The S_3 flavour symmetry and the reactor neutino mixing angle

arXiv:1205.4755

Félix González Canales
in colaboration with: A. Mondragón
Instituto de Física-UNAM
Facultad de Ciencias de la Electrónica-BUAP
México

PASCOS 2012

Experimental data of neutrino oscillations at 1 σ^1:

* The neutrino squared mass differences:

$$\Delta m_{21}^2 = 7.62 \pm 0.19 \times 10^{-5}\ \mathrm{eV}^2, \qquad \Delta m_{31}^2 = \begin{cases} -2.40^{+0.10}_{-0.07} \times 10^{-3}\ \mathrm{eV}^2, \\ +2.53^{+0.08}_{-0.10} \times 10^{-3}\ \mathrm{eV}^2. \end{cases}$$

* The solar and atmospheric mixing angles:

$$\sin^2\theta_{12}^l = 0.320^{+0.015}_{-0.017}, \quad \sin^2\theta_{23}^l = \begin{cases} 0.53^{+0.05}_{-0.07} \\ 0.49^{+0.08}_{-0.05} \end{cases},$$

- The reactor mixing angle

$$\sin^2\theta_{13}^l = \begin{cases} 0.027^{+0.003}_{-0.004} \\ 0.026^{+0.003}_{-0.004} \end{cases},$$

the upper (lower) row corresponds to inverted (normal) neutrino mass hierarchy.

[1] Pilar Coloma *et al* arXiv:1206.0475 [hep-ph]

Implications of neutrino oscillations

- The simple fact that neutrinos oscillate is a signal of physics beyond Standard Model.
  - Neutirnos massive.
  - Flavor mixing in leptonic sector.

Implications of neutrino oscillations

- The simple fact that neutrinos oscillate is a signal of physics beyond Standard Model.
  - Neutirnos massive.
  - Flavor mixing in leptonic sector.

But... What so beyond?

Implications of neutrino oscillations

- The simple fact that neutrinos oscillate is a signal of physics beyond Standard Model.
  - Neutirnos massive.
  - Flavor mixing in leptonic sector.

But... What so beyond?

"From a philosophical point of view we apply Ockham's Razor"

The group S_3 of permutations of three objects like flavor symmetry.

Implications of neutrino oscillations

- The simple fact that neutrinos oscillate is a signal of physics beyond Standard Model.
  - Neutirnos massive.
  - Flavor mixing in leptonic sector.

But... What so beyond?

"From a philosophical point of view we apply Ockham's Razor"

The group S_3 of permutations of three objects like flavor symmetry.

- Impose the symmetry directly in the mass matrix and break sequentially according to the chain $S_3 \supset S_3^{\rm diag} \supset S_2^{\rm diag}$.

  [J. Barranco, FGC and A Mondragón PRD 82 073010]

Implications of neutrino oscillations

- The simple fact that neutrinos oscillate is a signal of physics beyond Standard Model.
  - Neutirnos massive.
  - Flavor mixing in leptonic sector.

But... What so beyond?

"From a philosophical point of view we apply Ockham's Razor"

The group S_3 of permutations of three objects like flavor symmetry.

- Impose the symmetry directly in the mass matrix and break sequentially according to the chain $S_3 \supset S_3^{\rm diag} \supset S_2^{\rm diag}$.

  [J. Barranco, FGC and A Mondragón PRD 82 073010]

- The S_3 symmetry is left unbroken, and the concept of flavour is extended to the Higgs sector by introducing in the theory three Higgs fields which are $SU(2)$ doublets.

  [J. Kubo, A. Mondragón, M. Mondragón and E. Rodriguez-Jauregui Prog. Theor. Phys. 109 795-807]

arXiv:1205.4755

Irreducible representations of S_3

The group S_3 has two one-dimensional irreps (singlets) and one two-dimensional irrep (doublet)

- one dimensional:
  $\mathbf{1}_A$ antisymmetric singlet,
  $\mathbf{1}_s$ symmetric singlet.
- Two - dimensional:
  $\mathbf{2}$ doublet

Direct product of irreps of S_3

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A,$$
$$\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s, \mathbf{1}_s \otimes \mathbf{2} = \mathbf{2},$$
$$\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}}$$

