A new idea to search for charged lepton flavor violation using a muonic atom

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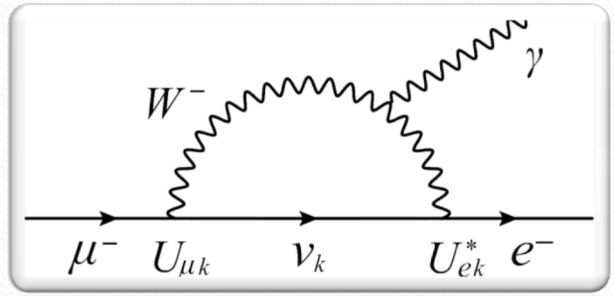
And work in progress

Introduction

Introduction

In Standard Model (SM)

Charged Lepton Flavor Violation (cLFV) via neutrino oscillation



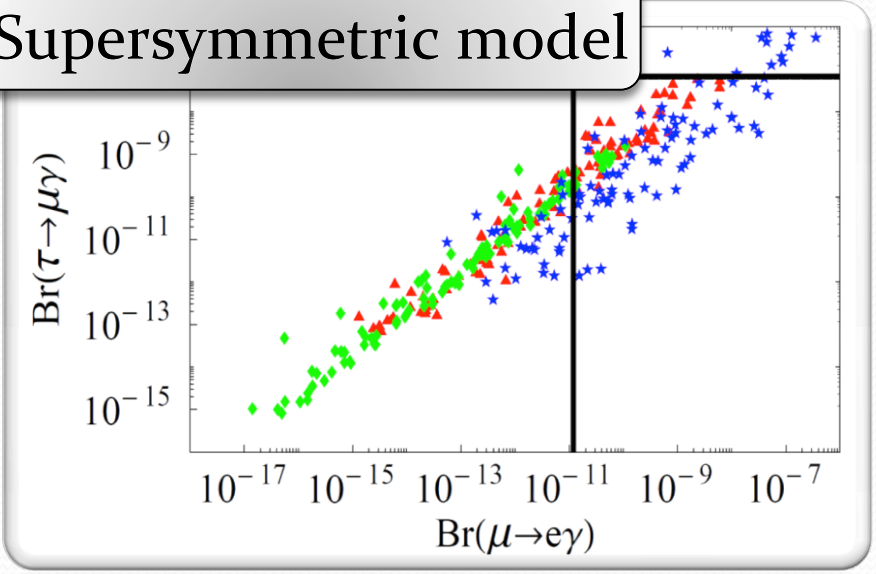
But … $\mathrm{BR}(\mu \to e\gamma) \sim \left(\frac{\delta m_\nu^2}{m_W^2}\right)^2 < 10^{-54}$ ⇨ Forever invisible

Discovery of the cLFV signal

⇨ One of the evidence for beyond the SM

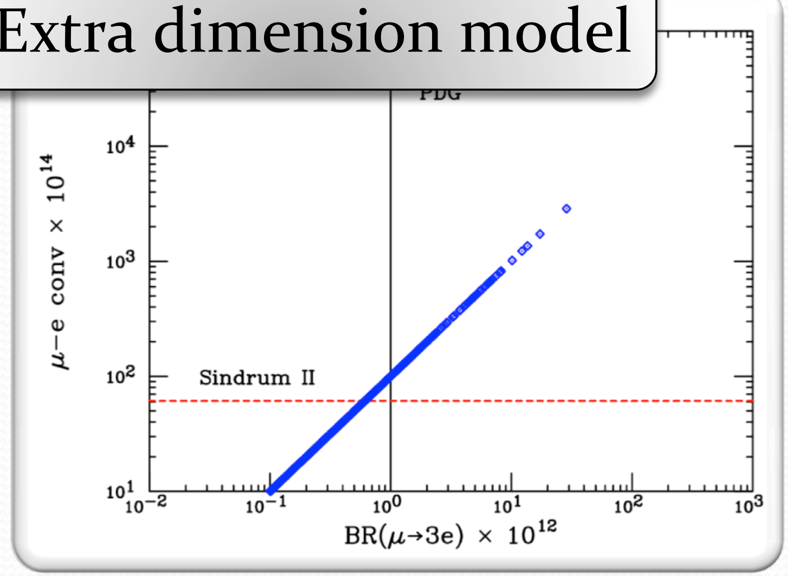
Introduction

Supersymmetric model



[M.~Raidal et al., Eur. Phys. J. C 57 (2008)]

Extra dimension model



[K.~Agashe, et al., Phys. Rev. D 74 (2006)]

Comparing LFV signals and their rates

⇨ Discrimination of new physics models

⇨ Prove for structure of new physics

Desire for many detectable cLFV processes

Introduction

New idea for cLFV search

$\mu^- e^- \longrightarrow e^- e^-$ in muonic atom

- What is target ?

