Different realizations of S_3 as a unified theory for quarks and leptons

U. J. Saldaña Salazar

Work supported by Mexican Grants: PAPIIT-IN113712 and CONACyT-132059

PASCOS 2012
Mérida, Yucatán.

June 7, 2012

In collaboration with:

Prof. A. Mondragón, L. Velasco-Sevilla and F. González Canales

Acknowledgements:

M. Mondragón

"What are the least things that I need in order to explain what I know?"

Talk of Prof. M. Lindner

1 Stage I: What we found...

2 Stage II: How we did it?

3 Stage III: Conclusions

Stage I: What we found...

$$\text{SM} + m_\nu + S_3|_3 S + 3H$$

$$=$$

- Mixing angles in terms of mass ratios.
- A "simple" and "natural" scenario with a unified treatment.
- A model with viable phenomenology. (Here only quarks are discussed)

$$\mathrm{SM} + m_\nu + S_3|_3 S + 3H$$

$$=$$

- Mixing angles in terms of mass ratios.

A desirable feature of any physical theory is predictivity.

SM + m_ν

$(6m_q,\ 3\theta_q,\ \delta^q_{cp},\ 6m_l\ (+3m_{\nu R}),\ 3\theta_l,\ \delta^l_{cp}\ (+2\ \text{Maj. phases}))$

20(+2 Maj.) free parameters

- Since the late 1960's:

$$\theta_C \approx \sqrt{m_d/m_s}$$

- Gatto, Sartori, and Tonin, Phys. Lett., B28:128-130, 1968.
- Cabibbo and Maiani, Phys. Lett., B28:131-135, 1968.
- Pagels, Phys. Rev., D11:1213, 1975.
- Weinberg, Trans. New York Acad. Sci., 38:185-201,1977.
- Wilczek and Zee, Phys. Lett., B70:418, 1977.
- Fritzsch, Phys. Lett., B70:436, 1977.
- Ebrahim, Phys. Lett., B73:181-184, 1978.
- Mohapatra and Senjanovic, Phys. Lett., B73:176, 1978.
- Fritzsch, Phys. Lett., B73:317-322, 1978.
- Pakvasa and Sugawara, Phys. Lett., B73:61, 1978.
- ...

We pursue a model with this feature.

$$\text{SM} + m_\nu + S_3|_3 S + 3H$$

$$=$$

- Mixing angles in terms of mass ratios.
- A "simple" and "natural" scenario with a unified treatment.

S_3

- "Simple" → Smallest non-abelian discrete symmetry group.
- "Natural" → Before Yukawa interactions 3 families are undistinguishable.
- Unified treatment → A universal mass matrix for quarks and leptons.

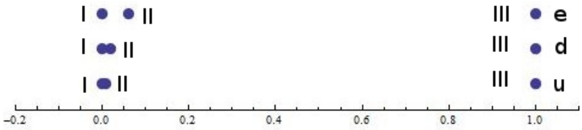
$$\text{SM} + m_\nu + S_3|_3 S + 3H$$

$$=$$

- Mixing angles in terms of mass ratios.
- A "simple" and "natural" scenario with a unified treatment.
- An incredible quark phenomenology.

Stage II: How we did it?

Relative values in each fermion sector



show a clear pattern: $\mathbf{2} \oplus 1$

Generic mass matrix for Dirac fermions

SM + m_ν + $S_3|_3S$ + $3H$

- $$\begin{pmatrix} f_{I(L,R)} \\ f_{II(L,R)} \end{pmatrix} \sim \mathbf{2}; \quad f_{III(L,R)} \sim 1_{\mathbf{S}},$$
- $$\begin{pmatrix} H_{1W} \\ H_{2W} \end{pmatrix} \sim \mathbf{2}; \quad H_{SW} \sim 1_{\mathbf{S}}; \quad H_{AW} \sim 1_{\mathbf{A}}.$$

After the Higgs mechanism:

