrix} \mathrm{H}^d \\ \mathbf{H}^d \end{pmatrix}, \\
\mathbf{H}^u &= \begin{pmatrix} h^{+u} \\ h^{0u} \end{pmatrix}, \; \mathbf{H}^d = \begin{pmatrix} h^{0d} \\ h^{-d} \end{pmatrix} \\
\langle H_{45} \rangle^{\alpha 5}_{\alpha} &= v_{45}, \; \langle H_{45} \rangle^{45}_{4} = -3 v_{45}; \\
\langle H_{\bar{45}} \rangle^{\alpha}_{\alpha 5} &= v_{\bar{45}}, \; \langle H_{\bar{45}} \rangle^{4}_{45} = -3 v_{\bar{45}}; \\
\langle \phi \rangle &= v_s, \; \langle \bar{\phi} \rangle = \bar{v}_s, \; \alpha, \beta = 1, 2, 3.
\end{aligned}
$$

$$
\begin{aligned}
W \;=\; & \sqrt{2}y_1^d\,(F_1\,T_2 - F_2\,T_1)\,H_3^d + \sqrt{2}y_2^d\left(F_1\,T_3H_2^d - F_2\,T_3H_1^d\right) \\
& +\sqrt{2}y_3^d F_3\left(T_1H_2^d - T_2H_1^d\right) + \sqrt{2}y_4^d F_3\,T_3H_3^d \\
& +\frac{y_1^u}{4}\,(T_1\,T_2 - T_2\,T_1)\,H_3^u + \frac{y_2^u}{4}\,(T_1\,T_3H_2^u - T_2\,T_3H_1^u) \\
& +\frac{y_3^u}{4}\,T_3\,(T_1H_2^u - T_2H_1^u) + \frac{y_4^u}{4}\,T_3\,T_3H_3^u \\
& +\sqrt{2}Y_1\,(F_1\,T_2 - F_2\,T_1)\,H_{\bar{45}} + \sqrt{2}Y_2F_3\,T_3H_{\bar{45}} \\
& +y_1^n\,(N_1^cF_2 - N_2^cF_1)\,H_3^u + y_2^n\,(N_1^cF_3H_2^u - N_2^cF_3H_1^u) \\
& +y_3^nN_3^c\,(F_1H_2^u + F_2H_1^u) + y_1^m\,(N_1^c\phi N_1^c + N_2^c\phi N_2^c) \\
& +y_2^mN_3^c\phi N_3^c.
\end{aligned}
$$

Masses and Mixings

In the $SU(5)$ basis, the mass term is written as

$-\mathcal{L} = \bar{f}_R M_f f_L + \bar{N}_R M_D \nu_{jL} + \frac{1}{2}\bar{N}_R M_R N_R^c + h.c.$

Quarks:

$$M_{u,d} = \begin{pmatrix} 0 & A_{u,d} & B_{u,d} \\ -A_{u,d} & 0 & -B'_{u,d} \\ C_{u,d} & -C'_{u,d} & D_{u,d} \end{pmatrix}$$

Charged leptons:

$$M_\ell = \begin{pmatrix} 0 & -A_\ell & B_\ell \\ A_\ell & 0 & -B'_\ell \\ C_\ell & -C'_\ell & D_\ell . \end{pmatrix}$$

Neutrinos

$$M_D = \begin{pmatrix} 0 & A_D & B_D \\ -A_D & 0 & -B'_D \\ C_D & C'_D & 0 \end{pmatrix}$$

$$M_R^{-1} = diag(x, x, y);$$

$$M_\nu = M_D^T M_R^{-1} M_D.$$

Assumming: $h_2^{0u} = h_1^{0u}$ and $h_2^{0d} = h_1^{0d}$

The free parameters are reduced

$$B_f = B'_f;\ C_f = C'_f.$$

$f = u, d, \ell, D.$

Therefore, we end up having

$$M_f = \begin{pmatrix} 0 & A_f & B_f \\ -A_f & 0 & -B_f \\ C_f & -C_f & D_f \end{pmatrix} ; \; M_\nu = \begin{pmatrix} A_\nu^2 + B_\nu^2 & B_\nu^2 & A_\nu C_\nu \\ B_\nu^2 & A_\nu^2 + B_\nu^2 & A_\nu C_\nu \\ A_\nu C_\nu & A_\nu C_\nu & 2C_\nu^2 \end{pmatrix}$$