  Flavor violation between μ and e

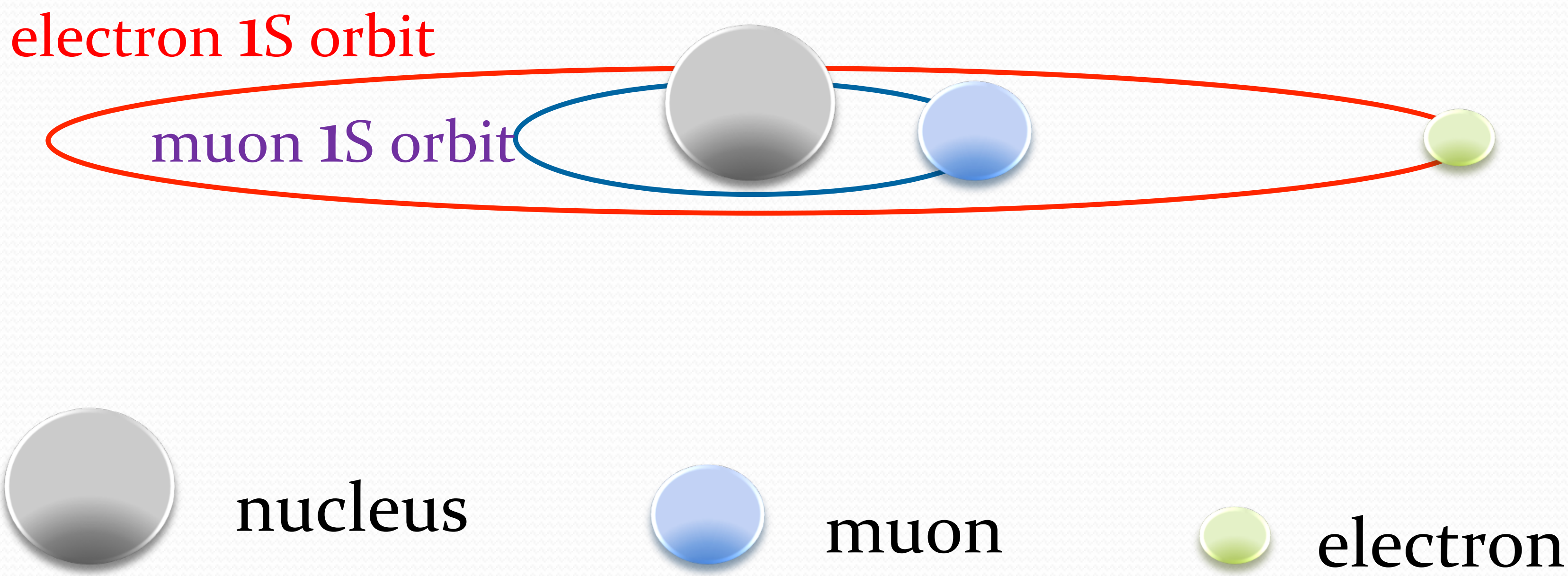
- What is advantage ?

  - Sensitive to both photonic dipole cLFV operator and 4-Fermi contact cLFV operator
  - Clean signal [back-to-back dielectron]

