$(SU_5 \times Q_6)$ Models of SUSY-FUT's

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Work supported by Mexican grants: PAPIIT-IN113712 and CONACyT-132059

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June 9, 2012

: Outline

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- $SU_5 \times Q_6$ SUSY-FUT's.
- Perspectives and Conclusions.

: Symmetry

- (Most of) SM parameters come from the Yukawa sector.
- Global symmetries to relate matrix elements.
- Local symmetries.
- SU_5, the simplest one (Rank 4), (Georgi, Glashow)

: (Super)symmetry

- Non-conventional symmetry.
- Possible solution to Hierachy Problem.
- No fermions in the adjoint of $SU_3 \times SU_2 \times U_1 \rightarrow$ Double the spectrum.
- Soft terms. 100 parameters.
- $N = 4$ is (all-loop) finite.

- Relations between couplings $\rightarrow$ symmetries?
- $\beta_g \frac{d\beta_{g_i}}{dg} = \beta_i$. (Zimmermann, Oehme, Sibold)
- (Counterexample) Dirac field + Pseudoscalar. (Sibold)
- Infrared ratio top Gauge-Yukawa (GYU) $\rightarrow M_t \sim 100 GeV$.... (Pendentlon, Ross)

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- General Field Theories ($4D$), $\beta^1 = 0$
- But $\beta^2 = 0 \rightarrow$ (Fermion-Boson) relations, (Lucha-Neinfeld)
- In SUSY, $\beta^1 = 0 \rightarrow \beta^2 = 0$. (Parkes, West, Jack, Jones, Mezincescu)

: Finiteness

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- Domino Effect?

Consider a (Renormalizable) SUSY Yang-Mills Theory with a simple group.

- Gauge anomaly is absent.
- $\beta_g^1 = 0 = \sum_i l(R_i) - 3C_2(G)$,
- $\gamma_j^{i\,1} = 0 = \frac{1}{32\pi}(C^{ilk}C_{jlk} - 2g^2C_2(R_i)\delta_j^i)$, $C_{ijk} = \rho_{ijk}g$
- isolote and non-degenerate, $\beta_{ijk}^1 = 0$.

$\rightarrow$ SUSY-Yang-Mills Theory with just one coupling constant (g) with $\beta_g = 0$.

: SUSY-FUT. The Strategy

We consider a SUSY-FUT which gauge group is broken at scale M_{GUT}, so at low energies (with respect to M_{GUT}) we are left with a $MSSM$ type theory. Then finiteness conditions work as boundary conditions.

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Finite Models in four ($SU(5)$) dimensions studied extensively.

Rajpoot, Taylor, Schwarz, Raby, Kazakov, León, Pérez-Mercader, Quirós, Mondragón, Kapetanakis, Zoupanos, Babu, etc.

- Higgs superpotential: $m_X\, X\bar{X} \,+\, h_X\, X\Sigma\bar{X}$
- Taking SU_5, $\langle\Sigma\rangle \,=\, w\, Diag\{-2,-2,-2,+3,+3\}$
- $m_H \,=\, m_X + 3\,(w) h_X, \quad m_T \,=\, m_X - 2\,(w) h_X,$
- $m_T \sim m_{GUT}$, at least one eigenvalue of $m_H \sim m_{EW}$ coupled to Yukawa Sector.

- Rotate Higgs sector. $U_X^*, U_{\bar{X}}$ (León, Pérez-Mercader)
- $v_{ew1} \sum_a (Y^u)^a (U_X^*)_{4a}$ (up) and $v_{ew2} \sum_a (Y^d)^a (U_{\bar{X}})_{a4}$ (down)
- $\det(m_T) \simeq 5w^4 h_X (U_X^*)_{44} (U_{\bar{X}})_{44} \quad \longrightarrow \quad (U_X^*)_{44} (U_{\bar{X}})_{44} \geq \frac{1}{50}$

: Matter

- Finiteness conditions impose $(4(\mathbf{5}), 7(\bar{\mathbf{5}}), (1)\mathbf{24}, (3)\mathbf{10})$
- $\mathbf{5} : H_a \,, \quad \bar{\mathbf{5}} : \bar{H}_a \,, \quad \mathbf{24} : \Sigma \,, \quad \mathbf{10} : 10_i \,, \quad \bar{\mathbf{5}} : \bar{5}_i \,.$ (Rajpoot, Taylor, Hamidi,Schwarz)
- $\frac{1}{2} Y_{1010H} \, 1010H + Y_{10\bar{5}\,\bar{H}} 10\bar{5}\bar{H} + Y_{H\bar{H}\Sigma} H\Sigma\bar{H} + \frac{1}{6} Y_{\Sigma^3} \Sigma^3 + \frac{1}{2} Y_{\bar{5}\,\bar{5}10} \bar{5}\bar{5}10 + \frac{1}{2} Y_{\bar{H}\bar{H}10} \bar{H}\bar{H}10 + Y_{H\Sigma\bar{H}} H\Sigma\bar{5}$

