Quantum and Medium Effects in Resonant Leptogenesis

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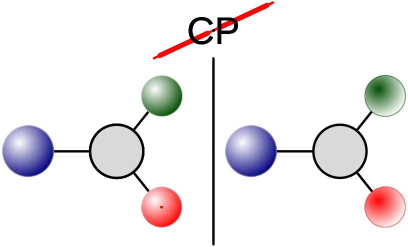
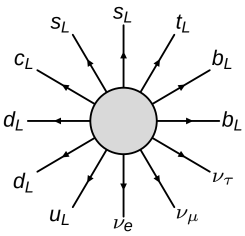
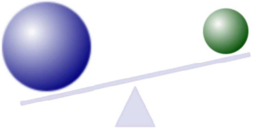
Leptogenesis





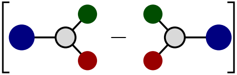
$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\bar{N}_i \left(i\partial\!\!\!/ - M_i\right) N_i$$
$$- h_{\alpha i}\bar{\ell}_\alpha \tilde{\phi} P_R N_i - h^\dagger_{i\alpha}\bar{N}_i \tilde{\phi}^\dagger P_L \ell_\alpha$$



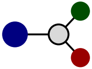
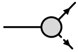
- kinetic equations needed, usually generalized Boltzmann equations (BEs)

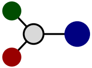
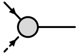
$$k^\mu \mathcal{D}_\mu f^{\ell}(X,k) = \sum_{\text{interactions of } \ell} C_k^{\ell+\cdot\leftrightarrow i+\cdot}[.f_{\ell}.](X,k)$$

- $2 \leftrightarrow 1$ collision term

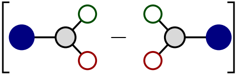
$$C^{\ell\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\phi}\, d\Pi_q^{N_i} (2\pi)^4 \delta(k+p-q)\Big[$$  $$\Big]$$

where

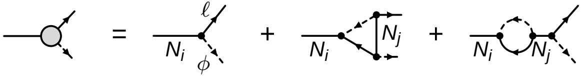
 $=\Big|$  $\Big|^2 (1 \pm f_k^{\ell})(1 \pm f_p^{\phi}) f_q^{N_i}$

 $=\Big|$  $\Big|^2 f_k^{\ell} f_p^{\phi} (1 \pm f_q^{N_i})$

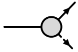
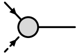
- $2 \leftrightarrow 1$ collision term antiparticles

$$C^{\bar\ell\bar\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\bar\phi}\, d\Pi_q^{N_i} (2\pi)^4 \delta(k+p-q)\Big[$$  $$\Big]$$

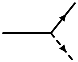
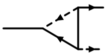
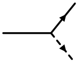
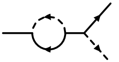
- one-loop vertex and self-energy contribution



- parametrize matrix elements by CP-violating parameter

 $\Big|^2 = \frac{1}{2}(1+\epsilon_i)\,|\mathcal{M}|^2_{N_i}$,  $\Big|^2 = \frac{1}{2}(1-\epsilon_i)\,|\mathcal{M}|^2_{N_i}$

$$\epsilon_i = \frac{|\mathcal{M}|^2_{N_i\to\ell\phi} - |\mathcal{M}|^2_{N_i\to\bar{\ell}\bar{\phi}}}{|\mathcal{M}|^2_{N_i\to\ell\phi} + |\mathcal{M}|^2_{N_i\to\bar{\ell}\bar{\phi}}}$$