$$\mu^- e^- \longrightarrow e^- e^-$$

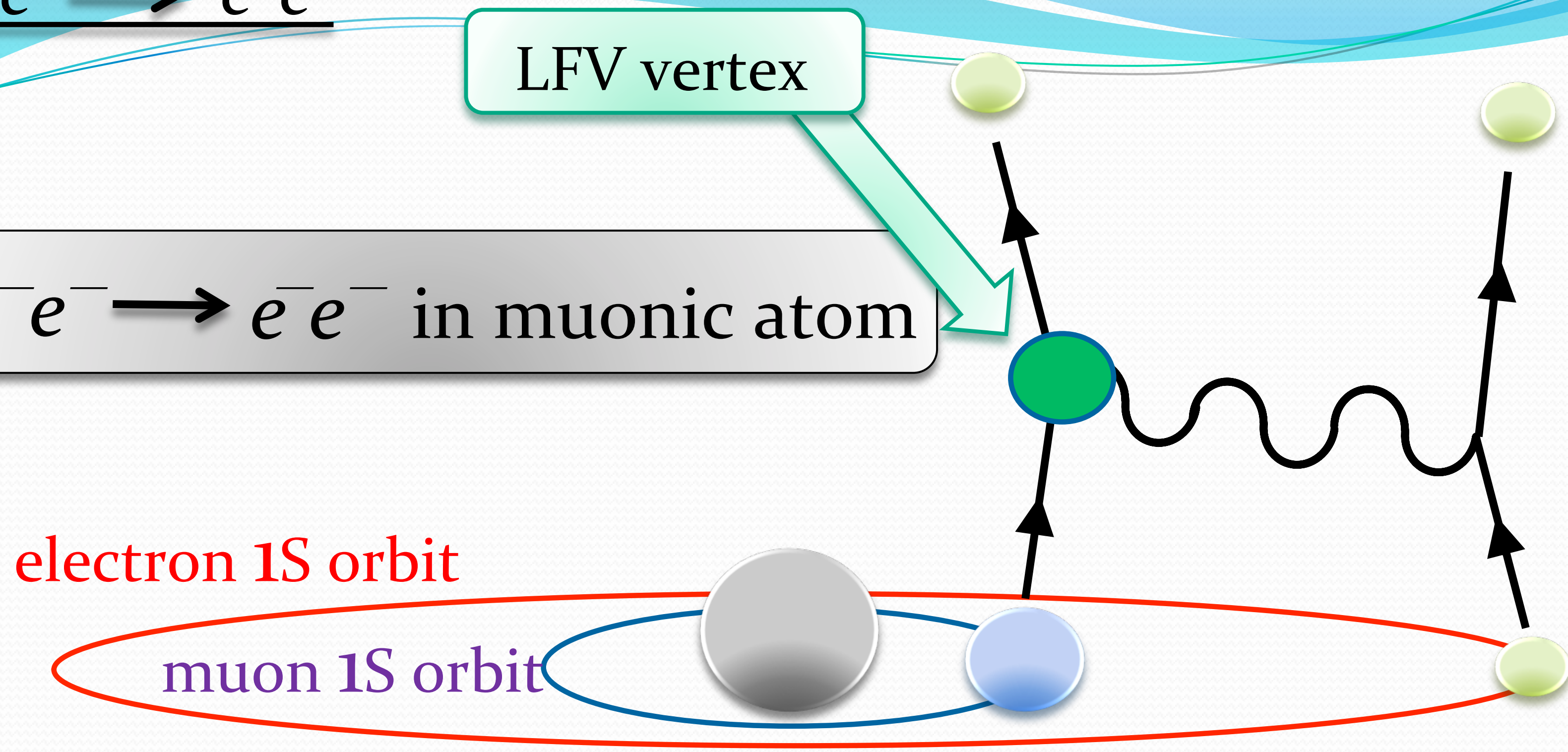
$\mu^- e \rightarrow e e$

Muonic atom



$\mu^- e^- \longrightarrow e^- e^-$

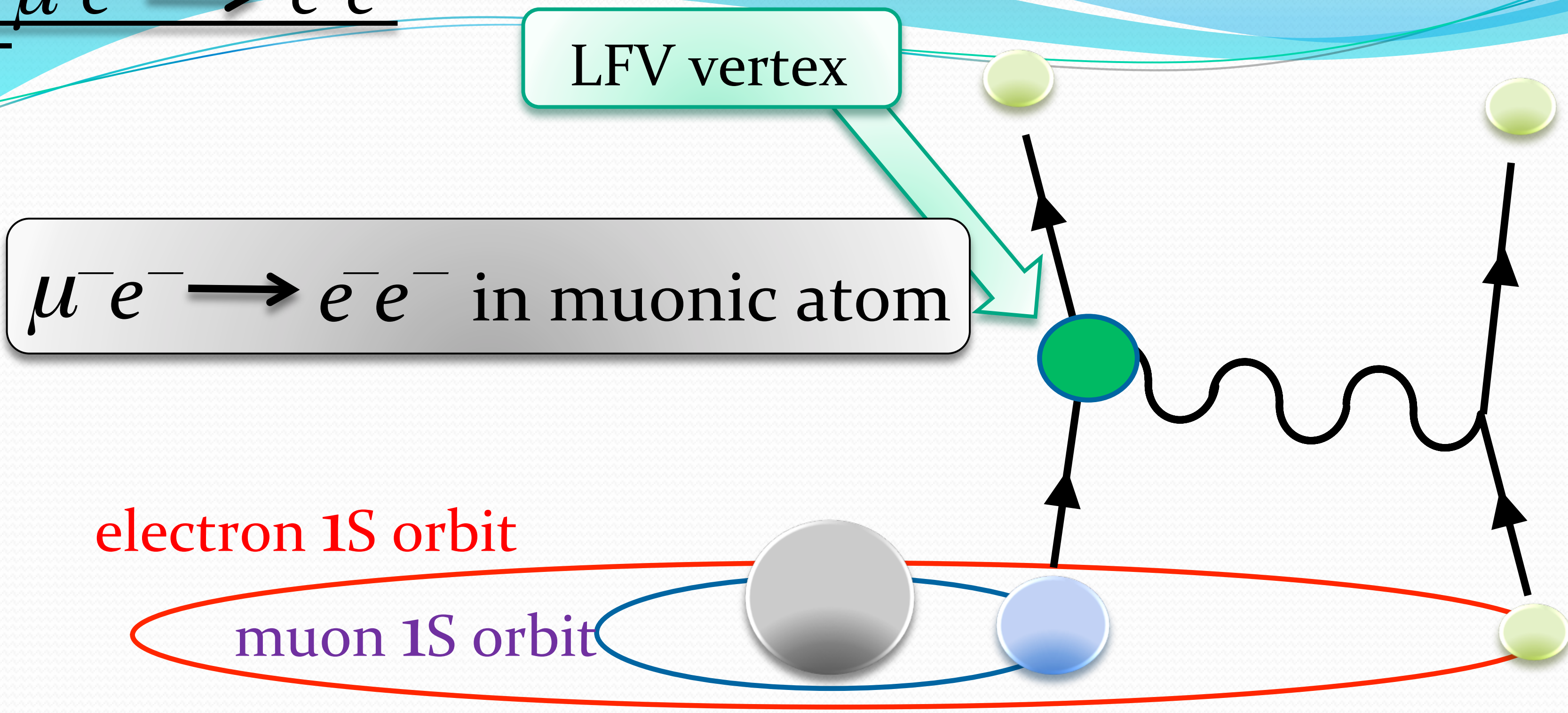
$\mu^- e^- \longrightarrow e^- e^-$ in muonic atom



Interaction rate

$$\Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-; Z) = 2\sigma v_{\mathrm{rel}} \left| \psi_{1\mathrm{S}}^{(\mathrm{e})}(0; Z-1) \right|^2$$

