

Yukawa unification :
MSSM at large $\tan\beta$

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DEPARTMENT OF
PHYSICS

Outline

- SO(10) Yukawa unification
- Large $\tan \beta \sim 50$
- $m_{\text{top}}, m_{\text{bottom}}, m_{\text{tau}}$
- Soft SUSY breaking masses
 - **Inverted scalar mass hierarchy**
- Higgs masses
- Fermion masses and mixing
- Flavor violation

Hierarchical $SO(10)$ Yukawas

$$W \supset 16_3 \mathbf{10} 16_3 + 16_3 \mathbf{10} \frac{45}{M} 16_2 + \dots$$

Albright, Anderson, Babu, Barr, Barbieri, Berezhiani,
Blazek, Carena, Chang, Dermisek, Dimopoulos, Hall,
Masiero, Murayama, Pati, Raby, Romanino, Rossi,
Starkman, Wagner, Wilczek, Wiesenfeldt,
Willenbrock

Effective higher dimension operators,

Small rep's + Many predictions !!

Possible UV completion to strings !!

$$\lambda \quad 16_3 \quad 10 \quad 16_3$$

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Note, CANNOT predict top mass due to large SUSY threshold corrections to bottom and tau mass

Hall, Rattazzi & Sarid

Carena, Olechowski, Pokorski & Wagner

So instead use Yukawa unification to predict soft SUSY breaking masses !!

Bottom mass corrections

$$\frac{\delta m_b}{m_b} \propto \frac{\alpha_3 \mu M_g \tan \beta}{m_{\tilde{b}}^2} + \frac{\lambda_t^2 \mu A_t \tan \beta}{m_{\tilde{t}}^2} + \log \text{corr.}$$

$$\frac{\delta m_b}{m_b} \leq -2\%$$

Needed to fit data

$$\mu M_g > 0 \quad \Rightarrow \quad \mu A_t < 0$$

Yukawa Unification & Soft SUSY breaking

Blazek, Dermisek & Raby PRL 88, 111804 (2002)

PRD 65, 115004 (2002)

Baer & Ferrandis, PRL 87, 211803 (2001)

Auto, Baer, Balazs, Belyaev, Ferrandis & Tata

JHEP 0306:023 (2003)

Tobe & Wells NPB 663, 123 (2003)

Dermisek, Raby, Roszkowski & Ruiz de Austri

JHEP 0304:037 (2003)

JHEP 0509:029 (2005)

Baer, Kraml, Sekmen & Summy

JHEP 0803:056 (2008)

JHEP 0810:079 (2008)

MSSO₁₀SMATM

$$\lambda \quad 16_3 \quad 10 \quad 16_3$$

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Fit t, b, tau requires

$$A_0 \approx -2m_{16} \quad m_{10} \approx \sqrt{2}m_{16}$$

$$m_{16} > \text{few TeV} \quad \mu, M_{1/2} \ll m_{16}$$

$$\tan \beta \approx 50$$

Radiative EWSB requires

$$\Delta m_H^2 \equiv \frac{(m_{H_d}^2 - m_{H_u}^2)}{2m_{10}^2} \approx 13\%$$

Roughly $\frac{1}{2}$ comes
From RG running from

$$M_G \rightarrow m_{\nu_\tau}$$

Blazek, Dermisek & Raby

"Just so" = "NUHM"

Inverted scalar mass hierarchy

Bagger, Feng, Polonsky & Zhang
PLB473, 264 (2000)

Third family scalars lighter than first two!