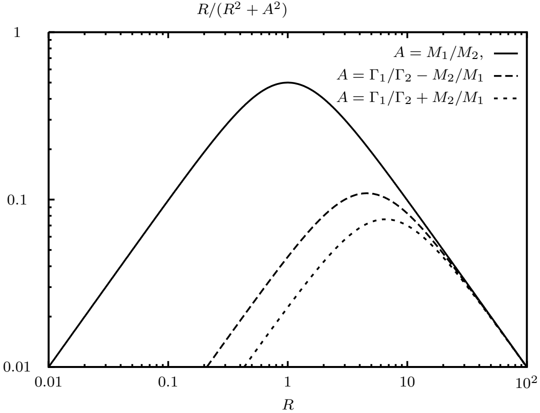
$\epsilon_i^{vac} = 2\frac{\mathrm{Im}\{(h^\dagger h)^2_{ij}\}}{(h^\dagger h)_{ii} M_i}\Big[\mathrm{Im}\Big\{$  $^* \times$  $\Big\} + 2\mathrm{Im}\Big\{$  $^* \times$  $\Big\}$

- self-energy contrib. completed by finite width (Paschos et. al., Pilaftsis et. al., Plümacher et. al., Covi et. al. . . .)

$$\epsilon_i^{vac} = -\frac{\mathrm{Im}\{(h^\dagger h)^2_{ij}\}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}}\frac{R}{R^2+A^2}\,, \quad \text{with } R \equiv \frac{M_j^2 - M_i^2}{M_i\Gamma_j}$$

Resonant Leptogenesis

- self-energy contribution is resonantly enhanced



- resonance conditions: $\epsilon_i^{vac} \sim 1$ if $\frac{\mathrm{Im}\left\{(h^\dagger h)_{ij}^2\right\}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \sim 1$ and $|R| \simeq A$
- require $|M_j - M_i| \sim \Gamma_{i,j}$

Nonequilibrium Quantum Field Theory

- repeat nonequilibrium quantum field analysis of resonant "leptogenesis" within toy-model 0911.4122 for SM+3ν_R

$$\left(\psi_i \to b\,b,\ \psi_i \to \bar{b}\,\bar{b}\right) \Longrightarrow \left(N_i \to \ell\,\phi,\ N_i \to \bar{\ell}\,\bar{\phi}\right)$$

- see also: Buchmüller, Anisimov, Drewes, Mendizabal and Garbrecht, Herannen, Schwaller, et. al.
- divergence of lepton current (flavour independent)

$$\partial_\mu j_L^\mu(x) = \frac{dn_L}{dt} + 3\,Hn_L$$

$$j_L^\mu(x) = \Big\langle \sum_{\alpha,a} \bar{\ell}^a_\alpha(x)\gamma^\mu \ell^a_\alpha(x)\Big\rangle = -\sum_{\alpha,a} \mathrm{Tr}\left[\gamma_\mu S_{\ell\,aa}^{\,\alpha\alpha}(x,x)\right]$$

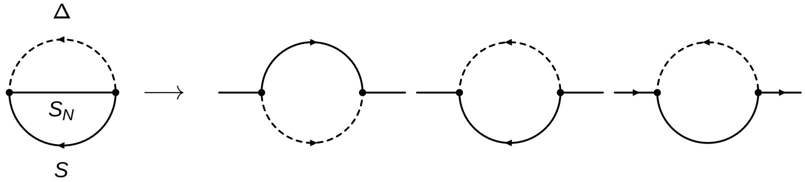
- Kadanoff-Baym equations

$$i\partial\!\!\!/_x S_{\ell_F}^{\alpha\beta}(x,y) = \int_0^{x^0} d^4z\, \Sigma_{\ell_\rho}^{\alpha\gamma}(x,z) S_{\ell_F}^{\gamma\beta}(z,y) - \int_0^{y^0} d^4z\, \Sigma_{\ell_F}^{\alpha\gamma}(x,z) S_{\ell_\rho}^{\gamma\beta}(z,y)$$

$$i\partial\!\!\!/_x S_{\ell_\rho}^{\alpha\beta}(x,y) = \int_{y^0}^{x^0} d^4z\, \Sigma_{\ell_\rho}^{\alpha\gamma}(x,z) S_{\ell_\rho}^{\gamma\beta}(z,y)$$