Irreducible representations of S_3

The group S_3 has two one-dimensional irreps (singlets) and one two-dimensional irrep (doublet)

- one dimensional:
  $\mathbf{1}_A$ antisymmetric singlet,
  $\mathbf{1}_s$ symmetric singlet.
- Two - dimensional:
  2 doublet

Direct product of irreps of S_3

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A,$$
$$\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s, \mathbf{1}_s \otimes \mathbf{2} = \mathbf{2},$$
$$\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}}$$

The direct (tensor) product of two doublets

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \text{ and } \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

has two singlets, r_s and r_A, and one doublet r_D^T

$r_s = p_{D1}q_{D1} + p_{D2}q_{D2}$ is invariant,

$r_A = p_{D1}q_{D2} - p_{D2}q_{D1}$ is not invariant

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

[J. Kubo, A. Mondragón, M. Mondragón and E. Rodriguez-Jauregui Prog. Theor. Phys. 109 795-807]

A Minimal S_3 invariant extension of the Standard Model

The Higgs sector is modified,

$$\Phi \rightarrow H = (\Phi_1, \Phi_2, \Phi_3)^T$$

H is a reducible $\mathbf{1_s} \oplus 2$ rep. of S_3

$$H_s = \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big)$$

$$H_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

A Minimal S_3 invariant extension of the Standard Model

The Higgs sector is modified,

$$\Phi \to H = (\Phi_1, \Phi_2, \Phi_3)^T$$

H is a reducible $\mathbf{1_s} \oplus 2$ rep. of S_3

$$H_s = \frac{1}{\sqrt{3}}\left(\Phi_1 + \Phi_2 + \Phi_3\right)$$

$$H_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Quark, lepton and Higgs fields

$$Q^T = (u_L, d_L), u_R, d_R,$$
$$L^\dagger = (\nu_L, e_L), e_R, \nu_R, H$$

All these fields have three species (flavours) and belong to a reducible $\mathbf{1} \oplus 2$ rep. of S_3

Leptons' Yukawa interactions

$$\begin{aligned}\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\ &\quad - Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.},\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} - Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} \\ &\quad - Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.},\end{aligned}$$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Leptons' Yukawa interactions

$$\mathcal{L}_{Y_E} = -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] - Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.},$$

$$\mathcal{L}_{Y_\nu} = -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} - Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.},$$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Furthermore, the Majorana mass terms for the right handed neutrinos are

$$\mathcal{L}_M = -\nu_R^T C \mathbf{M}_{\nu_R} \nu_R$$

where C is the charge conjugation matrix.

Mass matrices

We will assume that

$$< H_{D1} > \; = \; < H_{D2} > \; \neq 0 \qquad \text{and} \qquad < H_3 > \neq \; 0$$

and

$$< H_3 >^2 + < H_{D1} >^2 + < H_{D2} >^2 \;\approx\; \left(\frac{246}{2} GeV\right)^2$$

Then, the Yukawa interactions yield mass matrices of the general form

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

Mass matrices

We will assume that

$$< H_{D1} > \;=\; < H_{D2} > \;\neq 0 \qquad \text{and} \qquad < H_3 > \neq \; 0$$

and

$$< H_3 >^2 + < H_{D1} >^2 + < H_{D2} >^2 \;\approx\; \left(\frac{246}{2} GeV\right)^2$$

Then, the Yukawa interactions yield mass matrices of the general form

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$M_\nu = M_{\nu D} \tilde{\mathrm{M}}^{-1} (M_{\nu D})^T \quad \text{with} \quad \tilde{\mathrm{M}} = \mathrm{diag}(M_1, M_2, M_3)$$

The leptonic sector

To achieve a further reduction of the number of parameters, in the leptonic sector, we introduce an additional discrete Z_2 symmetry

$-$	$+$
$H_I,\ \nu_{3R}$	$H_S,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

then, $Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0.$ Hence, the leptonic mass matrices are

$$M_e = \begin{pmatrix} \mu_2^e & \mu_2^e & \mu_5^e \\ \mu_2^e & -\mu_2^e & \mu_5^e \\ \mu_4^e & \mu_4^e & 0 \end{pmatrix} \quad M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}$$

The unitary matrix that diagonalized the mass matrix of the charged leptons as function of its eigenvalues

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_e}{\tilde{m}_\mu} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\frac{\tilde{m}_e}{\tilde{m}_\mu} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & -\frac{\tilde{m}_e}{\tilde{m}_\mu} & 0 \end{pmatrix} + \mathcal{O}\left(10^{-5}\right)$$

$\tilde{m}_\mu = m_\mu / m_\tau$ and $\tilde{m}_e = m_e / m_\tau$.