Making a rotation: $f_{R,L} = U_{\pi/4} f_{1R,L} \Rightarrow M_{1f} = U_{\pi/4}^T M_f U_{\pi/4}$.

$$M_{1f} = \begin{pmatrix} 0 & A_f & 0 \\ -A_f & 0 & -\sqrt{2}B_f \\ 0 & -\sqrt{2}C_f & D_f \end{pmatrix} , \; U_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In order to obtain the U_{1fL}, we must build the bilineal form.

$$\begin{aligned} \tilde{M}_{1f} &= diag(\tilde{m}_{1f}, \tilde{m}_{2f}, 1) = U^{\dagger}_{1fR} \hat{M}_{1f} U_{1fL} \\ \tilde{M}^{\dagger}_{1f} \tilde{M}_{1f} &= U^{\dagger}_{1fL} \hat{M}^{\dagger}_{1f} \hat{M}_{1f} U_{1fL} = U^{\dagger}_{1fL} Q_f (\hat{M}^{\dagger}_{1f} \hat{M}_{1f}) Q^{\dagger}_f U_{1fL} \end{aligned}$$

We appropriately choose $U_{1fL} = Q_f O_{fL}$.

$$(\hat{M}^{\dagger}_{1f} \hat{M}_{1f}) = \begin{pmatrix} |\tilde{A}_f|^2 & 0 & |\tilde{A}_f||\tilde{B}_f| \\ 0 & |\tilde{A}_f|^2 + |\tilde{C}_f|^2 & |\tilde{C}_f||\tilde{D}_f| \\ |\tilde{A}_f||\tilde{B}_f| & |\tilde{C}_f||\tilde{D}_f| & |\tilde{B}_f|^2 + |\tilde{D}_f|^2 \end{pmatrix}$$

$$\begin{aligned} |\tilde{A}_f| &= \frac{q_f}{y_f}, \; |\tilde{B}_f| = \sqrt{\frac{1 + P_f - y_f^4 - R_f}{2} - \left(\frac{q_f}{y_f}\right)^2}; \\ |\tilde{C}_f| &= \sqrt{\frac{1 + P_f - y_f^4 + R_f}{2} - \left(\frac{q_f}{y_f}\right)^2}, \; |\tilde{D}_f| \equiv y_f^2. \\ R_f &= \sqrt{(1 + P_f - y_f^4)^2 - 4\left(P_f + q_f^4\right) + 8 q_f^2 y_f^2} \\ P_f &= \tilde{m}_{1f}^2 + \tilde{m}_{2f}^2, \; q_f = \sqrt[4]{\tilde{m}_{1f}^2 \tilde{m}_{2f}^2} \end{aligned}$$

K. Harayama et al. Prog. Theor. Phys., 97:781-790,1997; K. Harayama et al.Phys. Lett., B387:614-622,1996.

Therefore, $U_{fL} = U_{\pi/4} Q_f O_f$ then

$$V_{CKM} = U_{uL}^{\dagger} U_{dL} = O_{uL}^{T} Q_q O_{dL};\ Q_q = diag\left(1, \exp i\overline{\eta}_1, \exp i\overline{\eta}_2\right).$$

Taking into account neutrinos, we rotate $\nu_L = U_\nu \tilde{\nu}_L$.

$$\begin{aligned} \tilde{M}_\nu &= diag\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right) = U_\nu^T M_\nu U_\nu = U_{1\nu}^T u_{\pi/4}^T M_\nu u_{\pi/4} U_{1\nu} \\ &= U_{1\nu}^T M_{1\nu} U_{1\nu} = U_{1\nu}^T P_\nu \hat{M}_{1\nu} P_\nu U_{1\nu} = O_\nu^T \hat{M}_{1\nu} O_\nu. \end{aligned}$$

$$\begin{aligned} \hat{M}_{1\nu} &= \begin{pmatrix} |A_\nu|^2 + 2|B_\nu|^2 & \sqrt{2}|A_\nu||C_\nu| & 0 \\ \sqrt{2}|A_\nu||C_\nu| & 2|C_\nu|^2 & 0 \\ 0 & 0 & |A_\nu|^2 \end{pmatrix} \\ u_{\pi/4} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