Nonequilibrium Quantum Field Theory

- 1-loop self-energy



- gradient expansion, Wigner transformation ...

$$\partial_\mu j_L^\mu(t) = 2\int_0^\infty \frac{dk^0}{2\pi}\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}\Big\{\big[\Sigma_{\ell<}^{\alpha\beta}(t,\not{k})S_{\ell>}^{\beta\alpha}(t,\not{k}) - \Sigma_{\ell>}^{\alpha\beta}(t,\not{k})S_{\ell<}^{\beta\alpha}(t,\not{k})\big]$$
$$-\big[\bar{\Sigma}_{\ell<}^{\beta\alpha}(t,\not{k})\bar{S}_{\ell>}^{\alpha\beta}(t,\not{k}) - \bar{\Sigma}_{\ell>}^{\beta\alpha}(t,\not{k})\bar{S}_{\ell<}^{\alpha\beta}(t,\not{k})\big]\Big\}$$

- compare with conventional Boltzmann equations

$$\partial_\mu j_L^\mu(x) = \int\frac{d^3k}{(2\pi)^3 E_k^{\ell}}\Big[C^{\ell\phi\leftrightarrow N_i}(k) - C^{\bar{\ell}\bar{\phi}\leftrightarrow N_i}(k)\Big]$$

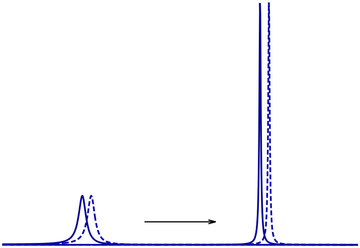
Nonequilibrium Quantum Field Theory

- Kadanoff-Baym ansatz and quasi-particle approximation for leptons and Higgs (lepton flavour-diagonal)

$$S_{\ell\,\rho}^{\alpha\beta}(t,\not{k}) = \delta^{\alpha\beta}\, P_L \not{k} P_R\, (2\pi)\,\mathrm{sign}(k^0)\,\delta(k^2)$$

$$\Delta_{\phi\,\rho}(t,p) = (2\pi)\,\mathrm{sign}(p^0)\,\delta(p^2)$$

- quasi-particle approximation insufficient for Majorana neutrinos



- overlap due to finite width neglected
- diagonal approximation neglects cross-correlations

$$S_{N\,\rho}^{ij}(t,q) = \delta^{ij}\,(\not{q} + M_i)\,(2\pi)\,\mathrm{sign}(q^0)\,\delta(q^2 - M_i^2)$$

- extended QP-approximation required

Nonequilibrium Quantum Field Theory

- equilibrium solution for $\hat{S}$ with off-diagonal components
- elegant analytic solution with off-shell dynamics possible (neglecting back-reaction and expansion) 1112.6428
- rewrite full $\hat{S}$ in terms of diagonal $\hat{\mathcal{S}}$ and off-diagonal $\hat{\Sigma}'$

$$\hat{S}^{-1}(x,y) = \hat{\mathcal{S}}^{-1}(x,y) - \hat{\Sigma}'(x,y)$$

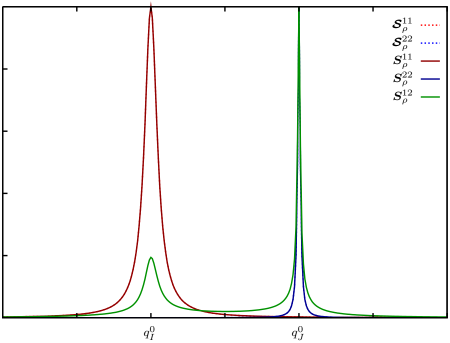
- solve for $\hat{S}$

$$\hat{S}_{F(\rho)} = \hat{\Theta}_R \left[\hat{\mathcal{S}}_{F(\rho)} - \hat{\mathcal{S}}_R \hat{\Sigma}'_{F(\rho)} \hat{\mathcal{S}}_A\right] \hat{\Theta}_A$$

where $\hat{\Theta}_A = \left(\mathbf{I} + \hat{\Sigma}'_A \hat{\mathcal{S}}_A\right)^{-1}$ and $\hat{\Theta}_R = \left(\mathbf{I} + \hat{\mathcal{S}}_R \hat{\Sigma}'_R\right)^{-1}$