$\mu^- e \longrightarrow e\ e$

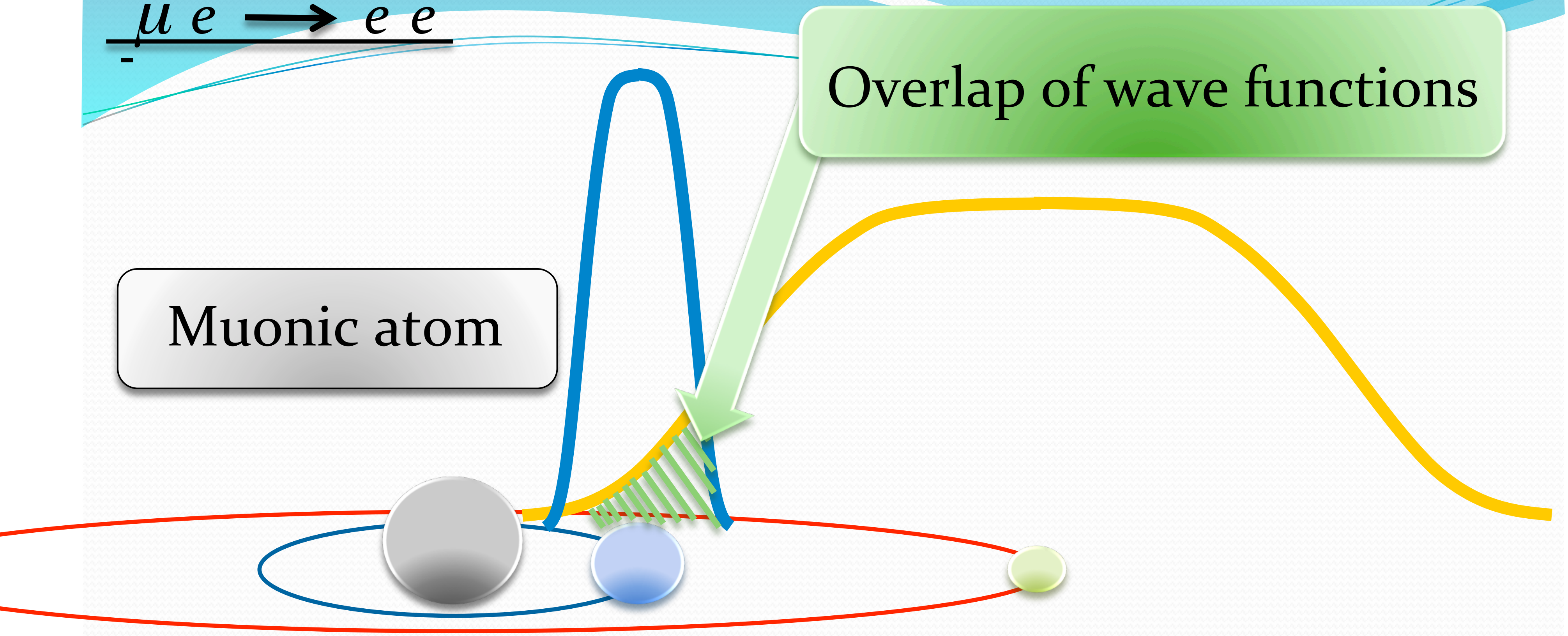


Interaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{rel} \left|\psi_{1S}^{(e)}(0; Z-1)\right|^2$$

