Renormalization group running of couplings and masses in the basic extension of the standard model

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Introduction

The Standard Model (SM) in high energy physics has been remarkably successful in : describing the properties of elementary particles, predicting the existence of the quarks *c*, *t* and *b*, and the third generation of leptons τ, $\nu\tau$, the existence of the eight gluons, and the bosons *W* , *Z* before their discovery, predicting parity violation neutral-weak-currents, and in being consistent with all the experimental results.

However, the SM falls short of being a complete theory of the fundamental interactions because of its lack of explanation of the probable unification of the fundamental interactions, the pattern and disparity of the particles masses (mass hierarchy), the origin of the CP violation in nature, the matter-antimatter asymmetry, the pattern of quark mixing, lepton mixing and the reason why there are 3 generation.

Potential 2HDM

This model is studied mainly for three reasons

- The firts one is that the 2HDM has a much richer Higgs spectrum (3 neutral and 2 charge Higgses) and a different high energy behavior. This makes that a lower mass than in the SM Higgs is permitted.

- Another reason may be that a different pattern of hierarchy of the Yukawa couplings is possible, because of the presence of two independent vacuum expectation values of the Higgs fields.

- The third reason, is that the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM) contains two Higgs doublets, so the Higgs sectors of the MSSM and the 2HDM are similar and the study of the 2HDM model may give important information on the properties of the Higgs sector in the MSSM.

Two Higgs Doublet Models

Reference: Mahmoudi, Stal, Phys. Rev. D 81

- Higgs Fields

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

- The Higgs potential

$$\begin{aligned} V(\Phi_1, \Phi_2) &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + (\mu_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.) \\ &+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_5^* (\Phi_2^\dagger \Phi_1)^2\right] \\ &- \{[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)](\Phi_1^\dagger \Phi_2) + h.c.\}. \end{aligned}$$

- Z_2 symmetry

$$\phi_1 \to \phi_1, \; \phi_2 \to -\phi_2$$

CP symmetry conservation

Reduces the number of parameters in the potential ($\lambda_6 = \lambda_7 = \mu^2_{12} = 0$)

Mass eigenstates and mass spectrum

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = U_\alpha \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \qquad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U_\beta \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = U_\gamma \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix},$$

- Where $\phi_3 = \frac{v_1}{\sqrt{2}} + h_1, \quad \phi_4 = \eta_1, \quad \phi_7 = \frac{v_2}{\sqrt{2}} + h_2, \quad \phi_8 = \eta_2$

$$U_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \qquad \tan 2\alpha = \frac{(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2}{(\lambda_1 v_1^2 - \lambda_2 v_2^2)}, \qquad U_\alpha^\dagger U_\alpha = 1$$

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}, \qquad \tan\beta = \frac{v_2}{v_1}, \quad v^2 = v_1^2 + v_2^2, \qquad 0 < \beta < \frac{\pi}{2}, \qquad \gamma = \beta$$

After the complete diagonalization, we obtain the following relations

- The mass eigenvalues for (H^0, h^0) are

$$M^2_{H^0,h^0} = \lambda_1 v_1^2 + \lambda_2 v_2^2 \pm \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + [v_1 v_2 (\lambda_3 + \lambda_4 + \lambda_5)]^2} > 0,$$