- insert $\hat{S}$ in self-energy
- analyse the pole structure of $\hat{S}$
- (extended) quasi-particle approximation for diagonal propagator

- spectral functions for $R = 100$, $T = 0.1\,M_1$, M_1



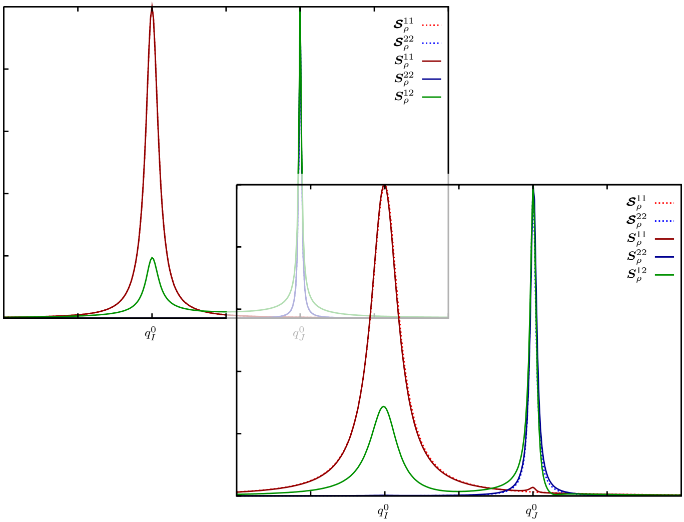
- consider 2RHN, $(M_1,\, M_2 \ll M_3)$

$$M_1 = 1 \cdot 10^3\,\mathrm{GeV}$$

$$h^\dagger h = \begin{pmatrix} (h^\dagger h)_{11} & (h^\dagger h)_{12} \\ (h^\dagger h)_{21} & (h^\dagger h)_{22} \end{pmatrix} = 10^{-12} \begin{pmatrix} 1.7 & 0.14 + 0.2i \\ 0.14 - 0.2i & 0.33 \end{pmatrix}$$

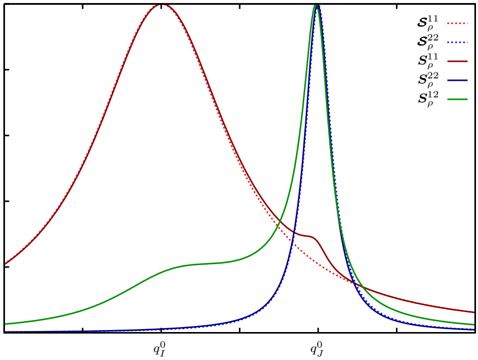
$M_2 > M_1,\ (h^\dagger h)_{11} > (h^\dagger h)_{22}$

