General four-zero texture mass matrix parametrizations in the neutrino and quark sector

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Outline

- Fritszch's matrices
- Parametrizations of mass matrices
- χ^2 analysis and some results.
- Conclusions

Fritszch's matrices

- Make able to generate strong hierarchy of quark masses with small flavor mixing angles
- Suitable to accommodate the neutrino oscillation data representing large mixing angles quite well
- Compatible with specific models of GUTs i.e. SO(10)
- Fritzsch-like texture 4 zero neutrino are seesaw invariant

Fritszch's matrices

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Some interesting issues

- Does exist a parallel structure between strongly hierarchical fermions and weakly hierarchical fermions?
- Can *weakly hierarchical* quark mass matrices are able to reproduce the mixing data which involves *strongly hierarchical* parameters?
- Does we have something new using non-hermitian mass matrices?

Mass matrix diagonalization

- Diagonal mass matrix

$$\tilde{M}_f \equiv \text{diag}(m_1^f, \ldots, m_n^f) = U_L^\dagger M_f U_R \quad \text{for} \quad f = u, d, \ell, \nu$$

- The hermitian squared mass matrix

$$\begin{aligned} H_f &\equiv M_f M_f^\dagger = U_L^f \tilde{M}_f^2 U_L^{f\dagger} \quad \text{(Unitary transformation)} \\ I_f &\equiv M_f^\dagger M_f = U_R^f \tilde{M}_f^2 U_R^{f\dagger} \quad (U_R^f \text{ can not be observed}) \end{aligned}$$

- Diagonalizing H_f gives a mixing matrix of the form $P_f^\dagger \mathcal{O}_f$
- Free parameters in the general case of n generations

$$N = 2n^2 - \underbrace{n^2}_{\text{mixing}} - \underbrace{n}_{\text{masses}} - \underbrace{n}_{\text{rephasing}} - \underbrace{X}_{\text{additional assumptions}}$$

The non-hermitian 4 zero mass matrix

- Fritszch matrix

$$M_f = \begin{pmatrix} 0 & C_f & 0 \\ C'_f & D_f & B_f \\ 0 & B'_f & A_f \end{pmatrix} \quad \text{with} \quad \begin{array}{lcl} |C_f| = |C'_f| & ; & \text{Arg}(C_f) \neq \text{Arg}(C'_f) \\ |B_f| = |B'_f| & ; & \text{Arg}(B_f) \neq \text{Arg}(B'_f) \end{array}$$

- Hierarchy

$$\begin{array}{ccc} |C_f| << |B_f| \quad ; \quad |D_f| << |A_f| & \text{Strong hierarchy} \\ |C_f| \lesssim |B_f| \quad ; \quad |D_f| \lesssim |A_f| & \text{Weak hierarchy} \end{array}$$

- Removing off-diagonal phases.

$$H_f = M_f M_f^\dagger = P_f^\dagger \tilde{H}_f P_f,$$

with $P_f = e^{-\frac{i}{2}\Xi}\text{diag}\left(e^{\frac{i}{2}\Xi}, e^{i(\phi_C - \phi_D)}, e^{i(\phi_C + \phi_{B'} + \Xi)}\right)$ and

$$\Xi = \arctan\left[\frac{a\sin(\phi_B + \phi_{B'} - \phi_A - \phi_D)}{d + a\cos(\phi_B + \phi_{B'} - \phi_A - \phi_D)}\right]$$

The mixing matrix

- Orthogonal matrix (as in the hermitian case)

$$\tilde{H} = \begin{pmatrix} c^2 & cd & bc \\ cd & c^2 + d^2 + b^2 & b|d + a\delta| \\ bc & b|d + a\delta^*| & a^2 + b^2 \end{pmatrix} \quad \text{with} \quad \delta = e^{i(\phi_D - \phi_{B'} - \phi_B + \phi_A)}$$

$$D_f = \text{diag}\left(m_1^2, m_2^2, m_3^2\right) = \mathcal{O}_f \tilde{H}_f \mathcal{O}_f^{-1}$$

- Mixing matrices

$$V_{\text{CKM}} = \mathcal{O}_d^{-1} P_d P_u^\dagger \mathcal{O}_u \quad ; \quad V_{\text{PMNS}} = \mathcal{O}_\ell^{-1} P_\ell P_\nu^\dagger \mathcal{O}_\nu$$

- Additional phases

$$P_f P_{f'}^\dagger = e^{-\frac{i}{2}(\Delta\Xi)} \text{diag}\left(1, e^{i\Phi_1}, e^{i\Phi_2} e^{i\Delta\Xi}\right)$$

thus $\Delta\Xi = 0$ is a good assumption, but not the only one.