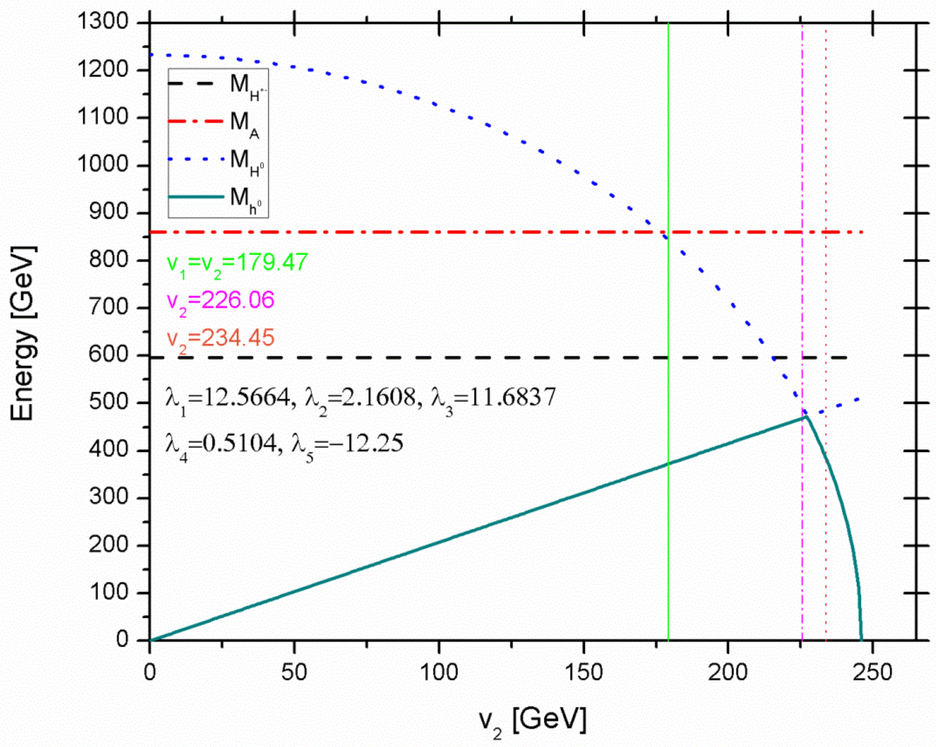
- The eigenvalues for the mass eigenstates $(H^\pm, G^\pm)$ are

$$M^2_{G^\pm} = 0, \qquad M^2_{H^\pm} = -\frac{1}{2}(\lambda_4 + \lambda_5) v^2 > 0,$$

- Finally, the mass eigenvalues for (G^0, A^0)

$$M^2_{G^0} = 0, \qquad M^2_{A^0} = -\lambda_5 v^2 > 0$$

Example



$M_{H^{\pm}}$
M_A
M_{H^0}
M_{h^0}
$v_1=v_2=179.47$
$v_2=226.06$
$v_2=234.45$
$\lambda_1=12.5664,\ \lambda_2=2.1608,\ \lambda_3=11.6837$
$\lambda_4=0.5104,\ \lambda_5=-12.25$
Energy [GeV]
v_2 [GeV]
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
0
50
100
150
200
250

Constraints from vacuum stability

As the values of the quartic interactions between the scalar doublets are not theoretically determined, it is of great interest to explore and constraint their values, therefore we analize the bounds from the vacuum stability

- Mass formulas

$$\lambda_1 > 0, \;\; \lambda_2 > 0, \;\; \lambda_4 + \lambda_5 < 0, \;\; \lambda_5 < 0, \;\; \lambda_4 < \left|\lambda_5\right| \;\; , \;\; \lambda_1\lambda_2 > \frac{1}{4} \left(\lambda_3 + \lambda_4 + \lambda_5\right)^2$$

- Lagrange multipliers method

$$L(x, \Lambda, \Theta) = f(x) + \sum_{i=1}^{n} \Lambda_i g_i(x) + \sum_{j=i}^{m} \Theta_j h_j(x),$$

$$g_i(x) = 0, \quad i = 1, 2, \ldots, n,$$

$$h_j(x) \leq 0 \; (\geq 0), \quad j = 1, 2, \ldots, m.$$

- Constraint equation

$$x_3^2 + x_4^2 \le x_1 x_2, \quad x_1 + x_2 - v^2 = 0$$

- The quartic potential

$$F = b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{44} x_4^2 + b_{12} x_1 x_2$$

- Where

$$x_1 = \left| \Phi_1 \right|^2 \;, \; x_2 = \left| \Phi_2 \right|^2 \;, \; x_3 = \frac{1}{2} \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \;, \; x_4 = \frac{1}{2i} \left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1 \right)$$

$$b_{11} = \lambda_1, \; b_{22} = \lambda_2, \; b_{12} = \lambda_3, \; b_{33} = (\lambda_4 + \lambda_5), \; b_{44} = (\lambda_4 - \lambda_5)$$