Suppresses flavor & CP violation

$$A_0 \approx -2m_{16} \quad m_{10} \approx \sqrt{2}m_{16}$$

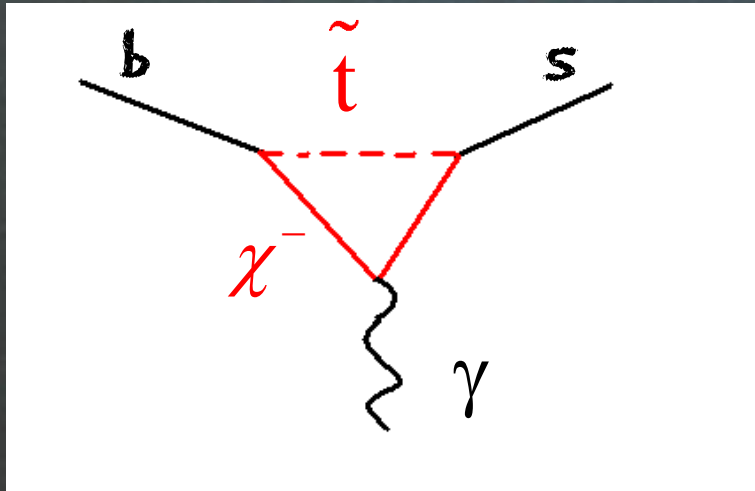
$$m_{16} > \text{few TeV} \quad \mu, M_{1/2} \ll m_{16}$$

$$\tan \beta \approx 50$$

Argue need Heavy scalars !

$$BR(B \rightarrow X_s \gamma) = (3.55 \pm 0.26) \times 10^{-4} \quad \text{Exp.}$$

$$BR(B \rightarrow X_s \gamma)_{SM} = (3.15 \pm 0.23) \times 10^{-4} \quad \text{NNLO Th.}$$



$$C_7^{eff} = C_7^{SM} + C_7^{SUSY}$$

$$C_7^{eff} \approx \pm C_7^{SM}$$

$$C_7^{\chi^+} \propto \frac{\mu A_t}{m} \tan \beta \times \text{sign}(C_7^{SM}) \approx \left\{ \begin{array}{c} -2C_7^{SM} \\ 0 \end{array} \right\}$$