Constructing parametrizations

- Invariants give 32 solutions, not all independent.

$$\begin{aligned}\mathsf{Tr}(\tilde{H}_f) &= \lambda_1^2+\lambda_2^2+\lambda_3^2 \\ \mathsf{Tr}^2(\tilde{H}_f)-\mathsf{Tr}(\tilde{H^2}_f) &= 2\lambda_1^2\lambda_2^2+2\lambda_1^2\lambda_3^2+2\lambda_2^2\lambda_3^2 \\ \mathsf{Det}(\tilde{H}_f) &= \lambda_1^2\lambda_2^2\lambda_3^2.\end{aligned}$$

- Chiral transformations makes able to choose $\lambda_i > 0$ ($|\lambda_i| = m_i$)
- Every solution define different parameters spaces.
- Masses, hierarchy and the value of δ define which one is phenomenologically compatible.

The orthogonal matrix

$$\mathcal{O}^f_{ij} = v_i((m^f_j)^2, a_f, \delta_f)$$

$$\begin{aligned}
v_1(m_i^2, a, \delta) &= (b^2 + c^2 - cd + d^2 - m_i^2)(a^2 - b|d + a\delta| + b^2 - m_i^2) \\
&\quad -b(c - |d + a\delta|)(-b|d + a\delta| + b^2 + c^2 + d^2 - m_i^2) \\
v_2(m_i^2, a, \delta) &= -m_i^2(a^2 + b^2 + c^2 - cd) + c(a^2 + bd)(c - d) \\
&\quad +b\left[c(b - c) + m_1^2\right]|d + a\delta| + m_i^4 \\
v_3(m_i^2, a, \delta) &= \left[c(d - c) - m_i^2\right](b^2 + c^2 + d^2 - b|d + a\delta| - m_i^2) \\
&\quad +c(b - d)(b^2 + c^2 - cd + d^2 - m_i^2)
\end{aligned}$$

- We have only one additional parameter with equations difficult to solve.
- If we assume $\Xi = 0$ we get $\delta = \pm 1$, that means $\phi_D - \phi_{B'} - \phi_B + \phi_A = m\pi$

Parametrizations

With $\delta = 1$ solutions of the invariants equations are

$$\tilde{m}_1 \leq a' \leq \tilde{m}_2 \quad \begin{cases} b' &= \sqrt{\frac{(a'-\tilde{m}_1)(-a'+\tilde{m}_2)(\tilde{m}_3+a')}{a'}} \\ d' &= -a' + \tilde{m}_1 + \tilde{m}_2 - \tilde{m}_3 \end{cases}$$

$$\tilde{m}_1 \leq a' \leq \tilde{m}_3 \quad \begin{cases} b' &= \sqrt{\frac{(a'-\tilde{m}_1)(a'+\tilde{m}_2)(\tilde{m}_3-a')}{a'}} \\ d' &= -a' + \tilde{m}_1 - \tilde{m}_2 + \tilde{m}_3 \end{cases}$$

$$\tilde{m}_2 \leq a' \leq \tilde{m}_3 \quad \begin{cases} b' &= \sqrt{\frac{(a'+\tilde{m}_1)(a'-\tilde{m}_2)(\tilde{m}_3-a')}{a'}} \\ d' &= -a' - \tilde{m}_1 + \tilde{m}_2 + \tilde{m}_3 \end{cases}$$

Here $c' = \sqrt{\frac{\tilde{m}_1 \tilde{m}_2 \tilde{m}_3}{a'}}$ with $X' = \frac{X}{m_3^f}$

- $a'(x_f) = \tilde{m}_{\text{max}} \left(1 - x_f \frac{\tilde{m}_{\text{max}} - \tilde{m}_{\text{min}}}{\tilde{m}_{\text{max}}}\right)$
- There are 3 independent forms to diagonalize H_f
- There are 9 possibilities to parametrize the mixing matrix.

χ^2 experimental inputs

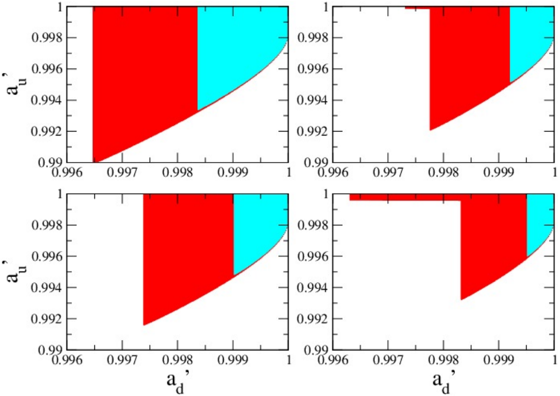
- For quarks
  - Masses (at the scale of Z)
  - Mixing angles (V_{CKM})
- For leptons
  - Limits on the mass difference of neutrinos
  - The angle θ_{13}

$$V_{\text{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}.$$

$$U_{\text{PMNS}} = \begin{pmatrix} 0.824^{+0.011}_{-0.010} & 0.547^{+0.016}_{-0.014} & 0.145^{+0.022}_{-0.031} \\ 0.500^{+0.027}_{-0.021} & 0.582^{+0.050}_{-0.023} & 0.641^{+0.061}_{-0.023} \\ 0.267^{+0.044}_{-0.027} & 0.601^{+0.048}_{-0.022} & 0.754^{+0.052}_{-0.020} \end{pmatrix}$$