Overlap of wave function of μ and e

$\mu^- e \rightarrow e\, e$

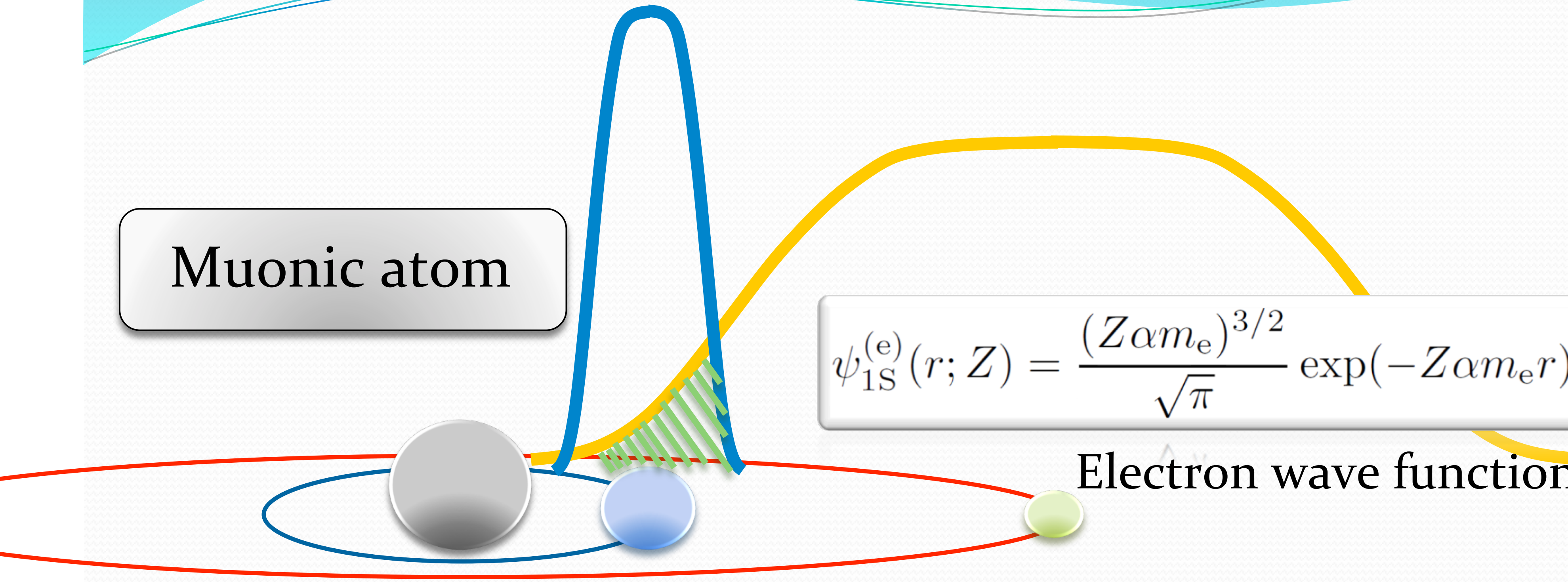


<u>Approximation</u>

Muon localization at nucleus position $\left(\because\ m_e \ll m_\mu \right)$

⇨ Overlap = electron wave function at nucleus

$\mu^- e \rightarrow e\, e$



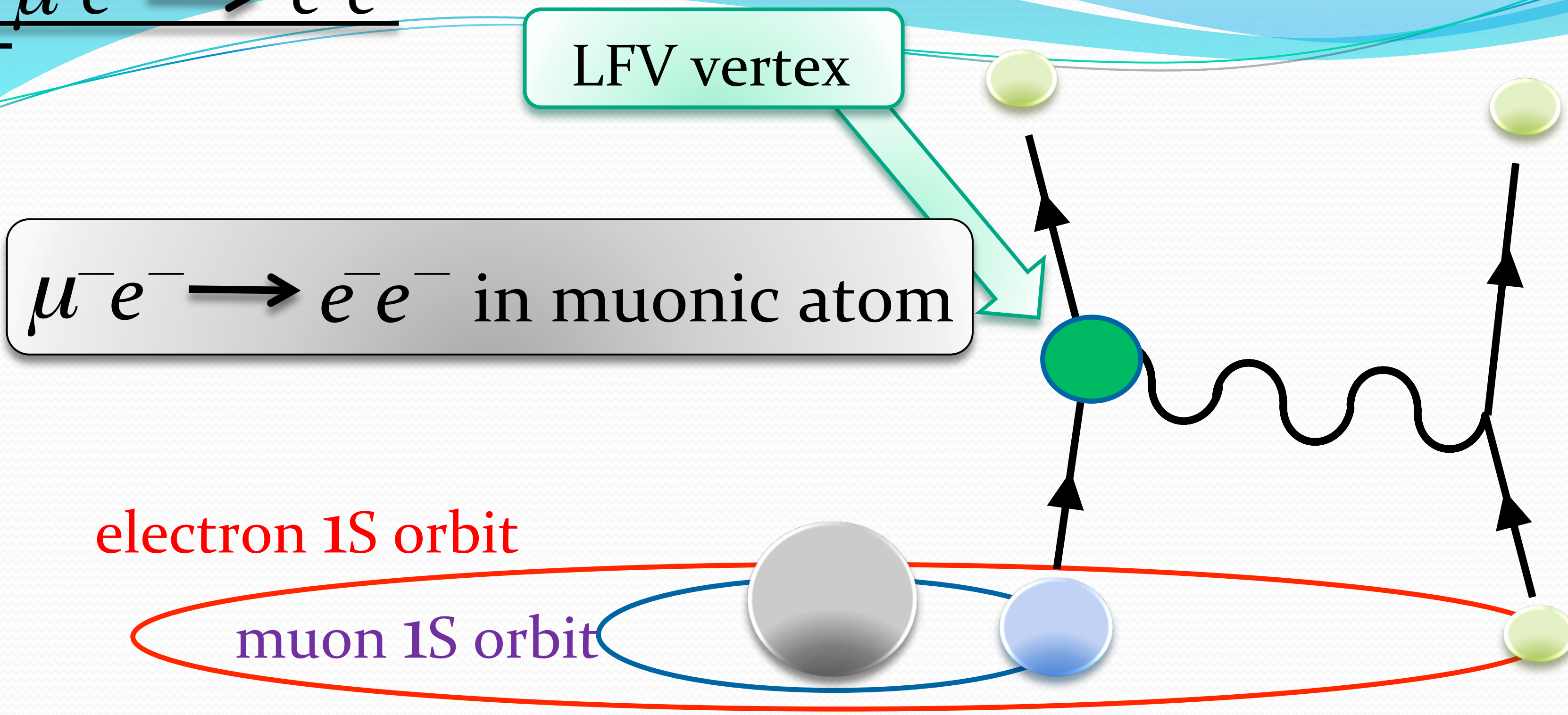
$$\psi^{(\mathrm{e})}_{1\mathrm{S}}(r;Z) = \frac{(Z\alpha m_{\mathrm{e}})^{3/2}}{\sqrt{\pi}}\exp(-Z\alpha m_{\mathrm{e}} r)$$

r : radial coordinate (distance from nucleus)

Z : atomic number of nucleus in muonic atom

Overlap of wave functions $\left|\psi^{(\mathrm{e})}_{1\mathrm{S}}(0;Z-1)\right|^2$

$\mu^- e \longrightarrow e\ e$



Interaction rate

$$\Gamma(\mu^- e^- \to e^- e^-; Z) = 2\sigma v_{\rm rel} |\psi_{1S}^{(e)}(0; Z-1)|^2$$