$$\mu M_g > 0 \Rightarrow \mu A_t < 0$$

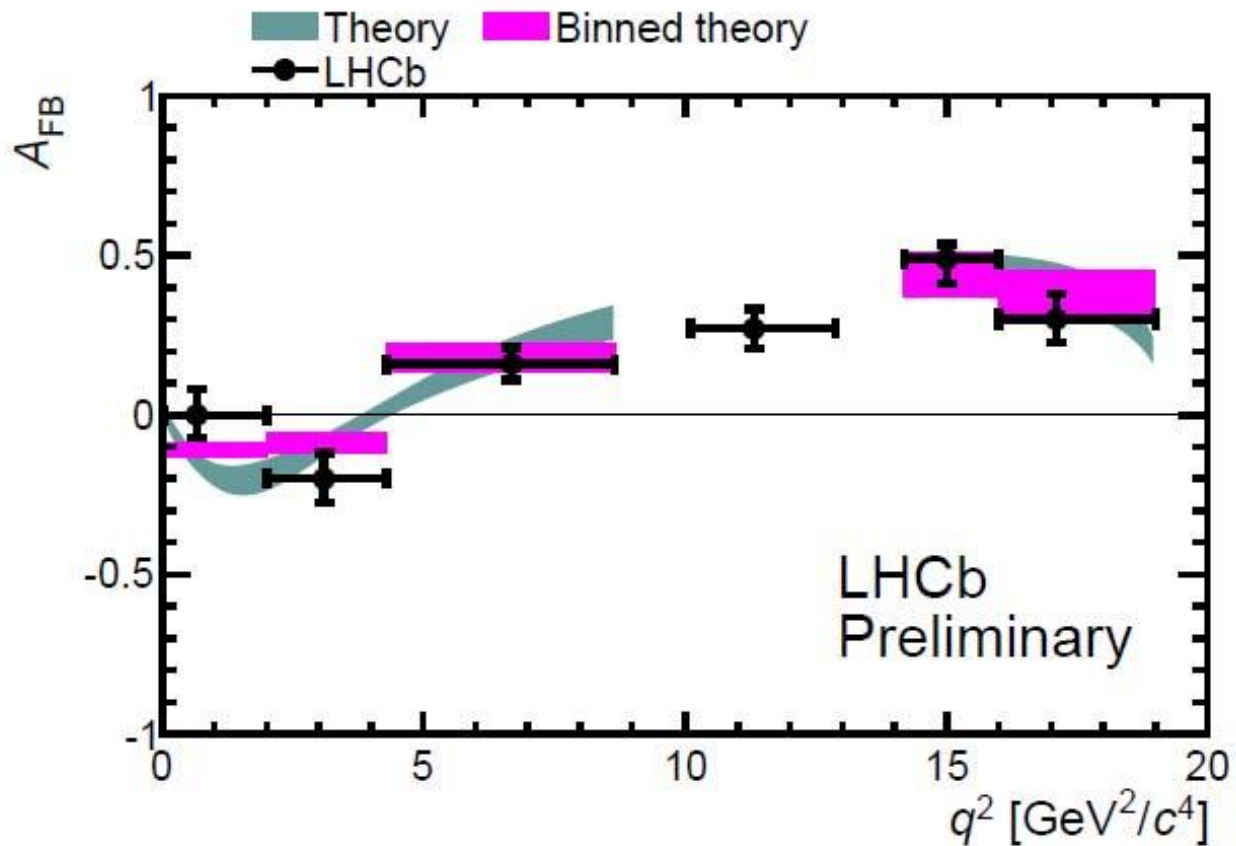
$$m_{16} \sim 4-5 \text{ TeV}$$

light squarks and sleptons !!

$$C_7^{\chi^+} \approx -2C_7^{\text{SM}} \quad \text{or}$$

$$C_7 = C_7^{\text{SM}} + C_7^{\chi^+} \approx -C_7^{\text{SM}}$$

LHCb BR(B \rightarrow K* μ^+ μ^-) favors $C_7 \approx +C_7^{\text{SM}}$



tension between $b \rightarrow s \gamma$ & $b \rightarrow s l^+ l^-$

Albrecht, Altmannshofer, Buras, Guadagnoli, & Straub
JHEP 0710:055 (2007)