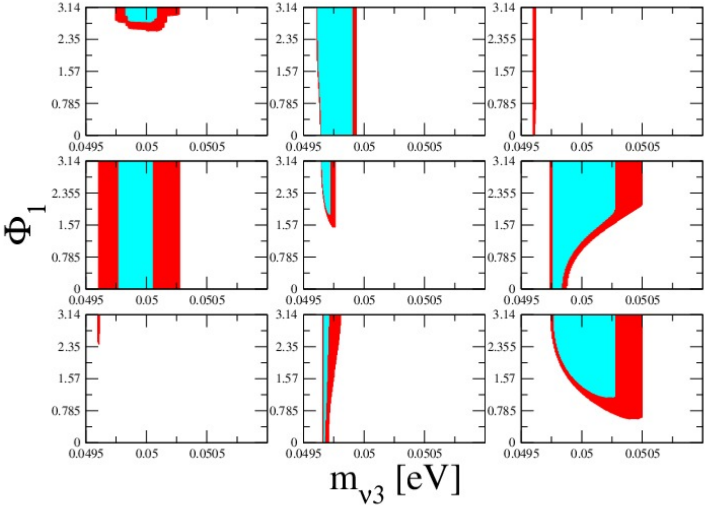
Four parametrizations for quarks sector

$$\chi^2_{\text{quarks}}(a_u, a_d, \phi) = \sum_{i=1}^{9} \left(\frac{V^{th}_{\text{CKM}}(a_u, a_d, \phi) - |V_{\text{CKM}}|}{\delta V_{\text{CKM}}} \right)^2 + \chi^2_{\text{angles}} \, .$$



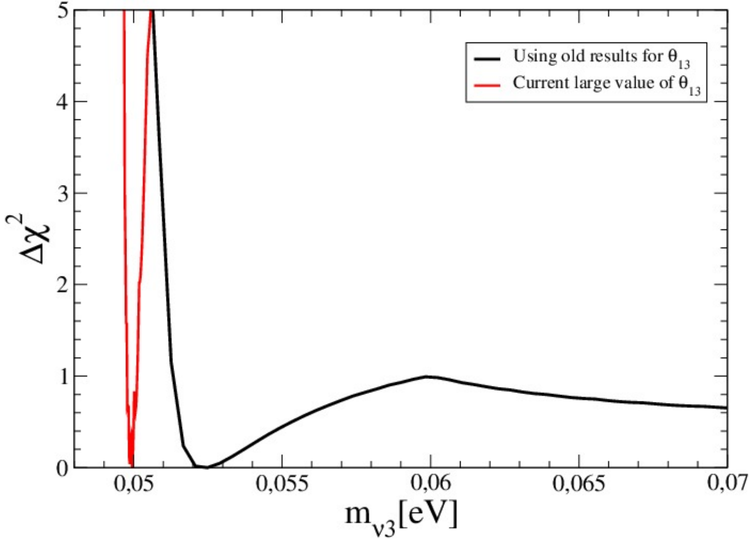
Parametrization	1	2	3
1	$\chi^2_{min} = 930.02$	$\chi^2_{min} = 3.55 \times 10^5$	$\chi^2_{min} = 3.55 \times 10^5$
2	$\chi^2_{min} = 1.43 \times 10^5$	$\chi^2_{min} = 1.270$	$\chi^2_{min} = 2.212$
3	$\chi^2_{min} = 4.50 \times 10^5$	$\chi^2_{min} = 1.44$	$\chi^2_{min} = 2.163$

Nine parametrizations for lepton sector



Parametrization	1	2	3
1	$\chi^2_{min} = 4.758$	$\chi^2_{min} = 0.009$	$\chi^2_{min} = 11.02$
2	$\chi^2_{min} = 4.798$	$\chi^2_{min} = 0.006$	$\chi^2_{min} = 3.119$
3	$\chi^2_{min} = 7.087$	$\chi^2_{min} = 0.456$	$\chi^2_{min} = 0.006$

Something about θ_{13}



Parallel parametrization (1,1)

Conclusions

- Some class of non-hermitian mass matrices can be analysed with the same number of free parameters
- Four zero mass matrix can deal with strong and weak hierarchy with the same mechanism
- With this analysis we found that the mass of the third neutrino is $m_{\nu_3} \sim 0.05\text{eV}$.
- Predictions is highly sensible to the experimental measurements