- The necessary conditions (karush-Kuhn-Tucker)

$$\frac{\partial f(x^*)}{\partial x_k} + \sum_{j=1}^{m} \Lambda_j \frac{\partial g_j(x^*)}{\partial x_k} + \sum_{j=1}^{m} \Theta_j \frac{\partial g_j(x^*)}{\partial x_k} = 0, \quad k = 1, 2, \ldots, n,$$

$$g_i(x^*) = 0, \quad i = 1, 2, \ldots, n,$$

$$\Lambda_i g_i(x) = 0, \quad i = 1, 2, \ldots, n,$$

$$h_j(x^*) \le 0, \quad j = 1, 2, \ldots, m,$$

$$\Theta_j h_j(x^*) = 0, \quad j = 1, 2, \ldots, m,$$

$$\Lambda_i \ge 0, \quad i = 1, 2, \ldots, m,$$

$$\Theta_j \ge 0, \quad j = 1, 2, \ldots, m.$$

- We obtain the following restrictions (stable vacuum requirements)

$$\lambda_1 + \lambda_2 > \lambda_3 + \lambda_4 + \lambda_5,$$
$$-2\sqrt{\lambda_1 \lambda_2} < \lambda_3 + \lambda_4 + \lambda_5$$

$$\lambda_1 + \lambda_2 > \lambda_3 + \lambda_4 - \lambda_5,$$
$$-2\sqrt{\lambda_1 \lambda_2} < \lambda_3 + \lambda_4 - \lambda_5$$

$$\lambda_1 + \lambda_2 > \lambda_3,$$
$$-2\sqrt{\lambda_1 \lambda_2} < \lambda_3$$

$$\lambda_1 + \lambda_2 > \lambda_3 + \lambda_4,$$
$$-2\sqrt{\lambda_1 \lambda_2} < \lambda_3 + \lambda_4, \quad \lambda_5 = 0$$

$$\lambda_T = \lambda_3 + \lambda_4 + \lambda_5$$

Extreme stability conditions

$$V_4 = \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_T x_1 x_2 = \left(\sqrt{\lambda_1}\, x_1 - \sqrt{\lambda_2}\, x_2\right)^2 + \left(\lambda_T + 2\sqrt{\lambda_1 \lambda_2}\right) x_1 x_2 \geq 0.$$

- In the extreme case, the condition to be satisfied is $V_4 = 0$, then

$$\sqrt{\frac{\lambda_1}{\lambda_2}} = \frac{x_2}{x_1} = \frac{v_2^2}{v_1^2}, \quad \lambda_T = -2\sqrt{\lambda_1 \lambda_2}, \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{v_2^4}{v_1^4} = (\tan\beta)^4$$

- In this case the Higgs masses become

$$M_{H^0} = \begin{cases} (4\lambda_1 \lambda_2)^{1/4} v, & \lambda_1 \neq \lambda_2, \quad v_1 \neq v_2, \\ (2\lambda)^{1/2} v, & \lambda_1 = \lambda_2 = \lambda, \quad v_1 = v_2 = v/\sqrt{2}. \end{cases}$$

- In another interesting case, which is the semi-extreme case, the $V_4 = (\sqrt{\lambda_1}\ x_1 - \sqrt{\lambda_2}\ x_2)^2 > 0$, and

$$\frac{\lambda_1}{\lambda_2} \neq (\tan\beta)^4, \quad \lambda_T = -2\sqrt{\lambda_1 \lambda_2},$$

- The masses become

$$M_{H^0} = \sqrt{2}\left(\lambda_1 v^2 + (\lambda_2 - \lambda_1) v_2^2\right)^{1/2} \quad .$$