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \sqrt{2}Y^f_2 v_S + Y^f_3 w_2 & Y^f_3 w_1 + \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_5 w_1 \\ Y^f_3 w_1 - \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_2 v_S - Y^f_3 w_2 & \sqrt{2}Y^f_5 w_2 \\ \sqrt{2}Y^f_6 w_1 & \sqrt{2}Y^f_6 w_2 & 2Y^f_1 v_S \end{pmatrix} \quad (1)$$

where the couplings Y^f_i are complex and the vev's (v_S, v_A, w_1, w_2) are real.

$$\theta_C \approx \sqrt{m_d/m_s}$$

1. $Tr[\mathcal{M}^f_{S_3}]$, $Det[\mathcal{M}^f_{S_3}]$, and $Tr[{\mathcal{M}^f_{S_3}}^2]$
2. $\mathcal{M}^f_{S_3} = \mathcal{M}^f_{S_3}(m_1, m_2, m_3)$
3. $U_{ij} = U_{ij}(\widetilde{m})$

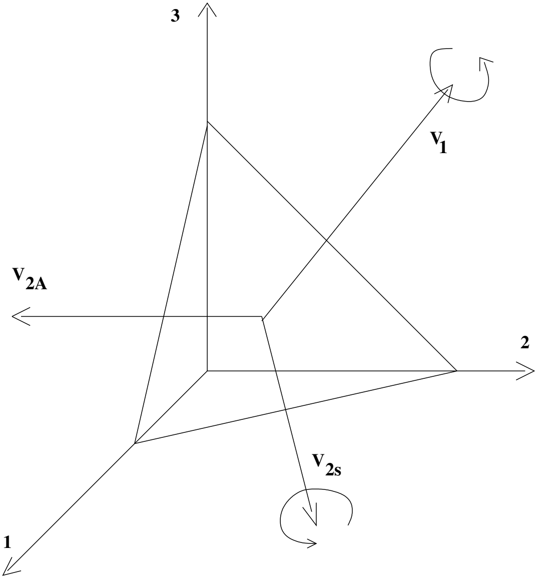

STOP
AND
THINK

4-zero Fritzsch-like texture

$$\mathcal{M}^{f}_{Her.} = \begin{pmatrix} 0 & A^{f} & 0 \\ A^{f*} & |B^{f}| & C^{f} \\ 0 & C^{f*} & |D^{f}| \end{pmatrix} \tag{2}$$

- Nishiura, Matsuda and Fukuyama, Phys. Rev., D60:013006, 1999.
- Fritzsch and Xing, Prog. Part. Nucl. Phys., 45:1-81, 2000.
- Matsuda and Nishiura, Phys. Rev., D74:033014, 2006.
- Barranco, Gonzalez Canales and Mondragón, Phys. Rev., D82:073010, 2010.

- Viable phenomenology.
- It allows a unified treatment.


3
V1
V2A
2
V2s
1

The rotated matrix:

$$\mathcal{M}^f_{Hier.} = \begin{pmatrix} \mu_1^f - \mu_4^f \mathrm{s}2\theta + \mu_2^f(\mathrm{c}^2\theta - \mathrm{s}^2\theta) & \mu_5^f + \mu_2^f \mathrm{s}2\theta + \mu_4^f(\mathrm{c}^2\theta - \mathrm{s}^2\theta) & \mu_6^f \mathrm{c}\theta - \mu_7^f \mathrm{s}\theta \\ -\mu_5^f + \mu_2^f \mathrm{s}2\theta + \mu_4^f(\mathrm{c}^2\theta - \mathrm{s}^2\theta) & \mu_1^f + \mu_4^f \mathrm{s}2\theta - \mu_2^f(\mathrm{c}^2\theta - \mathrm{s}^2\theta) & \mu_6^f \mathrm{s}\theta + \mu_7^f \mathrm{c}\theta \\ \mu_8^f \mathrm{c}\theta - \mu_9^f \mathrm{s}\theta & \mu_8^f \mathrm{s}\theta + \mu_9^f \mathrm{c}\theta & \mu_3^f \end{pmatrix}. \tag{3}$$

where we have denoted

$$\mu_1^f \equiv \sqrt{2} Y_2^f v_S, \;\; \mu_2^f \equiv Y_3^f w_2, \;\; \mu_3^f \equiv 2 Y_1^f v_S, \;\; \mu_4^f \equiv Y_3^f w_1, \;\; \mu_5^f \equiv \sqrt{2} Y_4^f v_A,$$
$$\mu_6^f \equiv \sqrt{2} Y_5^f w_1, \;\; \mu_7^f \equiv \sqrt{2} Y_5^f w_2, \;\; \mu_8^f \equiv \sqrt{2} Y_6^f w_1, \;\; \mu_9^f \equiv \sqrt{2} Y_6^f w_2.$$

What are the implied conditions...?