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- $\frac{1}{2}\,Y_{1010H}\,1010H \,+\, Y_{10\bar{5}\,\bar{H}}10\bar{5}\bar{H} \,+\, Y_{H\bar{H}\Sigma}H\Sigma\bar{H} \,+\, \frac{1}{6}\,Y_{\Sigma^3}\,\Sigma^3 \,+\, \frac{1}{2}\,Y_{\bar{5}\,\bar{5}10}\bar{5}\bar{5}10 \,+\, \frac{1}{2}\,Y_{\bar{H}\bar{H}10}\,\bar{H}\bar{H}10 \,+\, Y_{H\Sigma\bar{H}}\,H\Sigma\bar{5}$
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- Some violate baryon and lepton number.
- $Y_{\bar{5}\bar{5}10} \quad = \quad Y_{\bar{H}\bar{H}10} \quad = \quad Y_{H\Sigma\bar{5}} \quad = \quad 0$

: Finiteness Conditions

$$
\begin{array}{rcccc}
H_a^b & : & 3\, f_{ija}\, f^{ijb} + \frac{24}{5}\, h_{ac}\, h^{bc} & = & \frac{24}{5}\, g^2 \delta_a^b \\
\bar{5}_i^j & : & 4\, g_{kia}\, g^{kja} & = & \frac{24}{5}\, g^2\, \delta_i^j \\
\bar{H}_a^b & : & 4\, g_{ija}\, g^{ijb} + \frac{24}{5}\, h_{ca}\, h^{cb} & = & \frac{24}{5}\, g^2\, \delta_a^b \\
10_i^j & : & 3\, f_{ika}\, f^{jka} + 2\, g_{ika}\, g^{jka} & = & \frac{36}{5}\, g^2\, \delta_i^j \\
\Sigma & : & h_{ab}\, h^{ab} + \frac{21}{5} p^* p & = & \frac{36}{5}\, g^2\,,
\end{array}
$$

- Unitary transformations preserve finiteness conditions.
- $|p|^2 \quad = \quad \frac{15}{7} g^2$
- Number of Higgses in $10\bar{5}\bar{H} = 1010H$, (≥ 3)
- if $h_{ab} \neq 0$ $(\not{a}\not{b}) \rightarrow 3$ coupled Higgses.

Yukawas of the form:

$$f_y = g\,\{\,\sqrt{\tfrac{8}{5}}\sum_{ija}(\sum_{k}^{3}(\mathbb{U}_{10})_{ki}(\mathbb{U}_{10})_{kj}(\mathbb{U}_{H})_{ka})\,10_i 10_j H_a$$

$$+\sqrt{\tfrac{6}{5}}\sum_{ija}(\sum_{k}^{3}(\mathbb{U}_{10})_{ki}(\mathbb{U}_{\bar{5}})_{kj}(\mathbb{U}_{\bar{H}})_{ka})\,10_i \bar{5}_j \bar{H}_a$$

$$+\sum_{ab}((\mathbb{U}_{H})_{4a}(\mathbb{U}_{\bar{H}})_{4b})\,H_a \Sigma \bar{H}_b + \sqrt{\tfrac{15}{7}}\,\Sigma^3\,\}$$

satisfy finiteness conditions.

- All representations of (Bi)-Dihedral groups are (one and two)-dimensional
- Q_6 has two 2-dimensional and four 1-dimensional representations.
- $4^{14} + (2 \cdot 7)(4^{12}) + (2 \cdot 7 \cdot 3)(4^{10}) + \dots$
- $$10 \equiv \begin{pmatrix} 10_1 \\ 10_2 \end{pmatrix}$$
- Invariant tensors are proportonial to Pauli matrices.