- In both cases

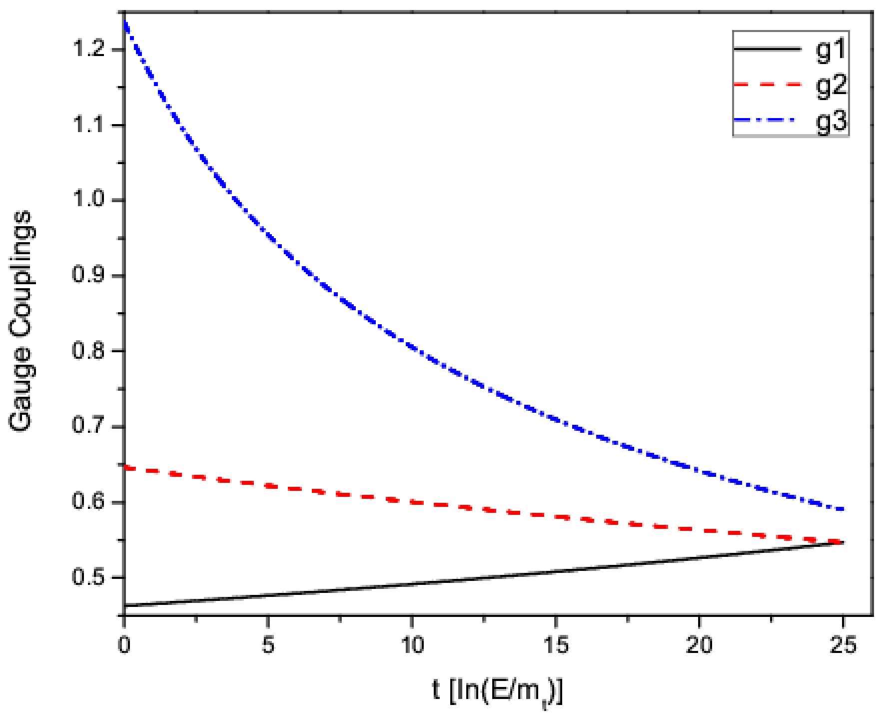
$$M_{h^0} = 0, \quad M_{H^\pm} = \left(\frac{1}{2}|\lambda_4 + \lambda_5|\right)^{1/2} v, \quad M_{A^0} = |\lambda_5|^{1/2} v.$$

Constraints from triviality

- RGE

The RGE determine the dependence of the coupling constants and other parameters of the Lagrangian on t, defined as $t = \ln(E/m_t)$, where E is the renormalization point energy. The RGE for the gauge couplings $g1$, $g2$, $g3$ with ($b1$, $b2$, $b3$) = (21/5,-3,-7) are

$$\frac{dg_i}{dt} = \frac{1}{(4\pi)^2} b_i g_i^3 \quad , i = 1, 2, 3$$



Constraints from triviality

- The RGE for the Yukawa couplings of the top and bottom quarks are

$$\frac{dy_t^2}{dt} = \frac{1}{(4\pi)^2} y_t^2 \left[9y_t^2 + y_b^2 - 2\left(\frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2 \right) \right],$$

$$\frac{dy_b^2}{dt} = \frac{1}{(4\pi)^2} y_d^2 \left[9y_b^2 + y_t^2 - 2\left(\frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2 \right) \right],$$

- And for the vacuum expectation values

$$\frac{dv_1}{dt} = \frac{1}{(4\pi)^2} v_1 \left[-3y_t^2 + \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) \right],$$

$$\frac{dv_2}{dt} = \frac{1}{(4\pi)^2} v_2 \left[-3y_b^2 + \left(\frac{9}{20} g_1^2 + \frac{9}{4} g_2^2 \right) \right],$$

- In the equations for the quartic couplings we include the quark Yukawa contributions of both sectors

$$\frac{d\lambda_1}{dt} = \frac{1}{(4\pi)^2}[24\lambda_1^2 + \lambda_3^2 + \lambda_5^2 + (\lambda_3 + \lambda_4)^2 - 9\lambda_1\left(\frac{1}{5}g_1^2 + g_2^2\right) + \frac{3}{8}\left(\frac{3}{5}g_1^2 + g_2^2\right)^2 + \frac{3}{4}g_2^4 + 12\lambda_1 y_t^2 - 6y_t^2],$$

$$\frac{d\lambda_2}{dt} = \frac{1}{(4\pi)^2}[24\lambda_2^2 + \lambda_3^2 + \lambda_5^2 + (\lambda_3 + \lambda_4)^2 - 9\lambda_2\left(\frac{1}{5}g_1^2 + g_2^2\right) + \frac{3}{8}\left(\frac{3}{5}g_1^2 + g_2^2\right)^2 + \frac{3}{4}g_2^4 + 12\lambda_2 y_b^2 - 6y_b^2],$$