The implied conditions:

1. Hermiticity:
   a) $Y_5^f = Y_6^{f*}$,
   b) $\arg(Y_4^f) = \pm\frac{\pi}{2}$,
   c) $\arg(Y_1^f) = \arg(Y_2^f) = \arg(Y_3^f) = 0$
2. Two null matrix elements: $(1,1)$ and $(1,3)$:
   a) $\tan\theta = \frac{w_1}{w_2}$,
   b) $\sqrt{2} Y_2^f v_s = -Y_3^f w_2 \cos^2\theta (1 - 3\tan^2\theta)$.

The resulting matrix:

$$\mathcal{M}^f_{Hier.} = \begin{pmatrix} 0 & |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta)+i|\mu_5^f| & 0 \\ |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta)-i|\mu_5^f| & -2|\mu_2^f|\cos^2\theta(1-3\tan^2\theta) & \mu_8^f\sec\theta \\ 0 & \mu_8^{f*}\sec\theta & |\mu_3^f| \end{pmatrix} \tag{4}$$

which is equivalent to:

$$\mathcal{M}^f_{Her.} = \begin{pmatrix} 0 & A^f & 0 \\ A^{f*} & |B^f| & C^f \\ 0 & C^{f*} & |D^f| \end{pmatrix}. \tag{5}$$

Quark mixing sector

Exact analytical forms (V_{CKM}):

$$V_{ud}^{th} = \sqrt{\frac{\widetilde{m}_c \widetilde{m}_s \xi_1^u \xi_1^d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{m}_u \widetilde{m}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1},$$

$$V_{us}^{th} = -\sqrt{\frac{\widetilde{m}_c \widetilde{m}_d \xi_1^u \xi_2^d}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{m}_u \widetilde{m}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u \xi_2^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_1^d}\, e^{i\phi_2} \right) e^{i\phi_1},$$

$$V_{ub}^{th} = \sqrt{\frac{\widetilde{m}_c \widetilde{m}_d \widetilde{m}_s \delta_d \xi_1^u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{m}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d \xi_1^u} - \sqrt{\delta_u \xi_2^u \xi_1^d \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1},$$

$$V_{cd}^{th} = -\sqrt{\frac{\widetilde{m}_u \widetilde{m}_s \xi_2^u \xi_1^d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{m}_c \widetilde{m}_d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} \left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_2^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_1^u \xi_2^d}\, e^{i\phi_2} \right) e^{i\phi_1}. \tag{6}$$

...

Studied cases:

1. ϕ_1 and ϕ_2 as free parameters. (S_3-SM, $w_1 \neq w_2$) (cyan dashed line)
   a. $\phi_2 = 0$ and ϕ_1 as a free parameter. (red dashed line)
   b. $\phi_1 = \frac{\pi}{2}$ and $\phi_2 = 0$. (blue solid line)
2. $\phi_1 = 0$ and ϕ_2 as a free parameter. ($H_{AW} = 0 \rightarrow MS_3$IESM, $w_1 = w_2$) (green dashed line)
   - Pakvasa and Sugawara, Phys. Lett. B73, 61, 1978.
   - Derman, Phys. Rev. D19, 317, 1979.
   - Wyler, Phys. Rev. D19, 330, 1979.
   - Yahalom, Phys. Rev. D29, 536, 1984.
   - Kubo, Mondragón, Mondragón, Rodriguez-J Prog. Theor. Phys. 109, 795 (2003)
   - Kubo, Okada and Sakamaki, Phys. Rev. D70:036007, 2004.
   - Mondragón, Mondragón and Peinado, Phys. Rev. D76:076003, 2007.
   - Teshima and Okumura, Phys. Rev. D84:016003, 2011.
   - Bhattacharyya et al, Phys. Rev. D83:011701, 2011.
   - ...

