Dynamical Mass Generation

Majorana Vs Dirac

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Summary of Basic Models :-

- NJL (1961)

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

- SNJL (1982) — dim 6 four-superfield interaction

$$\mathcal{L}_\psi = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1-\tilde{m}^2\theta^2\bar{\theta}^2) + \int d^4\theta\; g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-(1-\tilde{m}_c^2\theta^2\bar{\theta}^2)$$

- HSNJL (2010) — dim 5 four-superfield interaction

$$\mathcal{L}_\psi = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1-\tilde{m}^2\theta^2\bar{\theta}^2) - \int d^2\theta\; \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-(1+B\theta^2)$$

Application to EW Symmetry Breaking :-

- top mode SM   Miransky, W. Bardeen, · · · '89/'90 (Nambu)
  - — infared (quasi-)fixed point (IQFP)   (Pendleton-Ross), Hill, Marciano, · · ·
    prediction : top mass $>$ 200 GeV ; VEV – top condensate

- supersymmetric NJL (formal – '82, SSM – '90)
  - — $m_t = y_t \cdot v$, $m_b = y_b \cdot v'$; NJL predicts y not m ; $y_b < y_t$
  - — other not very nice features as MSSM
  - — lighter top fine, *but* . . .   (172.1 GeV top, tanβ $<$ 1.5)

- our **holomorphic SNJL** (alternative supersymmetrization)
  - — non-chiral symmetric 4-superfield interaction with t and b
  - — superfield condensate : both scalar and fermion condensate
  - — $y_t < y_b$;   nice , experimentally viable (LHC)

Nambu–Jona-Lasinio Model :-

- dynamical symmetry breaking
- four-fermion interaction

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

$$\longrightarrow \mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar{\psi}_+\bar{\psi}_-)$$

$$= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-)$$

- auxiliary scalar field ϕ (no kinetic term)
- EL-eq for $\phi^\dagger$ gives ϕ as composite

$$\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$$

- $\langle\phi\rangle \neq 0 \Longrightarrow$ symmetry breaking and fermion mass

Supersymmetrizing the NJL Model (Naively):-

- $i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \Phi_+^\dagger \Phi_+$
- $g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2 \Phi_+^\dagger \Phi_-^\dagger \Phi_+ \Phi_-$
- $-\mu g \, \phi \psi_+ \psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g \, \Phi \Phi_+ \Phi_-$
- $-\mu^2 \phi^* \phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2} \Phi \Phi$

BUT :-

- $\phi = -g/\mu \, \bar{\psi}_+ \bar{\psi}_-$ implies

$$\mu^2 \phi^* \phi = -\mu g \, \phi \psi_+ \psi_- = g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \qquad \text{(no SUSY !)}$$

- no nontrivial vacuum without SUSY breaking

The Supersymmetric NJL Model :-

Buchmüller & Love 82

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \Phi_+^\dagger\Phi_+ \;\; (1-\tilde{m}^2\theta^2\bar{\theta}^2)$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_- \quad -\longrightarrow \quad \int d^4\theta \;\; \Phi_1^\dagger\Phi_1$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi_2\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \mu\,\Phi_1\Phi_2$

BUT :-

- EL-eq for Φ_2 gives $\Phi_1 = -g\,\Phi_+\Phi_-$ implies

$$\int d^4\theta \;\; \bar{\Phi}_1\Phi_1 = \int d^4\theta \;\; g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-$$

- Φ_2 not the composite Φ_1 plays the Higgs superfield $\quad \langle\Phi_1\rangle = 0$

An Alternative Supersymmetrization ?

Jung, O.K., Lee 2010

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \Phi_+^\dagger\Phi_+ \;\; (1-\tilde{m}^2\theta^2\bar{\theta}^2)$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi_0\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2}\Phi_0\Phi_0$

$$\Longrightarrow \qquad \mathcal{L} = \int d^4\theta \left[(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-)(1-\tilde{m}^2\theta^2\bar{\theta}^2)\right] + \int d^2\theta \left[\frac{\mu}{2}\Phi_0^2 + \sqrt{\mu G}\Phi_0\Phi_+\Phi_-\right] + h.c.$$

- consider superpotential $\quad W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

$$\longrightarrow W - \frac{1}{2}(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)$$

With Holomorphic Four-Chiral Superfield Interaction :-

- $W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$ contains no $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$

- EL-eq for auxiliary superfield Φ_0 gives $\Phi_0 = -\sqrt{G/\mu}\,\Phi_+\Phi_-$

  implies $\quad \frac{\mu}{2}\Phi_0^2 = -\frac{\sqrt{\mu G}}{2}\Phi_0\Phi_+\Phi_- = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

- $\langle\Phi_0\rangle \quad \Longrightarrow \quad \frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ Dirac mass for Φ_+–Φ_-