$$\frac{d\lambda_3}{dt} = \frac{1}{(4\pi)^2}[4\lambda_3^2 + 4(3\lambda_3 + \lambda_4)(\lambda_1 + \lambda_2) - 3\lambda_3\left(3g_2^2 + \frac{3}{5}g_1^2 - 2(y_t^2 + y_b^2)\right) + 2\lambda_4^2 + 2\lambda_5^2 + \frac{3}{4}\left(g_2^2 - \frac{3}{5}g_1^2\right)^2 + \frac{3}{2}g_2^4 - 12y_t^2 y_b^2],$$

$$\frac{d\lambda_4}{dt} = \frac{1}{(4\pi)^2}[4\lambda_4^2 + 4\lambda_4(2\lambda_3 + \lambda_1 + \lambda_2) - 3\lambda_4\left(3g_2^2 + \frac{3}{5}g_1^2 - 2(y_t^2 + y_b^2)\right) + 8\lambda_5^2 + \frac{9}{5}g_1^2 g_2^2 + 12y_t^2 y_b^2],$$

$$\frac{d\lambda_5}{dt} = \frac{1}{(4\pi)^2}\lambda_5[4(\lambda_1 + \lambda_2) + 8\lambda_3 + 12\lambda_4 - 9\left(g_2^2 + \frac{1}{5}g_1^2\right) + 6(y_t^2 + y_b^2)]$$

- We evaluate the Higgs masses under different conditions for the quartic couplings in the energy scale E=Mt
- Several cases are discussed

A. Extreme case $\lambda_T = -2\sqrt{\lambda_1 \lambda_2}$ $\frac{\lambda_1}{\lambda_2} = (\tan\beta)^4$

B1. Semi-extreme case $\lambda_T = -2\sqrt{\lambda_1 \lambda_2}$ $\frac{\lambda_1}{\lambda_2} \neq (\tan\beta)^4$

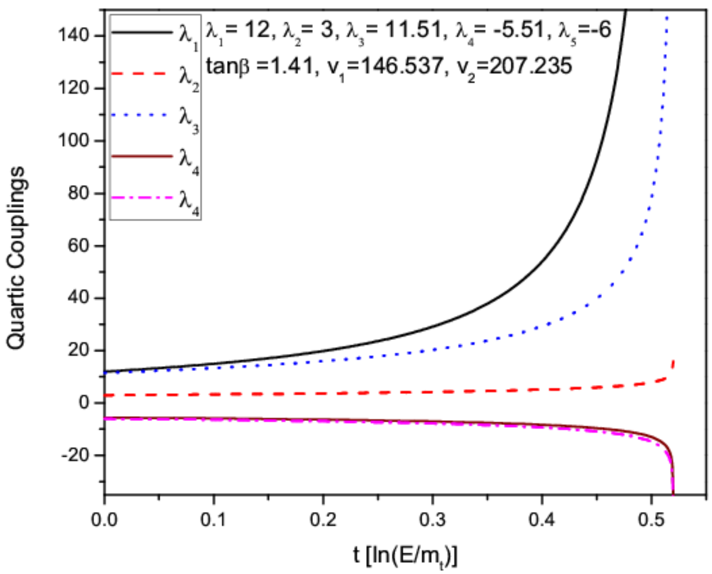
B2. Semi-extreme case $\lambda_T \neq -2\sqrt{\lambda_1 \lambda_2}$ $\frac{\lambda_1}{\lambda_2} = (\tan\beta)^4$

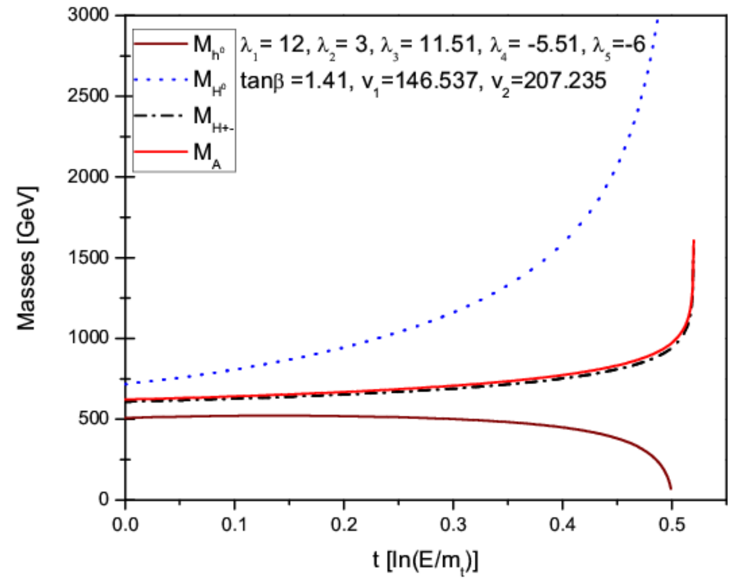
C. Lagrange inequality condition $\lambda_T \geq -2\sqrt{\lambda_1 \lambda_2}$