Cross section for elemental interaction

$\mu^- e \longrightarrow e\, e$

Effective Lagrangian [Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R\, \overline{\mu_R}\sigma^{\mu\nu}e_L F_{\mu\nu} + m_\mu A_L\, \overline{\mu_L}\sigma^{\mu\nu}e_R F_{\mu\nu}$$
$$+ g_1(\overline{\mu_R}e_L)(\overline{e_R}e_L) + g_2(\overline{\mu_L}e_R)(\overline{e_L}e_R)$$
$$+ g_3(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_R}\gamma_\mu e_R) + g_4(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_L}\gamma_\mu e_L)$$
$$+ g_5(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_L}\gamma_\mu e_L) + g_6(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_R}\gamma_\mu e_R) + (\text{H.c.})\Big]$$

cLFV effective coupling constant

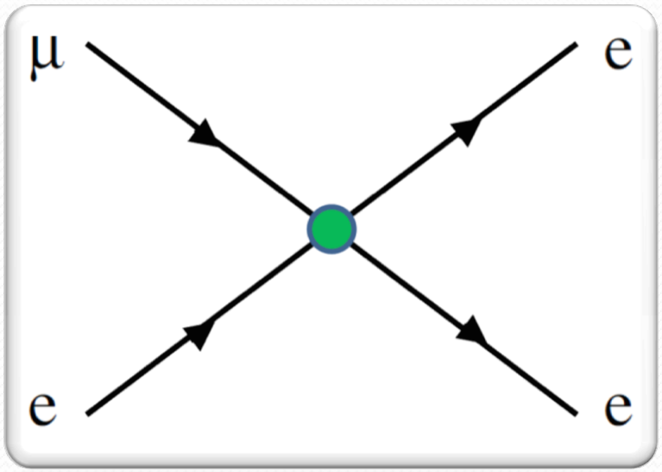
A_R A_L g_1 g_2 g_3 g_4 g_5 g_6

⇨ Sensitive to the structure of new physics

$\mu^- e \longrightarrow e\ e$

Effective Lagrangian [Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R \overline{\mu_R} \sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L \overline{\mu_L} \sigma^{\mu\nu} e_R F_{\mu\nu}$$
$$+ g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R)$$
$$+ g_3 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_R} \gamma_\mu e_R) + g_4 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_L} \gamma_\mu e_L)$$
$$+ g_5 (\overline{\mu_R} \gamma^\mu e_R)(\overline{e_L} \gamma_\mu e_L) + g_6 (\overline{\mu_L} \gamma^\mu e_L)(\overline{e_R} \gamma_\mu e_R) + (\text{H.c.})\Big]$$



4-Fermi interaction type

4-Fermi interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-)$$

$$= 24\pi(Z-1)^3\alpha^3\left(\frac{m_\mathrm{e}}{m_\mu}\right)^3\frac{\tilde{\tau}_\mu}{\tau_\mu}G$$

$$= (3.31\times 10^{-12})(Z-1)^3(\tilde{\tau}_\mu/\tau_\mu)G$$

τ_μ Lifetime of free muon (2.197×10^{-6} s)

$\tilde{\tau}_\mu$ Lifetime of bound muon

- 2.19×10^{-6} s for ^{1}H
- $(7-8)\times 10^{-8}$ s for ^{238}U

$\mu^- e^- \longrightarrow e^- e^-$

4-Fermi interaction dominant case

Branching ratio

$$\begin{aligned}
\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) &\equiv \tilde{\tau}_\mu \Gamma(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-) \\
&= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G \\
&= (3.31 \times 10^{-12})(Z-1)^3 (\tilde{\tau}_\mu / \tau_\mu) G
\end{aligned}$$

$$G \equiv G_{12} + 16 G_{34} + 4 G_{56} + 8 G'_{14} + 8 G'_{23} - 8 G'_{56}$$

$$G_{ij} \equiv |g_i|^2 + |g_j|^2$$

$$G'_{ij} \equiv \mathrm{Re}\left(g_i^* g_j\right)$$

$\mu^- e^- \longrightarrow e^- e^-$

4-Fermi interaction dominant case

Branching ratio ($\mu \longrightarrow e\ e\ e$)

$$\mathrm{Br}(\mu^+ \to \mathrm{e}^+\mathrm{e}^+\mathrm{e}^-) = \frac{1}{8}(G_{12} + 16G_{34} + 8G_{56})$$

Comparison two BRs ⇨ probe for CP violating

$$G \equiv G_{12} + 16G_{34} + 4G_{56} + 8G'_{14} \quad 8G'_{23} - 8G'_{56}$$

$$G_{ij} \equiv |g_i|^2 + |g_j|^2$$

$$G'_{ij} \equiv \mathrm{Re}\,(g_i^* g_j)$$

4-Fermi interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-)$$
$$= 24\pi(Z-1)^3\alpha^3\left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G$$
$$= (3.31\times 10^{-12})(Z-1)^3(\tilde{\tau}_\mu/\tau_\mu)G$$

Enhancement factor from overlap of wave functions

(∵ Positive charge attracts muon and electron toward the nucleus position.)