- kinetic term for Φ_0 from wave-function renormalization

  through Φ_+–Φ_- loop with Yukawa vertices

Towards the MSSM :-

- consider $W = G\,\varepsilon_{\alpha\beta}\,\hat{Q}^{\alpha}\hat{U}^{c}\hat{Q}'^{\beta}\hat{D}^{c}\,(1 + B\theta^2)$

$$W \;\longrightarrow\; W - \mu\,(\hat{H}_d - \lambda_u\hat{Q}\hat{U}^c)(\hat{H}_u - \lambda_d\hat{Q}'\hat{D}^c)(1 + B\theta^2)$$
$$= (-\mu\hat{H}_d\hat{H}_u + y_u\hat{Q}\hat{H}_u\hat{U}^c + y_d\hat{H}_d\hat{Q}'\hat{D}^c)(1 + B\theta^2)$$

- two composites — $\hat{H}_u = \frac{y_d}{\mu}\,\hat{Q}'\hat{D}^c$ and $\hat{H}_d = \frac{y_u}{\mu}\,\hat{Q}\hat{U}^c$

- low energy effective theory looks like MSSM ($A = B$)
- symmetric role for $\hat{H}_u$ and $\hat{H}_d$ $\left(\text{also}:\ \mu\lambda_\mathrm{u}\lambda_\mathrm{d} = \frac{\mathrm{y_u y_d}}{\mu} = \mathrm{G}\right)$

— numerical lifted through non-universal soft masses

— expect $\langle h_u \rangle \gtrsim \langle h_d \rangle$ (Vs UBB in D-flat)

Non-perturbative Analysis of DSB :-

- Dirac mass parameter ($\sim$ Higgs VEV) with SUSY breaking

e.g. Miller 83

$$\mathcal{M} = m - \theta^2 \eta$$

- superfield propagator with (soft) SUSY breaking

Scholl 84, Helayel-Neto 84

- superfield generating functional with SUSY breaking

$$\Gamma = \int \frac{d^4p}{2\pi^4} \int d^2\theta \, \Phi_+(-p,\theta) \, \Gamma^{(2)}_{+-}(p,\theta^2) \, \Phi_-(p,\theta) + h.c. + \cdots$$

$\Longrightarrow$ gap equation : $\qquad -\mathcal{M} = \Sigma^{(loop)}_{+-}(p,\theta^2)\Big|_{\text{on-shell}}$

$(from\ supergraphs)$

New Gap Equation Results (with nontrivial solutions) :-

- SNJL model

$$m = 2mg^2 I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$
$$\eta = -\eta\, g^2 \tilde{m}_C^2\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$

— solution known considering only m, as $\eta = 0$, or $\tilde{m}_C^2 = 0$

Büchmuller & Ellwanger 84

— interesting general solution

- HSNJL model

$$m = \frac{\bar{\eta} G}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)$$
$$\eta = \bar{m} G\, I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) + \frac{\bar{\eta} G B}{2}\, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2)\ .$$

— tightly coupled, cannot see solution when neglecting η part

Majorana Vs Dirac :-

- beyond Dirac mass generation

— HSNJL model has Majorana mass option

- $\frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ Dirac mass for $\Phi_+\Phi_-$
- $\frac{G}{2}\langle\Phi_+\Phi_+\rangle\ \Phi_-\Phi_-$ Majorana mass for $\Phi_-\Phi_-$
  and $\frac{G}{2}\langle\Phi_-\Phi_-\rangle\ \Phi_+\Phi_+$ for $\Phi_+\Phi_+$ mass

- $\mathcal{L} = \int d^4\theta \left[\Phi_+^\dagger \Phi_+(1-\tilde{m}_+^2\,\theta^2\bar{\theta}^2) + \Phi_-^\dagger\Phi_-(1-\tilde{m}_-^2\,\theta^2\bar{\theta}^2)\right]$
  $- \int d^2\theta\big[\frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_- - \mu(H_+ + \lambda_-\Phi_-\Phi_-)(H_- + \lambda_-\Phi_+\Phi_+)\big] + h.c.$

Gap Equations for Majorana Mass Analysis :-

$$m_+ = \frac{\bar{\eta}_- G}{2}\, I_2(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2)$$

$$\eta_+ = \bar{m}_- G\, I_1(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2) + \frac{\bar{\eta}_- G B}{2}\, I_2(|m_-|^2, \tilde{m}_-^2, |\eta_-|, \Lambda^2)$$

$$m_- = \frac{\bar{\eta}_+ G}{2}\, I_2(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2)$$

$$\eta_- = \bar{m}_+ G\, I_1(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2) + \frac{\bar{\eta}_+ G B}{2}\, I_2(|m_+|^2, \tilde{m}_+^2, |\eta_+|, \Lambda^2)$$

- $\tilde{m}_+^2 = \tilde{m}_-^2 \Longrightarrow m_+ = m_-\ , \eta_+ = \eta_-$ same equations as Dirac case

★ completing symmetry breaking/mass generation scenarios

- for $\tilde{m}_-^2 = 0$, no Majorana mass solution $(B = 0)$

Concluding Remarks :-

- **our HSNJL works** $\longrightarrow$

  dynamical symmetry breaking, mass generation

- **may provide more interesting version of MSSM**

- **key to analysis**

— **generating functional with SUSY breaking part**

— **maybe used for spontaneous SUSY breaking**

- **completing Majorana Vs Dirac**

— **splitted soft masses favor Dirac**

— **important for MSSM, also needed for** m_t **Vs** m_b

THANK YOU !