$$C_7^{\chi^+} \approx 0 \quad \text{or}$$

$$C_7 = C_7^{\text{SM}} + C_7^{\chi^+} \approx +C_7^{\text{SM}}$$

$$\Rightarrow m_{16} \geq 8 \text{ TeV}$$

Light Higgs mass

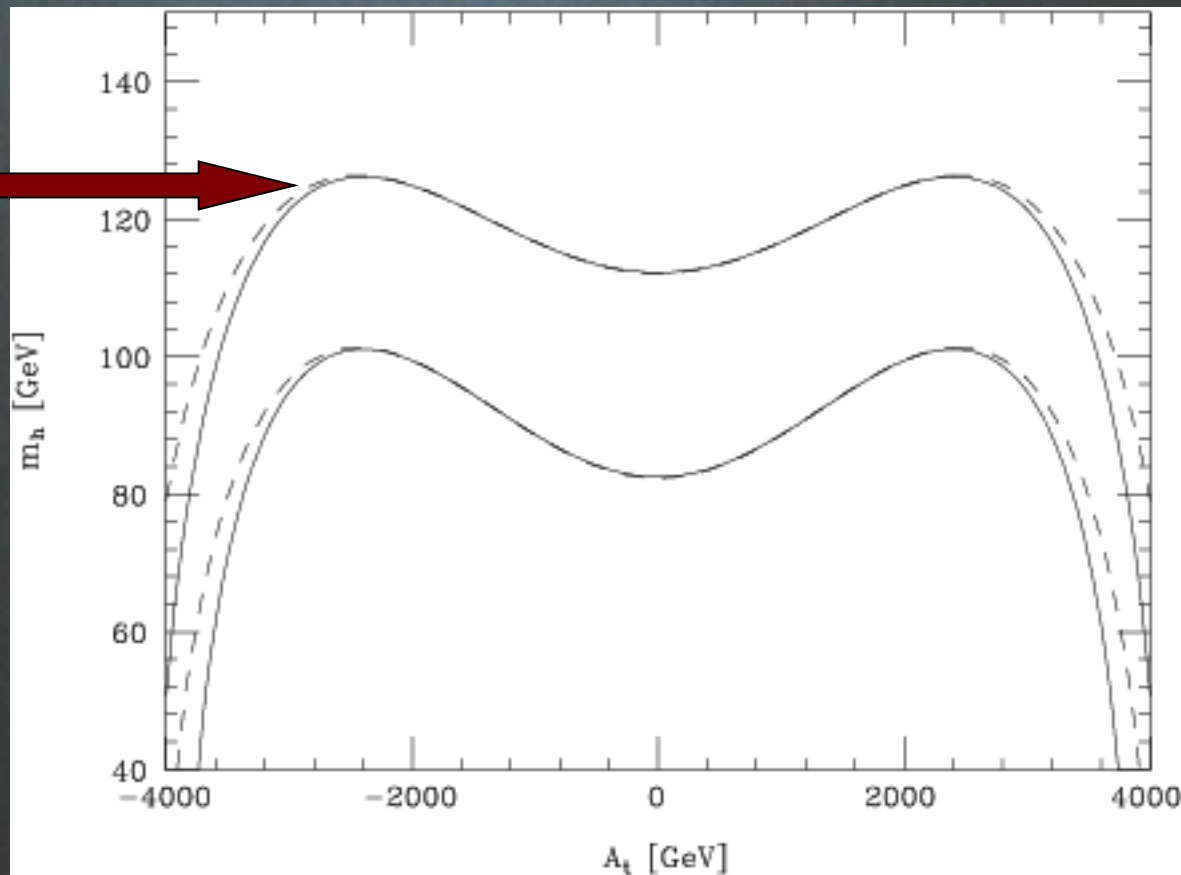
$$m_h^2 \approx M_Z^2 \cos^2 2\beta$$

$$+ \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left[\ln \left(\frac{M_{SUSY}^2}{m_t^2} \right) + \frac{X_t^2}{M_{SUSY}^2} \left(1 - \frac{X_t^2}{12M_{SUSY}^2} \right) \right]$$

$$X_t = A_t - \frac{\mu}{\tan \beta} \quad \frac{X_t}{M_{SUSY}} \sim \sqrt{6} \quad \text{Max mixing}$$

Large A_t & $M_{SUSY} \Rightarrow m_h \geq 125 \text{ GeV}$ Easy

Light Higgs mass



Carena,
Quiros &
Wagner '95

$\text{Br}(B_s \rightarrow \mu^+ \mu^-)$:
Light Higgs SM-like

SM: 3×10^{-9} MSSM: $\sim (\tan \beta)^6 / m_A^4$

CDF $1.8^{+1.8}_{-0.9} \times 10^{-8}$ (95% CL) w/ 7 fb^{-1}

LHCb bound $< 4.5 \times 10^{-9}$ (95% CL) w/ 1 fb^{-1}

$m_A \geq 1500 \text{ GeV}$

$m_A \sim m_H \sim m_{H^\pm} \Rightarrow h$ SM-like

3 Family $SO(10)$ + family symmetry

Dermisek & Raby

PLB 622:327 (2005)

Dermisek, Harada & Raby

PRD74, 035011 (2006)

Albrecht, Altmannshofer, Buras, Guadagnoli & Straub

JHEP 0710:055 (2007)

3 family SO_{10} SUSY Model

- $D_3 \times U(1)$ Family Symmetry
- Superpotential
- Yukawa couplings
- χ^2 analysis
- Charged fermion masses & mixing
- Neutrino masses & mixing

Superpotential for charged fermion Yukawa couplings

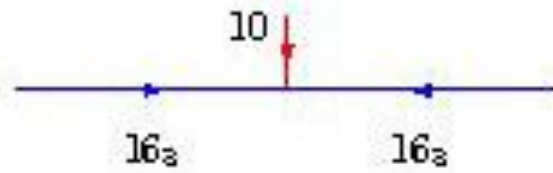
$$W_{ch.fermions} = 16_3 10 16_3 + 16_a 10 \chi_a$$

$$+ \overline{\chi}_a \left(M_\chi \chi_a + 45 \frac{\phi_a}{M} 16_3 + 45 \frac{\phi_a}{M} 16_a + A 16_a \right)$$

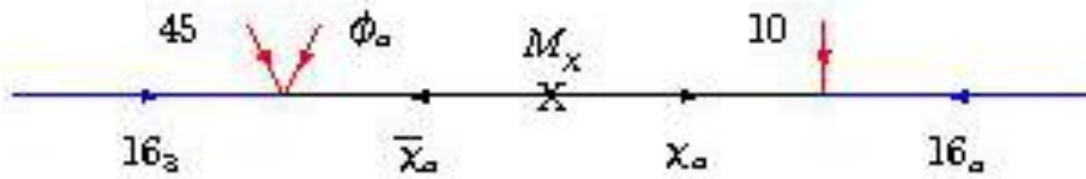
$$\langle \phi \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ \phi_2 \end{pmatrix} \quad \langle 45 \rangle = (B-L) M_G$$