D. Yukawa- Unification condition $\tan\beta = \frac{M_t}{M_b}$ $g_t = g_b$

Numerical solution to the RGE

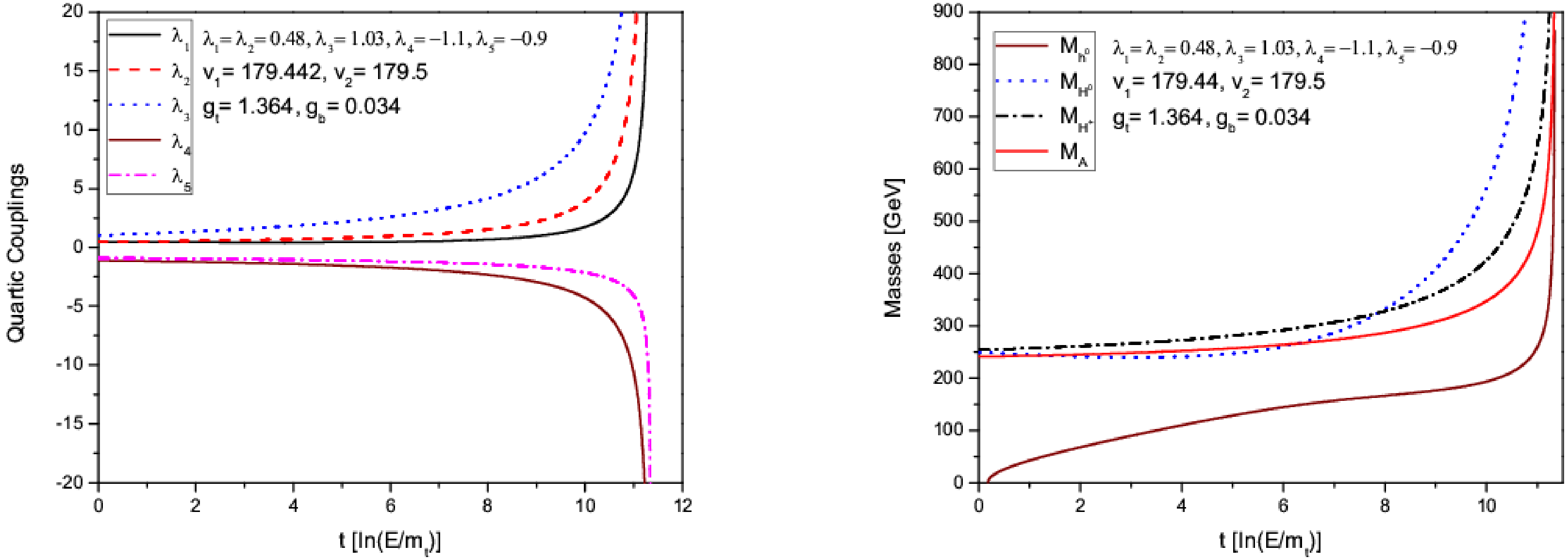
- Limit of validity of the model





The energy dependence of the quartic coupling and Higgs masses, case B2, with $\tan\beta = 1.41$