There are no free parameters in $\mathbf{M}_e$ other than the Dirac Phase δ!!.

The unitary matrix that diagonalized the mass matrix of the charged leptons as function of its eigenvalues

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_e}{\tilde{m}_\mu} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\frac{\tilde{m}_e}{\tilde{m}_\mu} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & -\frac{\tilde{m}_e}{\tilde{m}_\mu} & 0 \end{pmatrix} + \mathcal{O}\left(10^{-5}\right)$$

$\tilde{m}_\mu = m_\mu / m_\tau$ and $\tilde{m}_e = m_e / m_\tau$.

There are no free parameters in $\mathbf{M}_e$ other than the Dirac Phase δ!!.

The neutrino mass matrix I

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$\mathbf{M}_\nu = \mathbf{M}_{\nu D} \tilde{\mathbf{M}}_R^{-1} \mathbf{M}_{\nu D}^T$$

with

$$\tilde{\mathbf{M}}_R = diag\left[M_1, M_2, M_3\right] \qquad M_1 \neq M_2 \neq M_3$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$\mathbf{M}_{\nu_\mathrm{L}} = \begin{pmatrix} \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} \\ \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu \lambda}{\overline{M}} \\ \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu \lambda}{\overline{M}} & \frac{2\left(\mu_4^\nu\right)^2}{\overline{M}} + \frac{\left(\mu_3^\nu\right)^2}{M_3} \end{pmatrix}, \quad \lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \quad \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$\mathbf{M}_{\nu_L} = \begin{pmatrix} \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} & \frac{2\left(\mu_4^\nu\right)^2}{\overline{M}} + \frac{\left(\mu_3^\nu\right)^2}{M_3} \end{pmatrix}, \quad \lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \quad \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

when $\lambda = 0$, we recovers the mass matrix given by Kubo, A.Mondragón, M. Mondragón y E. Rodriguez-Jauregui Prog.Theor.Phys. 109 (2003) 795-807. In this case two of the right-handed neutrino masses are degenerate and θ_{13} is different from zero but very small.

The mass matrix with two texture zeros displaced

For simplify the analysis we will consider the case $\arg\{\mu_2^\nu\} = \arg\{\mu_3^\nu\}$.

$$\mathbf{M}_{\nu_L} = \mathbf{Q}\mathcal{U}_{\frac{\pi}{4}}\left(\mu_0 \mathbf{I}_{3\times 3} + \widehat{\mathbf{M}}\right)\mathcal{U}_{\frac{\pi}{4}}^{\dagger}\mathbf{Q},$$

where $\mathbf{Q} = e^{i\phi_2}\mathbf{diag}\left\{1, 1, e^{i\delta_\nu}\right\},\ \delta_\nu = \phi_4 - \phi_2 = \arg\{\mu_4^\nu\} - \arg\{\mu_2^\nu\}$,

$$\mathcal{U}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}, \quad \mu_0 = \frac{2\left|\mu_2^\nu\right|^2}{|\overline{M}|}(1 - |\lambda|),\ \widehat{\mathbf{M}} = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & 2d \end{pmatrix}$$

with

$$A = \sqrt{2}\frac{\left|\mu_2^\nu\right|\left|\mu_4^\nu\right|}{|\overline{M}|}(1 - |\lambda|),\ B = \frac{2\left|\mu_4^\nu\right|^2}{|\overline{M}|} + \frac{\left|\mu_3^\nu\right|^2}{M_3} - \frac{2\left|\mu_2^\nu\right|^2}{|\overline{M}|}(1 - |\lambda|),$$

$$C = \sqrt{2}\frac{\left|\mu_2^\nu\right|\left|\mu_4^\nu\right|}{|\overline{M}|}(1 + |\lambda|) \text{ and } d = \frac{2|\lambda|\left|\mu_2^\nu\right|^2}{|\overline{M}|}.$$