Familon VEVs assumed

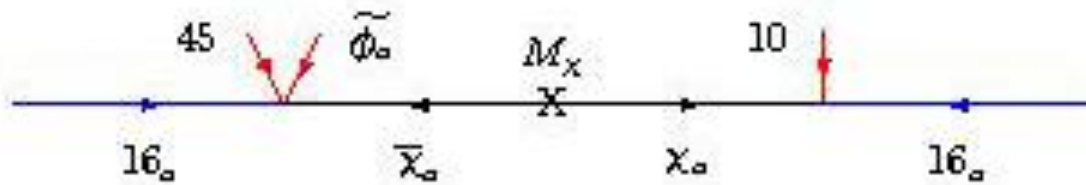
Effective higher dimension operators



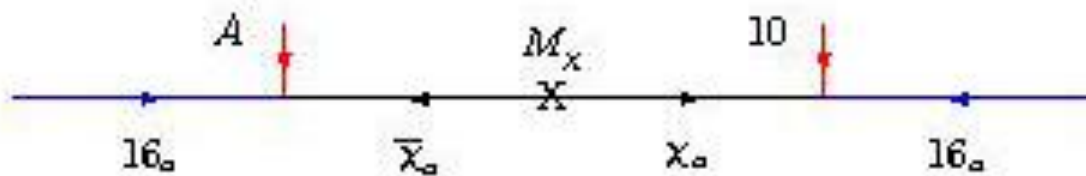
(3,3)



(3,2) (2,3)
(3,1) (1,3)



(2,2)



(1,2) (2,1)

$SO(10) \times (D_3 \times U(1)$ family sym.) Yukawa Unification for 3rd Family

7 real para's
+ 4 phases

+ 3 real Majorana
Neutrino masses

Dermisek & Raby
PLB 622:327 (2005)

$$Y_u = \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon \xi \\ -\epsilon' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon \xi \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 3 \epsilon \xi \\ \epsilon' & 3 \tilde{\epsilon} & 3 \epsilon \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \omega & \frac{3}{2} \epsilon \xi \omega \\ \epsilon' \omega & 3 \tilde{\epsilon} \omega & \frac{3}{2} \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

Georgi - Jarlskog relation

$$\begin{array}{l} \mathbf{M}_{GUT} \\ m_\mu \sim 3m_s \\ m_e \sim \frac{1}{3}m_d \end{array} \quad \rightarrow \quad \begin{array}{l} \mathbf{M}_Z \\ m_s \sim m_\mu \\ m_d \sim 9m_e \end{array}$$

Renormalization group invariant

$$200 \sim \frac{m_\mu}{m_e} = 9 \frac{m_s}{m_d}$$

Extend to neutrino sector

$$W_{\text{neutrino}} = \overline{16}(\lambda_2 N_a 16_a + \lambda_3 N_3 16_3) \\ + \frac{1}{2}(S_a N_a N_a + S_3 N_3 N_3)$$

$$\langle S_a \rangle = M_a \quad \langle S_3 \rangle = M_3 \quad \langle \overline{16} \rangle = \nu_{16}$$



Assume 3 new real para's

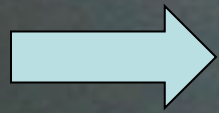
$$W_{\text{neutrino}} = \nu m_\nu \bar{\nu} + \bar{\nu} V N + \frac{1}{2} N M_N N$$

$$Y_\nu = \begin{pmatrix} 0 & -\varepsilon'\omega & \frac{3}{2}\varepsilon\xi\omega \\ \varepsilon'\omega & 3\varepsilon\omega & \frac{3}{2}\varepsilon\omega \\ -3\varepsilon\xi\sigma & -3\varepsilon\sigma & 1 \end{pmatrix} \lambda \quad \omega = \frac{2\sigma}{(2\sigma - 1)}$$

$$m_\nu = Y_\nu \frac{\nu}{\sqrt{2}} \sin \beta$$

$$V = \nu_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$M_N = \text{diag} (M_1 \quad M_2 \quad M_3)$$