Highlights of the fit:

1. By considering unitary the V_{CKM} we need to fit to 4 observables $(V_{ud}, V_{us}, V_{ub}, \mathcal{J}_q)$.
2. The χ^2 is constructed

$$\chi^2 = \frac{\left(V_{ud}^{\rm th} - V_{ud}\right)^2}{\sigma_{V_{ud}}^2} + \frac{\left(V_{us}^{\rm th} - V_{us}\right)^2}{\sigma_{V_{us}}^2} + \frac{\left(V_{ub}^{\rm th} - V_{ub}\right)^2}{\sigma_{V_{ub}}^2} + \frac{\left(\mathcal{J}_q^{\rm th} - \mathcal{J}_q\right)^2}{\sigma_{\mathcal{J}_q}^2} \tag{7}$$

3. The minimization of χ^2 depends in:
   - mass ratios* (2σ regions),

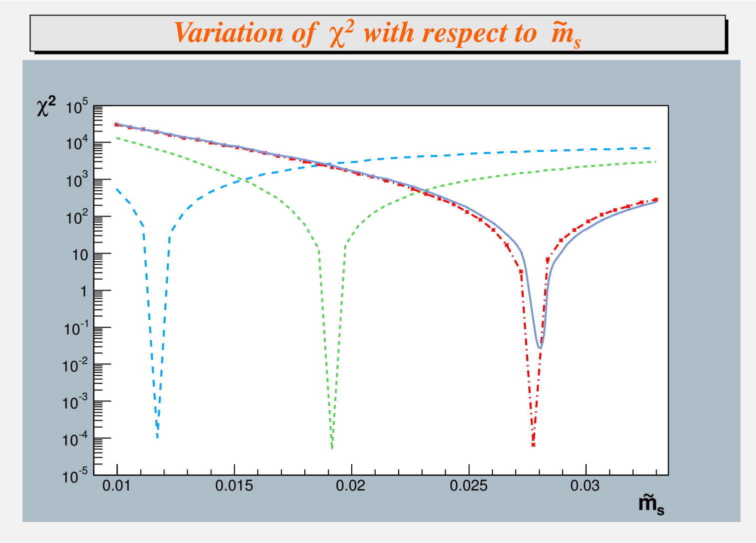
	Mass ratios at m_z
$\widetilde{m_u}(m_z)$	0.0000081 ± 0.0000025
$\widetilde{m_c}(m_z)$	0.0036 ± 0.0004
$\widetilde{m_d}(m_z)$	0.000998 ± 0.000188
$\widetilde{m_s}(m_z)$	0.021 ± 0.006

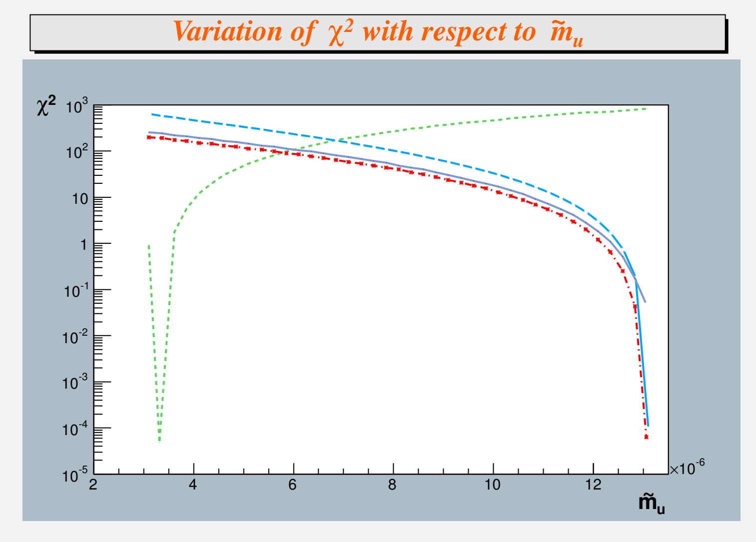
   - δ_u and δ_d,
   - ϕ_1 and ϕ_2.