The unitary matrix that diagonalizes the mass matrix of the neutrinos as function of its eigenvalues

$$\mathbf{U}_\nu = \mathbf{Q}^\nu \mathcal{U}_{\frac{\pi}{4}} \mathbf{O}_\nu^{N[I]}$$

where

$$\begin{pmatrix} \sqrt{\frac{[-1]\left(m_{\nu_3}-\mu_0\right)\left(m_{\nu_2}-\mu_0\right)f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{\left(m_{\nu_{3[1]}}-\mu_0\right)\left(\mu_0-m_{\nu_{1[3]}}\right)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & -\sqrt{\frac{[-1]\left(\mu_0-m_{\nu_1}\right)\left(m_{\nu_2}-\mu_0\right)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ \sqrt{\frac{[-1]2d\left(\mu_0-m_{\nu_1}\right)f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{2d\left(m_{\nu_2}-\mu_0\right)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & \sqrt{\frac{[-1]2d\left(m_{\nu_3}-\mu_0\right)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ -\sqrt{\frac{[-1]\left(\mu_0-m_{\nu_1}\right)f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{\left(m_{\nu_2}-\mu_0\right)f_1 f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}} & -\sqrt{\frac{\left(m_{\nu_3}-\mu_0\right)f_1 f_2^{N[I]}}{\mathcal{D}_1^{N[I]}}} \end{pmatrix}$$

$$\mathcal{D}_1^{N[I]} = 2d\left(m_{\nu_2}-m_{\nu_1}\right)\left(m_{\nu_{3[1]}}-m_{\nu_{1[3]}}\right), \qquad \mathcal{D}_2^{N[I]} = 2d\left(m_{\nu_2}-m_{\nu_1}\right)\left(m_{\nu_{3[2]}}-m_{\nu_{2[3]}}\right),$$

$$\mathcal{D}_3^{N[I]} = 2d\left(m_{\nu_{3[1]}}-m_{\nu_{1[3]}}\right)\left(m_{\nu_{3[2]}}-m_{\nu_{2[3]}}\right), \qquad f_1 = \left(2d+\mu_0-m_{\nu_1}\right),$$

$$f_2^{N[I]} = [-1]\left(2d+\mu_0-m_{\nu_2}\right), \qquad f_3^{N[I]} = [-1]\left(m_{\nu_3}-\mu_0-2d\right),$$

$m_{\nu_{2[1]}} > \mu_0 > m_{\nu_{1[3]}}$ and $m_{\nu_{3[2]}} > 2d+\mu_0 > m_{\nu_{2[1]}}$. The superscripts N and I denote the normal and inverted hierarchies respectively.

The neutrino mixing matrix II

From a comparison of V_{PMNS}^{th} with V_{PMNS}^{PDG}, we obtain the neutrino mixing angles as function of the lepton masses

$$\sin^2\theta_{12}^l = \frac{|(V_{PMNS})_{12}|^2}{1-|(V_{PMNS})_{13}|^2},\ \sin^2\theta_{23}^l = \frac{|(V_{PMNS})_{23}|^2}{1-|(V_{PMNS})_{13}|^2},\ \sin^2\theta_{13}^l = |(V_{PMNS})_{13}|^2$$

$$\sin^2\theta_{12}^l = \frac{\left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2\left(O_{11}^{N[l]}\right)^2+\left(O_{21}^{N[l]}\right)^2-2\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{11}^{N[l]}O_{21}^{N[l]}\cos\delta_l}{1-\left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2\left(O_{23}^{N[l]}\right)^2-\left(O_{33}^{N[l]}\right)^2+2\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{23}^{N[l]}O_{33}^{N[l]}\cos\delta_l}$$

$$\sin^2\theta_{23}^l = \frac{\left(O_{13}^{N[l]}\right)^2+\left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2\left(O_{23}^{N[l]}\right)^2+2\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{13}^{N[l]}O_{23}^{N[l]}\cos\delta_l}{1-\left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2\left(O_{23}^{N[l]}\right)^2-\left(O_{33}^{N[l]}\right)^2+2\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{23}^{N[l]}O_{33}^{N[l]}\cos\delta_l}$$

$$\sin^2\theta_{13}^l = \left(\frac{\tilde{m}_e}{\tilde{m}_\mu}\right)^2\left(O_{23}^{N[l]}\right)^2+\left(O_{33}^{N[l]}\right)^2-2\frac{\tilde{m}_e}{\tilde{m}_\mu}O_{23}^{N[l]}O_{33}^{N[l]}\cos\delta_l$$