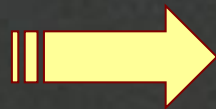


$$M_\nu = U_e^T \left(m_\nu (V^T)^{-1} M_N V^{-1} m_\nu^T \right) U_e$$

Using χ^2 analysis, fit

15 charged fermion & 5 neutrino
low energy observables with

11 arbitrary Yukawa & 3 Majorana mass
parameters



4 & 2 d. o. f. or 6 predictions

Global χ^2 analysis

Sector	#	Parameters
gauge	3	$\alpha_G, M_G, \epsilon_3,$
SUSY (GUT scale)	5	$m_{16}, M_{1/2}, A_0, m_{H_u}, m_{H_d},$
textures	11	$\epsilon, \epsilon', \lambda, \rho, \sigma, \tilde{\epsilon}, \xi,$
neutrino	3	$M_{R_1}, M_{R_2}, M_{R_3},$
SUSY (EW scale)	2	$\tan \beta, \mu$

24 parameters at GUT scale

compared to SM - 27 parameters

CMSSM - 32 parameters

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.403	80.6	0.5
M_Z	91.1876	90.7	1.1
$G_F \times 10^5$	1.16637	1.16	0.3
$1/\alpha_{em}$	137.036	136.8	0.4
$\alpha_s(M_Z)$	0.1176	0.117	0.2
M_t	170.9	170.6	0.2
$m_b(m_b)$	4.2	4.22	0.3
$m_c(m_b)$	1.25	1.14	1.2
$m_s(2 \text{ GeV})$	0.095	0.107	0.5
$m_d(2 \text{ GeV})$	0.005	0.00741	1.2
$m_u(2 \text{ GeV})$	0.00225	0.00461	3.1
M_τ	1.777	1.78	0.1
M_μ	0.10566	0.106	0.1
M_e	0.000511	0.000511	0.0
$ V_{us} $	0.2258	0.225	0.6
$ V_{ub} \times 10^3$	4.1	3.26	2.1
$ V_{cb} $	0.0416	0.0416	0.1
$\sin 2\beta$	0.675	0.639	1.4
$\Delta m_{31}^2 \times 10^{21}$	2.6	2.6	0.0
$\Delta m_{21}^2 \times 10^{23}$	7.9	7.9	0.0
$\sin^2 2\theta_{12}$	0.852	0.852	0.0
$\sin^2 2\theta_{23}$	0.996	1.0	0.2
$\epsilon_K \times 10^3$	2.229	2.33	0.4
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	3.55	2.86	1.3
$\text{BR}(B \rightarrow X_s \ell^+ \ell^-) \times 10^6$	1.6	1.62	0.0
$\Delta M_s / \Delta M_d$	35.05	31.1	1.1
$\text{BR}(B^+ \rightarrow \tau^+ \nu) \times 10^4$	1.31	0.517	1.7
total χ^2 :			27.4

Albrecht et al.

JHEP 0710:055 (2007)

m_{16}	10000
μ	1200
$\text{BR}(B_s \rightarrow \mu^+ \mu^-) \times 10^8$	2.1
\hat{s}_0	0.14
$\text{BR}(\mu \rightarrow e \gamma) \times 10^{13}$	0.0026
$\delta a_\mu^{\text{SUSY}} \times 10^{10}$	+0.52
M_{h_0}	129
M_A	842
$m_{\tilde{t}_1}$	1903
$m_{\tilde{b}_1}$	2366
$m_{\tilde{\tau}_1}$	3933
$m_{\tilde{\chi}_1^0}$	60
$m_{\tilde{\chi}_1^\pm}$	120
$m_{\tilde{g}}$	506

Required $C_7 = +C_7^{\text{SM}}$

Yukawa unification : MSSM at Large $\tan \beta$

Work in progress with

Archana Anandakrishnan

Christopher Plumberg

Using

3 family GUT code and support from

Radovan Dermisek + Aditi Raval,

Higgs code of Pietro Slavich &

MicrOMEGAS ($B \rightarrow s \gamma$, $B_s \rightarrow \mu^+ \mu^-$)