: $H's$

- All H's in a singlet irrep. of Q_6 are not allowed.
- $H^\intercal = (H_1, H_2)$, **2**
- Similar for $10\bar{5}\bar{H}$

$$\Psi^A = 10, 10. \quad \Psi^B = 10, \bar{5}\,. \quad \Psi^A_3 = 10_3, 10_3\,. \quad \Psi^C_1 = H, \bar{H}.$$
$$.\quad \Psi^B_3 = 10_3, \bar{5}_3 \quad \Psi^C_3 = H_3, \bar{H}_3$$

$$a\,\Psi^A\Psi^B\Psi_3^C \;+\; b\,\Psi^A\Psi_3^B\Psi^C \;+\; c\,\Psi_3^A\Psi^B\Psi^C \;+\; d\,\Psi^A\Psi^B\Psi^C \;+\; e\,\Psi_3^A\Psi_3^B\Psi_3^C$$

$(2^4 \cdot 4^6)$ possibilities. After Finiteness Conditions:

$$|a_{f,g}| \;=\; |b_{f,g}| \;=\; |c_{f,g}| \;=\; \sqrt{\tfrac{1}{2}}\,(l_{f,g})\,\mathrm{sen}(\theta_{f,g}),$$

$$|e_{f,g}| \;=\; \sqrt{2}|\,d_{f,g}| \;=\; (l_{f,g})\cos(\theta_{f,g}),$$

$[(a,b,c) \;=\; 0] \vee [(d,e) = 0] \Rightarrow$ All loop finite

We look for the assignments that allow a, b, c, e, d; Then mass matrices are :

$$\frac{1}{2}\begin{bmatrix} -dv_1 + av_3 & dv_2 & b\,v_1 \\ dv_2 & dv_1 + av_3 & bv_2 \\ c\,v_1 & cv_2 & ev_3 \end{bmatrix}, \quad 10 = \mathbf{2}', \quad 10_3 = \mathbf{1},\ H_3 = \mathbf{1},$$

$$\frac{1}{2}\begin{bmatrix} dv_2 + av_3 & -dv_1 & b\,v_1 \\ -v_1 & -dv_2 + av_3 & bv_2 \\ c\,v_1 & cv_2 & ev_3 \end{bmatrix}, \quad 10 = \mathbf{2}, \quad 10_3 = \mathbf{1}'',\ H_3 = \mathbf{1}',$$

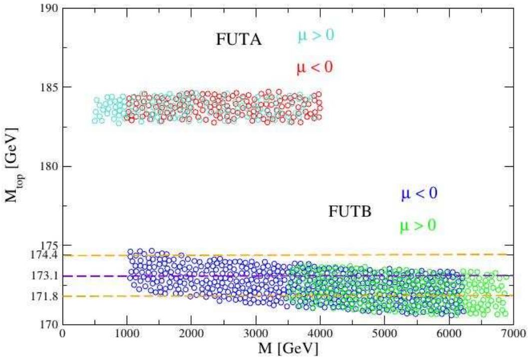
Additionaly for the $10\bar{5}\bar{H}$ sector:

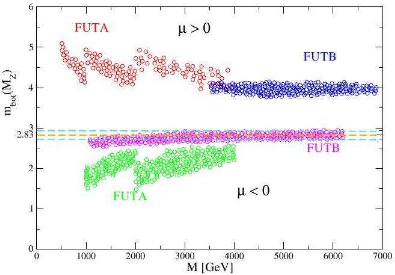
$$\begin{bmatrix} dv_2 + av_3 & dv_1 & bv_1 \\ -dv_1 & dv_2 - av_3 & bv_2 \\ cv_1 & -cv_2 & ev_3 \end{bmatrix}, \quad 10_3 = \mathbf{1}'', \quad \bar{5} = \mathbf{1}', \quad \bar{H} = \mathbf{1}''$$

$$\begin{bmatrix} dv_2 + av_3 & dv_1 & bv_2 \\ -dv_1 & dv_2 - av_3 & -bv_1 \\ cv_2 & cv_1 & ev_3 \end{bmatrix}, \quad 10_3 = \mathbf{1}''', \quad \bar{5}_3 = \mathbf{1}_3, \quad \bar{H}_3 = \mathbf{1}''$$

These mass matrices have been proved to be phenomenologically succesful. (Canales,Kubo)

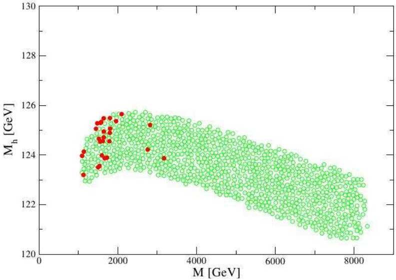
Finite SU_5 (one family) .





FUTA
μ > 0
FUTB
FUTB
FUTA
μ < 0
$m_{bot}(M_Z)$
M [GeV]
2.83

$$(m_i^2 + m_j^2 + m_k^2)/(MM^\dagger) = 1 + \frac{g^2}{16\pi^2}\Delta^{(1)} O(g^4) \quad (1)$$



: Perspectives and Conclusions

- By introducing soft terms and imposing finiteness (at least at two-loops) we will generate mass sum rules in the soft sector. (Mondragón, Kazakov, Zoupanos).
- Threshold corrections.
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- Could be a link between the MSSM and a more fundamental theory.