⇒ Notable advantage for heavy nuclei

$\mu^- e^- \rightarrow e^- e^-$

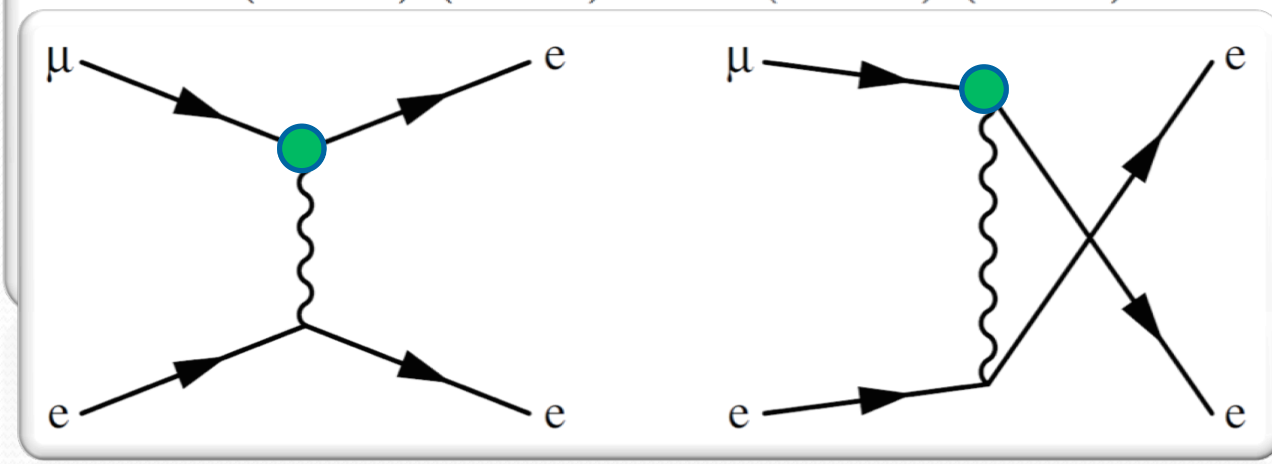
Effective Lagrangian [Y. Kuno and Y. Okada Rev. Mod. Phys. **73** (2001)]

$$\mathcal{L}_{\mu^- e^- \to e^- e^-} = -\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R\, \overline{\mu_R}\sigma^{\mu\nu} e_L F_{\mu\nu} + m_\mu A_L\, \overline{\mu_L}\sigma^{\mu\nu} e_R F_{\mu\nu}$$

$$+ g_1 (\overline{\mu_R} e_L)(\overline{e_R} e_L) + g_2 (\overline{\mu_L} e_R)(\overline{e_L} e_R)$$

$$\ldots {}_\mu e_L)$$

$$\ldots {}_\mu e_R) + (\mathrm{H.c.})\Big]$$



Photonic interaction type

Photonic interaction dominant case

Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) = 1536\pi^2 (Z-1)^3 \alpha^4 \left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right) \frac{m_\mathrm{e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}$$

$$= 2.08 \times 10^{-9} (Z-1)^3 \left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right) (\tilde{\tau}_\mu / \tau_\mu)$$

Photon propagator in non-relativistic limit

$$\frac{1}{q^2} \simeq \frac{1}{m_\mu m_e}$$

Enhancement factor compared with 4-Fermi case

$$\frac{m_\mu^2}{m_e^2}$$

$\mu^{-} e \longrightarrow e\ e$

Case : same order cLFV coupling

$$A_{L(R)} \cong g_i \quad \left(i = 1, 2, \ldots 6 \right)$$

Ratio between photonic and 4-Fermi cross section

$$\sigma v_{\text{photonic}} \sim \alpha \frac{m_\mu^2}{m_e^2} \times \sigma v_{\text{4Fermi}} \sim 10^3 \times \sigma v_{\text{4Fermi}}$$

One of the distinct features for the process

Discovery reach

Discovery reach

How to get upper limit for BR($\mu^- e^- \longrightarrow e^- e^-$)

⇨ Calculate ratio of the BR to other limited cLFV BR

4-Fermi interaction dominant case

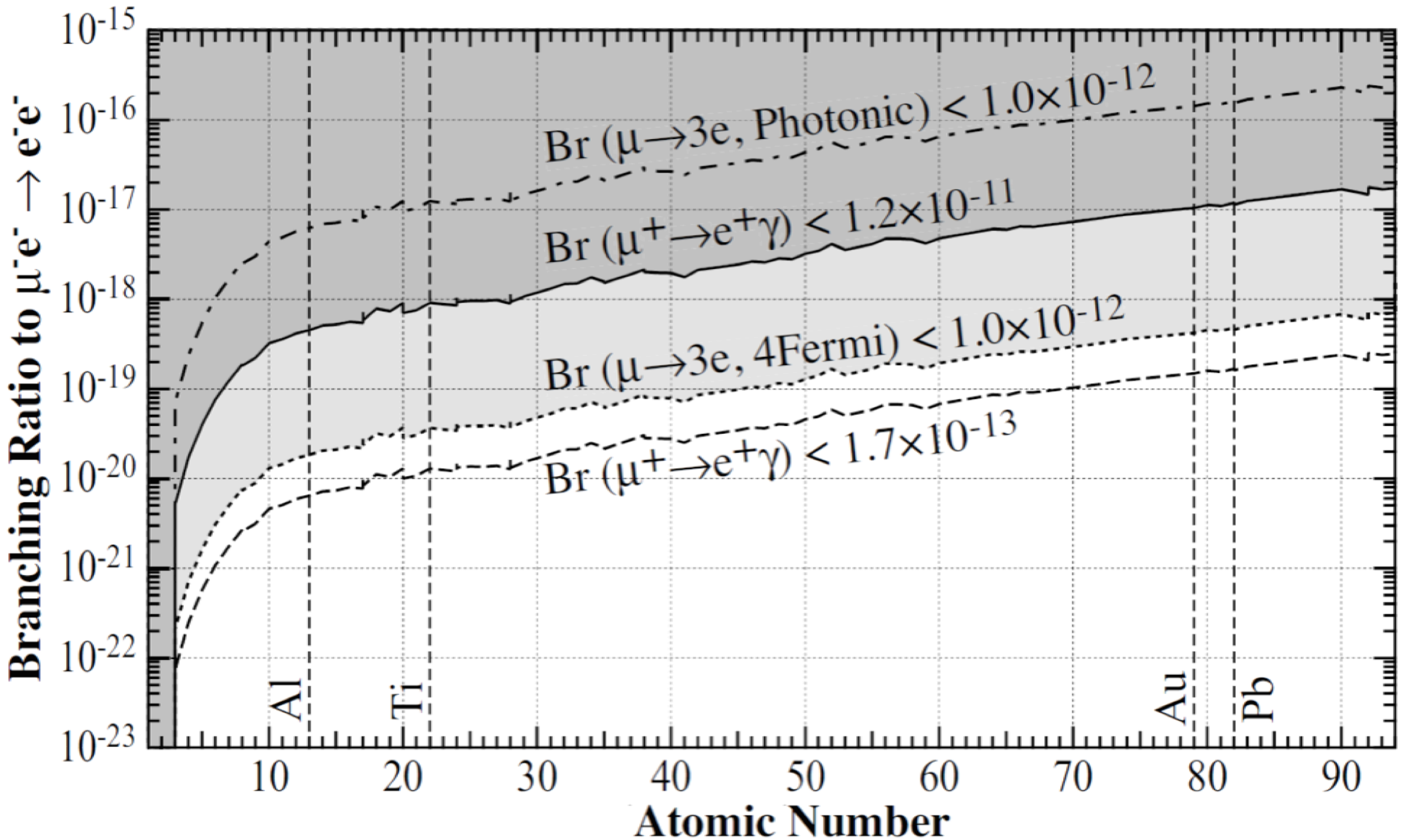
$$\frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ e^+ e^-)}$$