- spectral functions for $R = 100$, $T = 0.1\,M_1$, M_1

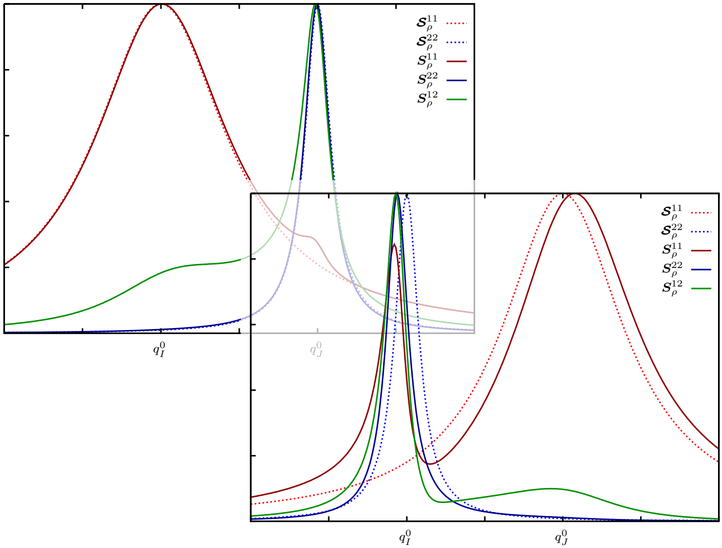


Nonequilibrium Quantum Field Theory

- spectral functions for $R = 10$, $T = 0.1\, M_1$, M_1



- spectral functions for $R = 10$, $T = 0.1\,M_1$, M_1



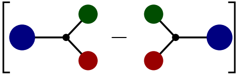
- N_1, N_2 undergo level-crossing

Nonequilibrium Quantum Field Theory

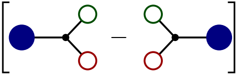
- quantum-corrected rate equation

$$\partial_\mu j^\mu_L =$$

$$\sum_i \int d\Pi_k^{\ell}\, d\Pi_p^{\phi}\, d\Pi_q^{N_i} (2\pi)^4 \delta(k+p-q) \Big\{ |\mathcal{M}|^2_{N_i \to \ell\phi}$$



$$- |\mathcal{M}|^2_{N_i \to \bar{\ell}\bar{\phi}}$$



$$\Big\}$$

- corrected *CP*-violating parameter in $R \gg 1$ limit

$$\epsilon_i = \frac{\mathrm{Im}\{(h^\dagger h)^2_{ij}\}}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j)^2 + (\Gamma_j / M_j\, q \cdot L_\rho)^2} \frac{k \cdot L_\rho}{k \cdot q}$$

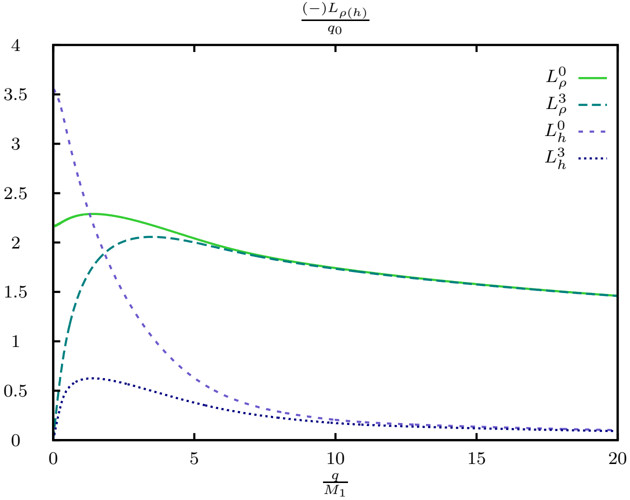
- medium effects

$$L_\rho(t, \not{q}) = 16\pi \int d\Pi_k^{\ell}\, d\Pi_p^{\phi} (2\pi)^4 \delta(q - p - k) \not{k} (1 + f_{\phi} - f_{\ell})$$

- retrieve conventional amplitudes in $R \to \infty$, $T \to 0$ limit

- $L_{h(\rho)}$ capture medium effects (two components for each sufficient)

$$m_\ell = m_\phi = 0, T = M_1$$



Rate Equations

- rate equations

$$\frac{dY_{N_i}}{dz} = -\kappa_i\, z\, \frac{\langle\gamma^D_{N_i}\rangle}{2\Gamma_i^{vac} n^{eq}_{N_i}} \left(Y_{N_i} - Y^{eq}_{N_i}\right)$$

$$\frac{dY_L}{dz} = \sum_i \kappa_i\, z \left[\frac{\left\langle\epsilon_i\gamma^D_{N_i}\right\rangle}{2\Gamma_i^{vac} n^{eq}_{N_i}} \left(Y_{N_i} - Y^{eq}_{N_i}\right) - (1 + c_{\phi\ell})c_\ell \frac{\langle\gamma^W_{N_i}\rangle}{2\Gamma_i^{vac} 2n^{eq}_\ell} Y_L\right]$$

- medium corrected reaction densities (compare Garbrecht et. al., Wong et. al., Hannestad et. al., Pastor et. al.)