Studied cases:

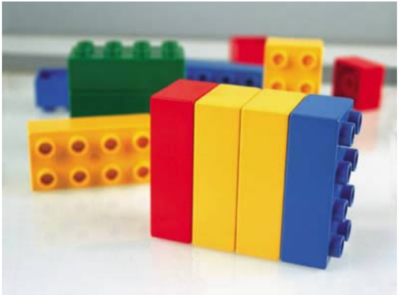
1. 4 free parameters: δ_u, δ_d, $cos\phi_1$, and $cos\phi_2$. (S_3-SM, $w_1 \neq w_2$) (cyan dashed line)
   a. 3 free parameters: δ_u, δ_d, and $cos\phi_1$.. (red dashed line)
      - Mondragón and Rodriguez-Jauregui, Phys. Rev., D59:093009, 1999.
      - Fritzsch and Xing, Prog. Part. Nucl. Phys., 45:1-81, 2000.
      - Barranco, Gonzalez Canales and Mondragón, Phys. Rev., D82:073010, 2010.

   b. 2 free parameters: δ_u and δ_d. (blue solid line)
2. 1 free parameter: ϕ_2. ($H_{AW} = 0 \rightarrow$ MS_3IESM, $w_1 = w_2$) (green dashed line)


Variation of χ^2 with respect to $\tilde{m}_s$
χ^2
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.01
0.015
0.02
0.025
0.03
$\tilde{m}_s$


Variation of χ^2 with respect to $\tilde{m}_u$
χ^2
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2
4
6
8
10
12
$\times 10^{-6}$
$\tilde{m}_u$

Stage III: Conclusions



"What are the least things that I need in order to explain what I know?" (Prof. M. Lindner)

- Massive neutrinos (Dirac or Majorana).
- S_3 (smallest...)
- A *minimally* extended Higgs sector.

Thanks for your **attention**.

"Wine is valued by its price, not its flavour."

A. Trollope

"Is your model better than no model?"

Prof. A. de Gouvea

Backup (Aaahh!)

Quark sector experimental status:

1. An incredible development in the last decade

$$|V_{CKM}| = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}. \quad (8)$$

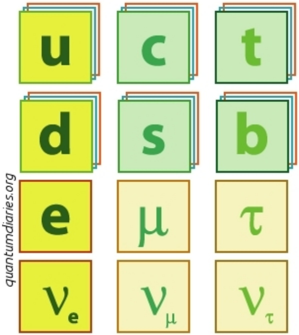
2. Any quark model should present a detailed analysis about their predictions.

Unknown scalar sector $\rightarrow$ possibility of adding more H

Recall S_3 irreps: $\mathbf{2}$, $1_{\mathbf{S}}$, and $1_{\mathbf{A}}$

General case allowed $\rightarrow + 3H$ ($H_{\mathbf{D}}$, $H_{\mathbf{S}}$, and $H_{\mathbf{A}}$)

SM + m_ν + Flavour Symm.|.mmyS ruovalF



In the spirit of constructing the simplest model...

1st way

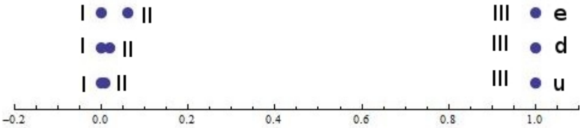
- No new particles $\rightarrow$ Discrete symmetry groups
- Mixing angles in terms of mass ratios $\rightarrow$ Non-abelian $+ \geq 3H$ *

  * D. Wyler, Phys. Rev. **D19**, 330 (1979)
- The smallest group $\rightarrow$ S_3

In the spirit of taking seriously some facts...

2nd way

- Experiments have not found evidence of more families.
- Universal interactions among 3 massless families $\rightarrow$ S_3
- Mass distribution pattern: $\mathbf{2} \oplus 1$



$\rightarrow$ S_3 irreps: $\mathbf{2}$, $1_\mathbf{S}$ and $1_\mathbf{A}$

- 1H:

$$\mathcal{M}_f = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$