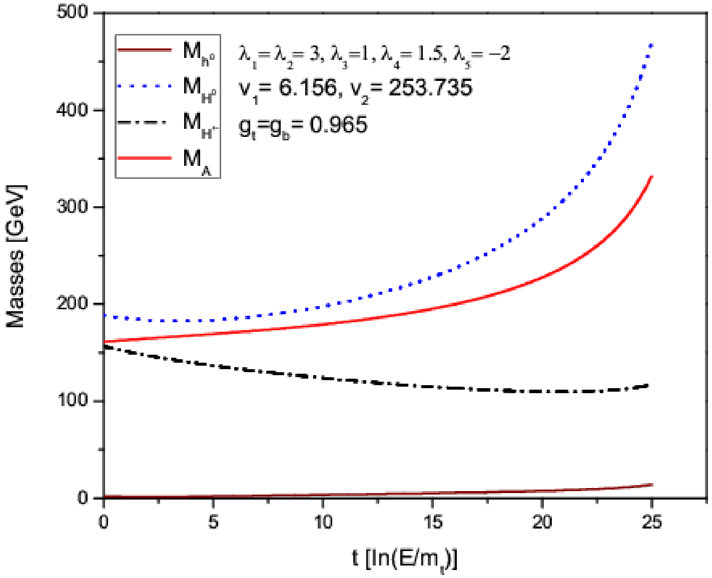
$$173.2 < E \ \left[GeV \right] < 292, \quad 0 < t < 0.52$$

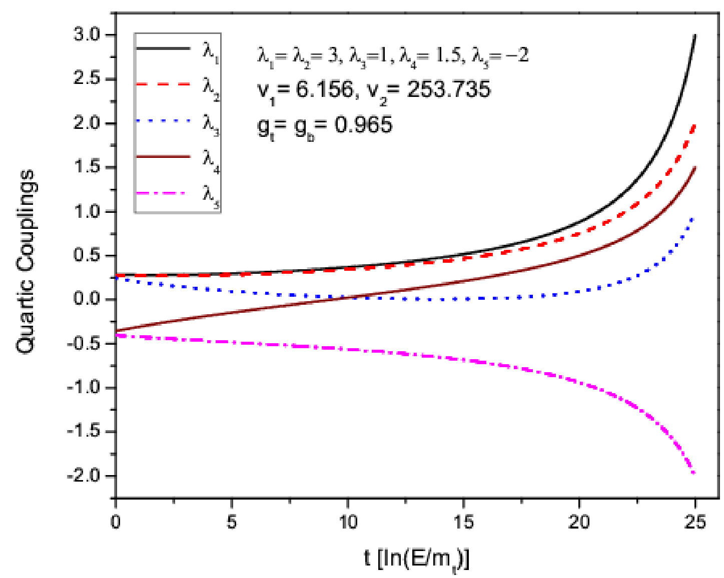


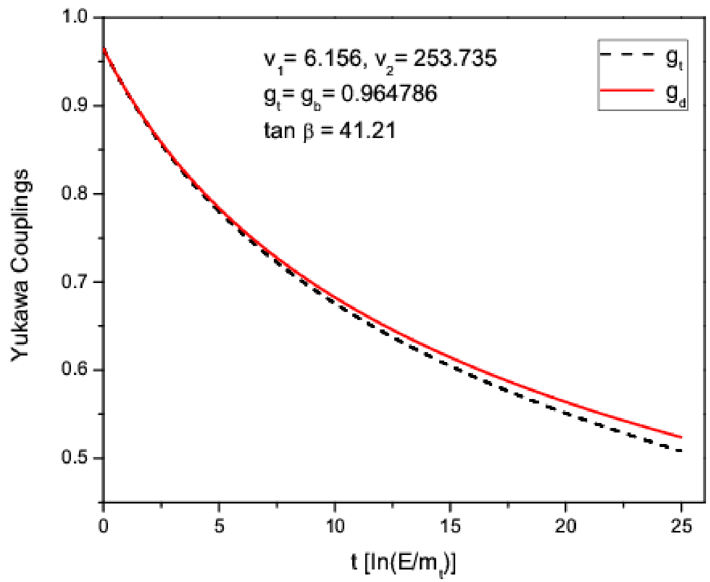
The energy dependence of the quartic coupling and the Higgs masses, case A with $\tan\beta = 1$