Preliminary
 $m_{16} = 10 \text{ TeV}$

Anandakrishnan,
 Plumberg,
 Raby

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.44	80.55	
M_Z	91.1876	91.1876	
$G_F \times 10^5$	1.16637	1.166	0.293
$1/\alpha_{\text{em}}$	137.036	136.987	0.359
$\alpha_s(M_Z)$	0.1184	0.1182	0.247
M_t	172.9	172.4	0.51
$m_b(m_b)$	4.19	4.49	1.69
$M_b - M_c$	3.43	3.46	0.642
$m_c(m_b)$	1.29	1.299	0.08
$m_s(2 \text{ GeV})$	0.1	0.092	0.27
$m_d(2 \text{ GeV})/m_s(2 \text{ GeV})$	0.052	0.07	2.72
$1/Q^2(2 \text{ GeV})$	0.0019	0.00189	0.35
M_τ	1.777	1.777	0.09
M_μ	0.10566	0.10566	0.07
M_e	0.000511	0.000511	0.03
$ V_{us} $	0.2252	0.2249	0.29
$ V_{ub} \times 10^3$	3.89	2.995	2.0
$ V_{cb} $	0.0406	0.0397	0.67
$\sin 2\beta$	0.673	0.615	2.5
$\Delta m_{31}^2 \times 10^{21}$	2.46	2.457	0.017
$\Delta m_{21}^2 \times 10^{23}$	7.54	7.52	0.039
$\sin^2 \theta_{12}$	0.307	0.295	0.32
$\sin^2 \theta_{23}$	0.398	0.433	0.47
$\sin^2 \theta_{13}$	0.0245	0.0132	1.7
$\epsilon_K \times 10^3$	2.228	2.43	0.92
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	3.55	2.99	1.7
$\text{BR}(B \rightarrow \mu^+ \mu^-) \times 10^9$	< 4.5	4.6	0.13
m_h	126	127.8	0.88
total χ^2 :			30.4

Preliminary
 $m_{16} = 15 \text{ TeV}$

Anandakrishnan,
 Plumberg,
 Raby

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.44	80.61	
M_Z	91.1876	91.1876	
$G_F \times 10^5$	1.16637	1.16604	0.27
$1/\alpha_{em}$	137.036	136.95	0.58
$\alpha_s(M_Z)$	0.1184	0.1191	1.13
M_t	172.9	171.5	0.34
$m_b(m_b)$	4.19	4.21	0.13
$M_b - M_c$	3.43	3.38	0.87
$m_c(m_b)$	1.29	1.11	1.58
$m_s(2 \text{ GeV})$	0.1	0.096	0.11
$m_d(2 \text{ GeV})/m_s(2 \text{ GeV})$	0.052	0.067	2.33
$1/Q^2(2 \text{ GeV})$	0.0019	0.00187	0.50
M_τ	1.777	1.776	0.07
M_μ	0.10566	0.10566	0.0
M_e	0.000511	0.000511	0.0
$ V_{us} $	0.2252	0.2249	0.25
$ V_{ub} \times 10^3$	3.89	3.05	1.90
$ V_{cb} $	0.0406	0.0400	0.45
$\sin 2\beta$	0.673	0.623	2.13
$\Delta m_{31}^2 \times 10^{21}$	2.46	2.45	0.01
$\Delta m_{21}^2 \times 10^{23}$	7.54	7.53	0.02
$\sin^2 \theta_{12}$	0.307	0.287	0.035
$\sin^2 \theta_{23}$	0.398	0.531	1.73
$\sin^2 \theta_{13}$	0.0245	0.0158	1.32
$\epsilon_K \times 10^3$	2.228	2.25	0.14
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	3.55	3.41	0.41
$\text{BR}(B \rightarrow \mu^+ \mu^-) \times 10^9$	< 4.5	1.65	0
m_h	126	123.7	1.11
total χ^2 :			25.35