The Reactor Mixing Angle

In a first, preliminary analysis for the reactor mixing angle θ^l_{13} and for an normal neutrino mass hierarchy

$$m_{\nu_1} = 3.22 \times 10^{-3}\ \mathrm{eV},\ m_{\nu_2} = 9.10 \times 10^{-3}\ \mathrm{eV},\ m_{\nu_3} = 4.92 \times 10^{-2}\ \mathrm{eV}.$$

the parameter values $\delta_l = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV, we get

$$\sin^2\theta^l_{13} \approx 0.029 \longrightarrow \theta^l_{13} \approx 10.8^\circ,$$

in good agreement with experimental data.

The Reactor Mixing Angle

In a first, preliminary analysis for the reactor mixing angle θ^l_{13} and for an normal neutrino mass hierarchy

$$m_{\nu_1} = 3.22 \times 10^{-3}\ \mathrm{eV},\ m_{\nu_2} = 9.10 \times 10^{-3}\ \mathrm{eV},\ m_{\nu_3} = 4.92 \times 10^{-2}\ \mathrm{eV}.$$

the parameter values $\delta_l = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV, we get

$$\sin^2\theta^l_{13} \approx 0.029 \longrightarrow \theta^l_{13} \approx 10.8^\circ,$$

in good agreement with experimental data.

The solar and atmospheric mixing angles:

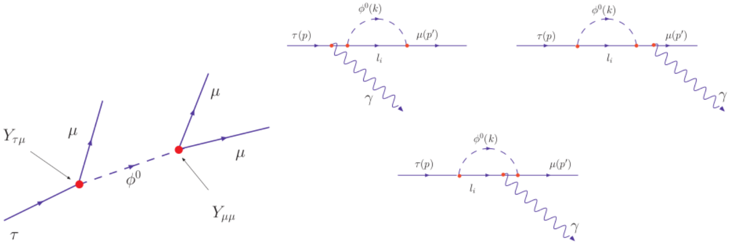
$$\theta^{l^{th}}_{12} = 35^\circ, \quad \theta^{l^{th}}_{23} = 46^\circ,$$

FCNC I

In the Standard Model the FCNC at tree level are suppressed by the GIM mechanism.
Models with more than one Higgs $SU(2)$ doublet have tree level FCNC due to the exchange of scalar fields.
The mass matrix written in terms of the Yukawa couplings is

$$\mathcal{M}^e_Y = Y^{E1}_w H^0_1 + Y^{E2}_w H^0_2,$$

FCNC processes: The left contributes to the process $\tau^- \to 3\mu$. The right contribute to the process $\tau \to \mu\gamma$.



Yukawa matrices in the mass representation[2]

The Yukawa matrices in the mass basis defined by

$$\tilde{Y}_m^{EI} = U_{eL}^{\dagger} Y_w^{EI} U_{eR}$$

and

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m , \quad \tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m$$

$x = m_e/m_\mu$.

All off diagonal terms give rise to FCNC processes!!

[2] A. Mondragon *et al* Phys. Rev. D 76 076003

Branching ratios

We define the partial branching ratio (only leptonic decays)

$$Br(\tau \to \mu e^+ e^-) = \frac{\Gamma(\tau \to \mu e^+ e^-)}{\Gamma(\tau \to e\nu\bar{\nu}) + \Gamma(\tau \to \mu\nu\bar{\nu})}, \; \Gamma(\tau \to \mu e^+ e^-) \approx \frac{m_\tau^5}{3 2^{10} \pi^3} \frac{(Y^{1,2}_{\tau\mu} Y^{1,2}_{ee'})^2}{M^4_{H_{1,2}}}$$

thus

$$Br(\tau \to \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2} \right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}} \right)^4,$$