Orthogonal neutrino matrix is given by

$$O_\nu = \begin{pmatrix} \sqrt{\frac{m_{\nu_3}\left(m_{\nu_2}+m_{\nu_1}-m_{\nu_3}+R_\nu\right)}{\left(m_{\nu_2}-m_{\nu_1}\right)\left(m_{\nu_2}-m_{\nu_1}+m_{\nu_3}-R_\nu\right)}} & \sqrt{\frac{m_{\nu_3}\left(m_{\nu_2}+m_{\nu_1}-m_{\nu_3}+R_\nu\right)}{\left(m_{\nu_2}-m_{\nu_1}\right)\left(m_{\nu_2}-m_{\nu_1}-m_{\nu_3}+R_\nu\right)}} & 0 \\ -\sqrt{\frac{m_{\nu_2}-m_{\nu_1}+m_{\nu_3}-R_\nu}{2\left(m_{\nu_2}-m_{\nu_1}\right)}} & \sqrt{\frac{m_{\nu_2}-m_{\nu_1}-m_{3\nu}+R_\nu}{2\left(m_{\nu_2}-m_{\nu_1}\right)}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_\nu = \sqrt{(m_{\nu_2}+m_{\nu_1}-m_{\nu_3})^2 - 4m_{\nu_2}m_{\nu_1}}.$$

- An inverted ordering, $m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$, is predicted.
- There is a sum rule: $m_{\nu_3} \leq (\sqrt{m_{\nu_2}} - \sqrt{m_{\nu_1}})^2$

$$\sqrt{m_{\nu_3}} = \sqrt{m_{\nu_2}} - \sqrt{m_{\nu_1}}$$

Then, $U_\nu = u_{\pi/4} P_\nu^\dagger O_\nu$. Therefore, the PMNS mixing matrix is given by

$$V = U_\ell^\dagger U_\nu K = O_{\ell L}^T Q_\ell^\dagger S_{23} P_\nu^\dagger O_{\nu L} K$$

$$V = \begin{pmatrix} O_{11\ell}O_{11\nu} + O_{31\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{11\ell}O_{12\nu} + O_{31\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{21\ell}\exp\left(i\eta_{2e}\right) \\ O_{12\ell}O_{11\nu} + O_{32\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{12\ell}O_{12\nu} + O_{32\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{22\ell}\exp\left(i\eta_{2e}\right) \\ O_{13\ell}O_{11\nu} + O_{33\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{13\ell}O_{12\nu} + O_{33\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right) & O_{23\ell}\exp\left(i\eta_{2e}\right) \end{pmatrix} K$$

Comparing with the PDG parametrization, we obtain

$$\begin{aligned} \left|\sin\theta_{13}\right| &= \left|O_{21\ell}\right|, \quad \left|\sin\theta_{23}\right| = \frac{\left|O_{22\ell}\right|}{\sqrt{1-\left|O_{21\ell}\right|^2}} \\ \left|\tan\theta_{12}\right|^2 &= \frac{\left|O_{11\ell}O_{12\nu} + O_{31\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right)\right|^2}{\left|O_{11\ell}O_{11\nu} + O_{31\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right)\right|^2}. \end{aligned}$$

A. Mondragón et al. Phys. Rev., D76:076003,2007

Results

Quarks: χ^2 analysis

$$
\begin{aligned}
y_u &= 0.953555^{+0.02538}_{-0.056013}, y_d = 0.9602600^{+0.020121}_{-0.050659} \\
\overline{\beta}_1 &= 1.448186^{+0.075394}_{-0.153936}, \overline{\beta}_2 = 1.461911^{+0.113089}_{-0.177181}
\end{aligned}
$$

$\chi^2_{min} = 0.06136717.$

$$
|V|_{CKM} = \begin{pmatrix} 0.9742741^{+0.0001849}_{-0.0001781} & 0.2253394^{+0.0007666}_{-0.0008004} & 0.003492633^{+0.000347107}_{-0.000395203} \\ 0.2252032^{+0.0007668}_{-0.0008032} & 0.9734453^{+0.0001937}_{-0.0001843} & 0.04108302^{+0.00132368}_{-0.00149022} \\ 0.008577626^{+0.000639874}_{-0.000534336} & 0.04032911^{+0.00133919}_{-0.00154291} & 0.9991496^{+0.0000604}_{-0.0000556} \end{pmatrix}
$$

For the Jarlskog invariant, $J = 2.916157^{+0.3509}_{-0.3919} \times 10^{-5}$.