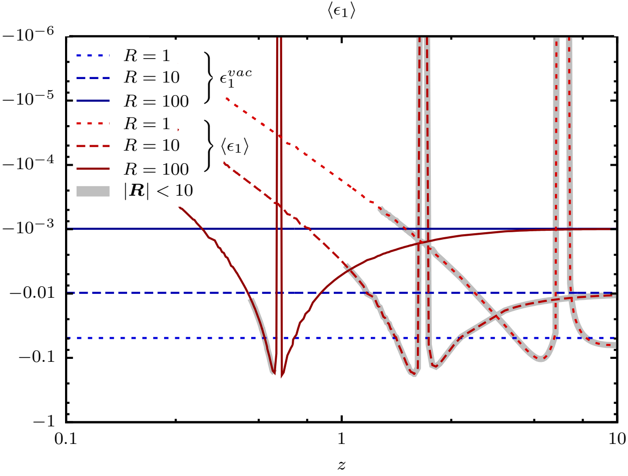
$$\langle\gamma^D_{N_i}\rangle \equiv \int d\Pi_q^{N_i} d\Pi_k^{\ell} d\Pi_p^{\phi} (2\pi)^4 \delta(q - k - k)$$
$$\times |\mathcal{M}|^2_{N_i} (1 + f_\phi - f_\ell) f_{N_i}$$

$$\left\langle\epsilon_i\gamma^D_{N_i}\right\rangle \equiv \int d\Pi_q^{N_i} d\Pi_k^{\ell} d\Pi_p^{\phi} (2\pi)^4 \delta(q - k - k)$$
$$\times \epsilon_i\, |\mathcal{M}|^2_{N_i} (1 + f_\phi - f_\ell) f_{N_i}$$

$$\langle\gamma^W_{N_i}\rangle \equiv \int d\Pi_q^{N_i} d\Pi_k^{\ell} d\Pi_p^{\phi} (2\pi)^4\, \delta(q - p - k)$$
$$\times |\mathcal{M}|^2_{N_i} (1 + f_\phi - f_\ell)(1 - f_{N_i}) f_{N_i}$$

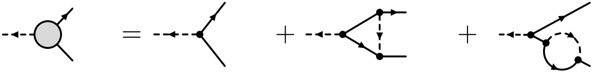
Preliminary Results

- averaged *CP*-violating parameter



Thermal Masses and Modified Dispersion Relations

- $L_{h(\rho)}$ depend on the kinematics in the decay
- thermal masses of leptons and Higgs $m_{\ell} \sim 0.2T$, $m_{\phi} \sim 0.4T$,
  $\phi \rightarrow \ell + N_i$ for $m_{\phi} > m_{\ell} + M_i$



ϵ_i for Higgs $(1 + f_{\phi} - f_{\ell}) \longrightarrow (f_{\phi} + f_{\ell})$

- leptons have modified dispersion relations (Weldon; Giudice et.al.; Plümacher, Kießig)

$$d\Pi_q^{\ell} = \int \frac{d^3\mathbf{k}\, dk_0}{(2\pi)^3} \delta(k^2 - m_{\ell}^2) \longrightarrow \int \frac{d^3\mathbf{k}\, dk_0}{(2\pi)^3} \delta(\delta_{\ell}(k))$$

- collinear enhancement can enable Majorana decay with enhanced rate at high T (Bödecker et. al., Laine et. al.)

Conclusions

- Nonequilibrium quantum field theory can put Leptogenesis on solid ground
- improved rate equations for quasi-degenerate case
- quantitative corrections can be significant
- need quantum analysis with off-shell dynamics in degenerate case
- need to include further medium effects