Similar computations lead to

$$Br(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H} \right)^4, \; Br(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau} \right)^2 \left(\frac{m_\tau}{M_H} \right)^4,$$

$$Br(\tau \to 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H} \right)^4, \; Br(\mu \to 3e) \approx 18 \left(\frac{m_e m_\mu}{m_\tau^2} \right)^2 \left(\frac{m_\tau}{M_H} \right)^4,$$

$$Br(\mu \to e\gamma) \approx \frac{27\alpha}{64\pi} \left(\frac{m_e}{m_\mu} \right)^4 \left(\frac{m_\tau}{M_H} \right)^4.$$

Leptonic processes via FCNC

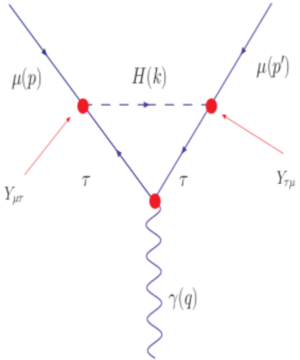
FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \to 3\mu$	8.43×10^{-14}	5.3×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	8×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert *et. al.*(2005)
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert *et. al.* (2006)
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt *et al.* (1998)
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks *et al.* (1999)

Small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the **gravitational core collapse and shock generation** in the explosion stage of a supernova

Muon Anomalous Magnetic Moment

The anomalous magnetic moment of the muon is related to the gyroscopic ratio by

$$a_\mu = \frac{\mu_\mu}{\mu_B} - 1 = \frac{1}{2}(g_\mu - 2)$$



In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour changing neutral scalars also contribute to the anomalous magnetic moment of the muon

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu}}{16\pi^2} \frac{m_\mu m_\tau}{M_H^2} \left(log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right)$$

From our results: $Y_{\mu\tau} Y_{\tau\mu} = \frac{m_\mu m_\tau}{4 v_1 v_2}$

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246\ GeV)^2} \frac{(2 + \tan^2\beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right),$$

$\tan\beta = \frac{v_s}{v_1}$

From the experimental upper bound on $(\mu \to 3e)$, we get $\tan\beta \leq 14$, Hence

$$\delta a_{\mu} = 1.7 \times 10^{-10}$$

From the experimental upper bound on $(\mu \to 3e)$, we get $\tan\beta \leq 14$, Hence

$$\delta a_\mu = 1.7 \times 10^{-10}$$

Contribution to the anomaly of the muon's magnetic moment

The difference between the experimental value and the Standard Model prediction for the anomaly is

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.79.1)10^{-10}$$

$\Delta a_\mu \sim 3\sigma$ (three standard deviations) !!

But, the uncertainty in the computation of higher order hadronic effects is large

$$\delta a_\mu^{LBL}(3, had) \approx 1.5910^{-9}; \quad \delta a_\mu^{VP}(3, had) \approx -1.8210^{-9}$$

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx \frac{1.7}{28} \approx 6\% \quad \text{and} \quad \delta a_\mu^{(H)} < \delta a_\mu(3, had)$$

The contribution of the exchange of flavour changing scalars to the anomaly of the muon's magnetic moment, $\delta a_\mu^{(H)}$, is small but not negligible, and it is compatible with the best, state of the art, measurements and theoretical predictions.

Summary

- By introducing three $SU(2)_L$ Higgs doublet fields, in the theory, we extended the concept of flavour and generations to the Higgs sector and formulated a minimal S_3-invariant Extension of the SM
- The neutrino mixing angles θ_{12}, θ_{23} and θ_{13}, are determined by an interplay of the $S_3 Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy
- The fit of, $\sin^2\theta_{13}^{th}$ to $\sin^2\theta_{13}^{exp}$ breaks the mass degeneracy of the right handed neutrinos.
- The branching ratios of all flavour changing neutral processes in the leptonic sector are strongly suppressed by the $S_3 Z_2$ symmetry and powers of the small mass ratios m_e/m_τ, m_μ/m_τ, and $\left(m_\tau/M_{H_{1,2}}\right)^4$, but could be important in astrophysical processes
- The anomalous magnetic moment of the muon gets a small but non-negligible contribution from the exchange of flavor changing scalar fields