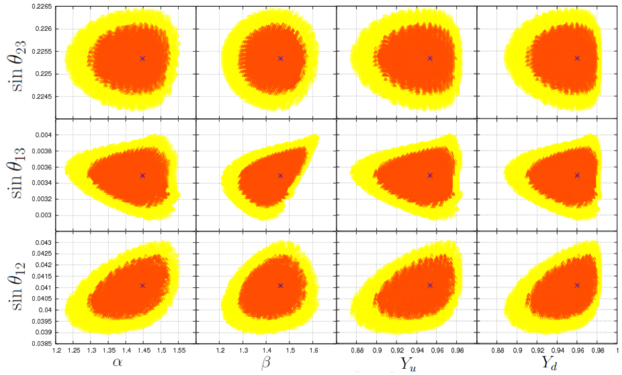

Figure: Allowed region for the three CKM mixing angles as function of the four free parameters in the quark sectoe at 65% (orange color) and 90% (yellow color) of C.L.

Neutrinos: χ^2 analysis for $\sin^2 2\theta_{13}^{\ell}$ and $\sin^2 \theta_{23}^{\ell}$

$$
\begin{aligned}
y_e &= 0.8478^{+0,0045}_{-0.0046} \quad 1\sigma \\
\theta_{23}^{\ell th} &= 46.18^{+0.66}_{-0.65}, \quad \theta_{13}^{\ell th} = 3.38^{+0.03}_{-0.02}, \\
\chi^2_{min} &= 0.85
\end{aligned}
$$

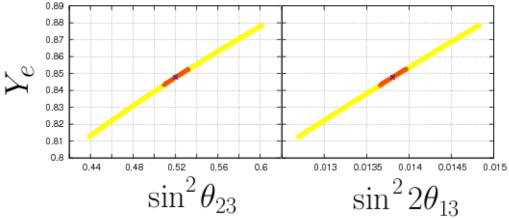

Figure: Atmospheric and reactor mixing angle values at 65% (orange color) and 95% (yellow color) of C.L.

For the solar angle:

$$m_{\nu_3} \leq \left(\sqrt[4]{m_{\nu_3}^2 + \Delta m_{\odot}^2 + \Delta m_{ATM}^2} - \sqrt[4]{m_{\nu_3}^2 + \Delta m_{ATM}^2} \right)^2$$

$0 \leq m_{\nu_3} \leq 4 \times 10^{-6}$ eV. As a result, we obtain

$$\begin{aligned} m_{\nu_2} &\approx 5.0 \pm 0.087 \times 10^{-2} eV, \\ m_{\nu_1} &\approx 4.90 \pm 0.089 \times 10^{-2} eV \end{aligned}$$

In order to calculate the solar angle we consider: $m_{\nu_2} = 0.05080$ eV, $m_{\nu_1} = 0.04987$ eV and $m_{\nu_3} = 3.9 \times 10^{-6}$ eV; $\bar{\eta}_{3e} = \pi$ and using the y_e value at 90% at C.L, we obtain for the solar mixing angle

$$\theta_{12}^{\ell th} = 36.62 \pm 4.06.$$

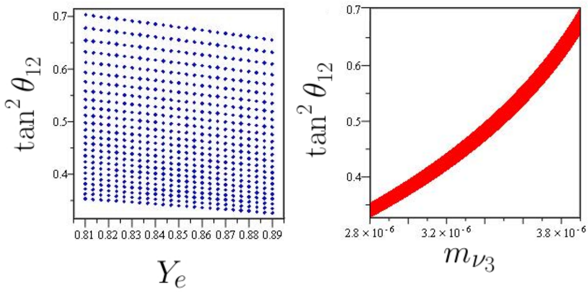

Figure: Solar mixing angle values as function of the y_e parameter and the m_{ν_3} neutrino mass considering $m_{\nu_2} = 0.05080$ eV, $m_{\nu_1} = 0.04987$ eV and $\bar{\eta}_{3e} = \pi$.

Conclusions

1. We have reproduced successfully the CKM mixing matrix which is consistent with the experimental results.
2. The model predicts a notable inverted hierarchy among the neutrino masses and an appealing sum rule of the neutrino masses appears such that it turns out fundamental to determine the neutrino masses, and therefore, the solar mixing angle.
3. As main result, we have that the atmospheric ($\theta_{23}^{\ell th} = 46.18^{+0.66}_{-0.65}$) and solar angle ($\theta_{12}^{\ell th} = 36.62 \pm 4.06$) are in good agreement with the experimental data, however, the reactor angle is not consistent with the global fits but it is not completely negligible and this is so large in comparison to the tribimaximal scenario, $\theta_{13}^{\ell th} = 3.38^{+0.03}_{-0.02}$.