Preliminary
 $m_{16} = 20 \text{ TeV}$

Anandakrishnan,
 Plumberg,
 Raby

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.44	80.7	
M_Z	91.1876	91.1876	
$G_F \times 10^5$	1.16637	1.1670	0.56
$1/\alpha_{em}$	137.036	136.108	0.52
$\alpha_s(M_Z)$	0.1184	0.1188	0.670
M_t	172.9	170.5	2.15
$m_b(m_b)$	4.19	4.27	0.49
$M_b - M_c$	3.43	3.44	0.37
$m_c(m_b)$	1.29	1.12	1.48
$m_s(2 \text{ GeV})$	0.1	0.10	0.0
$m_d(2 \text{ GeV})/m_s(2 \text{ GeV})$	0.052	0.068	2.43
$1/Q^2(2 \text{ GeV})$	0.0019	0.0019	0.0
M_τ	1.777	1.776	0.04
M_μ	0.10566	0.10565	0.006
M_e	0.000511	0.000511	0.0
$ V_{us} $	0.2252	0.2251	0.08
$ V_{ub} \times 10^3$	3.89	3.27	1.39
$ V_{cb} $	0.0406	0.0399	0.51
$\sin 2\beta$	0.673	0.663	0.39
$\Delta m_{31}^2 \times 10^{21}$	2.46	2.51	0.32
$\Delta m_{21}^2 \times 10^{23}$	7.54	7.48	0.11
$\sin^2 \theta_{12}$	0.307	0.290	0.45
$\sin^2 \theta_{23}$	0.398	0.456	0.76
$\sin^2 \theta_{13}$	0.0245	0.0055	2.87
$\epsilon_K \times 10^3$	2.228	2.45	0.99
$\text{BR}(B \rightarrow X_s \gamma) \times 10^4$	2.99	3.46	1.68
$\text{BR}(B \rightarrow \mu^+ \mu^-) \times 10^9$	< 4.5	4.5	0.03
m_h	126	125.5	1.39
total χ^2 :			29.74

m_{16}	10 TeV	15 TeV	20 TeV
A_0	-20 TeV	-30.5 TeV	-41 TeV
μ	1279	775	1169
$M_{1/2}$	243	219	249
χ^2	30	25	29
$\text{BR}(\mu \rightarrow e\gamma) \times 10^{13}$	0.00224	0.000045	0.000024
M_{h_0}	128	124	125
M_A	1741	751	2252
$m_{\tilde{t}_1}$	1811	1655	2526
$m_{\tilde{b}_1}$	2251	2739	3782
$m_{\tilde{\tau}_1}$	4043	5591	7813
$m_{\tilde{\chi}_1^0}$	129	136	162
$m_{\tilde{\chi}_1^+}$	260	280	341
$m_{\tilde{g}}$	850	883	1041

Yukawa unification & Large $\tan \beta$

Good

Gauge coupling unif.

$m_h \geq 125$ GeV & SM-like

Inverted scalar mass hierarchy

- Suppresses FCNC
- $g \rightarrow t \bar{t} \chi^0, b \bar{b} \chi^0, t \bar{b} \chi^-, \dots$

Bad

Fine-tuned 1/1000

To do list -

- Is there an upper bound on m_{16} ??
- Chi2 contours for gluino mass?
- Other UV BCs ??

Other boundary conditions at M_{GUT}

- DR3
- Non-universal gaugino masses
 - a. Gauge kinetic function
 - b. "Mirage mediation"

DR3

Baer, Kraml & Sekmen

arXiv:0908.0134

$$m_{16}^{(3)} \leq m_{16}^{(1,2)}$$

$$m_i^2 = Q_X^i D_X + (m_i^0)^2$$

	Q	U	D	L	E	N		H _U	H _D
Q_X^i	1	1	-3	-3	1	5		-2	2
m_i^0			m_{16}					m_{10}	

Non-univ. Gauginos – Gaugino kinetic function

Badziak, Olechowski & Pokorski

arXiv:0907.4709 (hep-ph)

$$\mu < 0, \quad M_3 : M_2 : M_1 = 1 : -\frac{3}{2} : -\frac{1}{2}$$

+ D -term splitting for scalars

Totally different spectrum

Non-univ gauginos - Mirage mediation BCs

$$\mu < 0, \quad M_i = \left(1 + \frac{\log \left(\frac{M_{Pl}}{m_{3/2}} \right)}{4 \pi} \alpha_{GUT} b_i \alpha \right) M_0$$

+ D -term splitting for scalars

Extra slide