$$173.2 < E \ \left[GeV \right] < 1.45612\times10^{7}, \ \ 0 < t < 12$$

Case: low energy unification

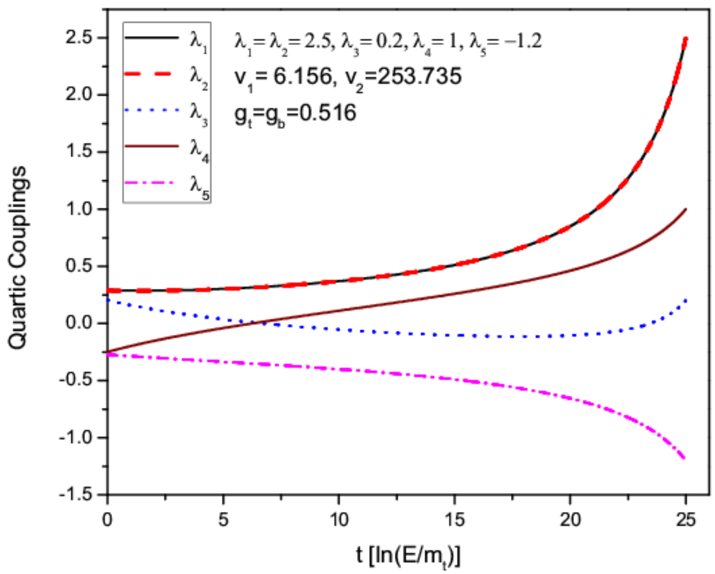


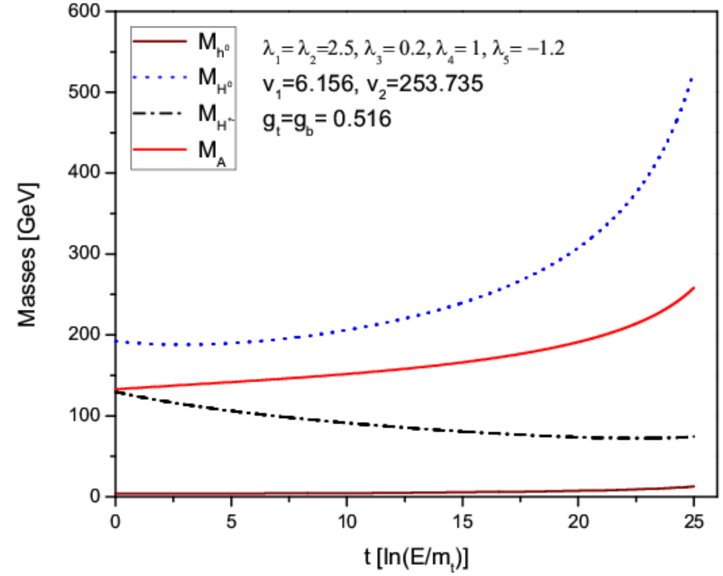


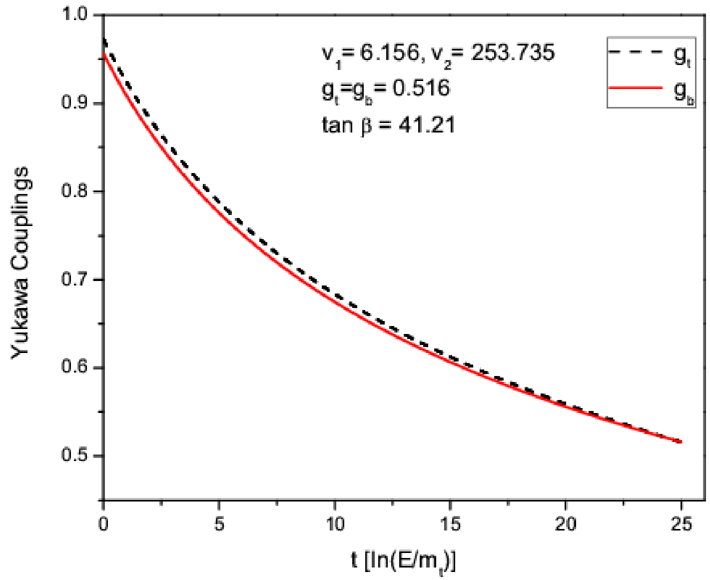


$$173.2 < E \ \left[GeV \right] < 1.234\times 10^{13}, \quad 0 < t < 25$$

Case: unification at high energies







Conclusions:

- Through the former results one can establish the region of validity of the model under several circunstances considered in the literature.
- We have obtained new restrictions to be satisfied by the quartic couplings through the Lagrange multipliers method.
- We have considered different cases under the RGE.

Thank You