Photonic interaction dominant case

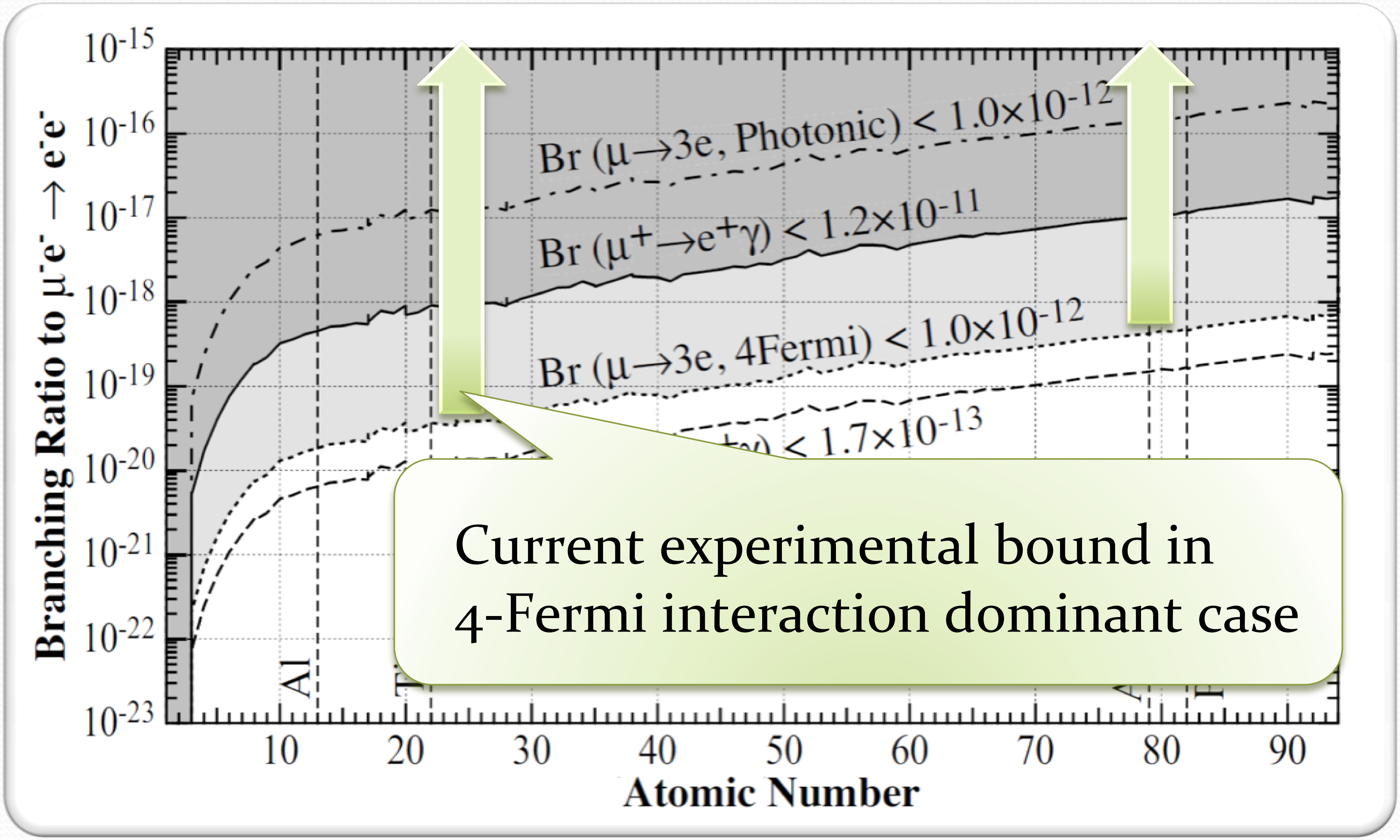
$$\frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ e^+ e^-)} \qquad \frac{\mathrm{Br}(\mu^- e^- \to e^- e^-)}{\mathrm{Br}(\mu^+ \to e^+ \gamma)}$$

[These ratios are independent on cLFV effective coupling]

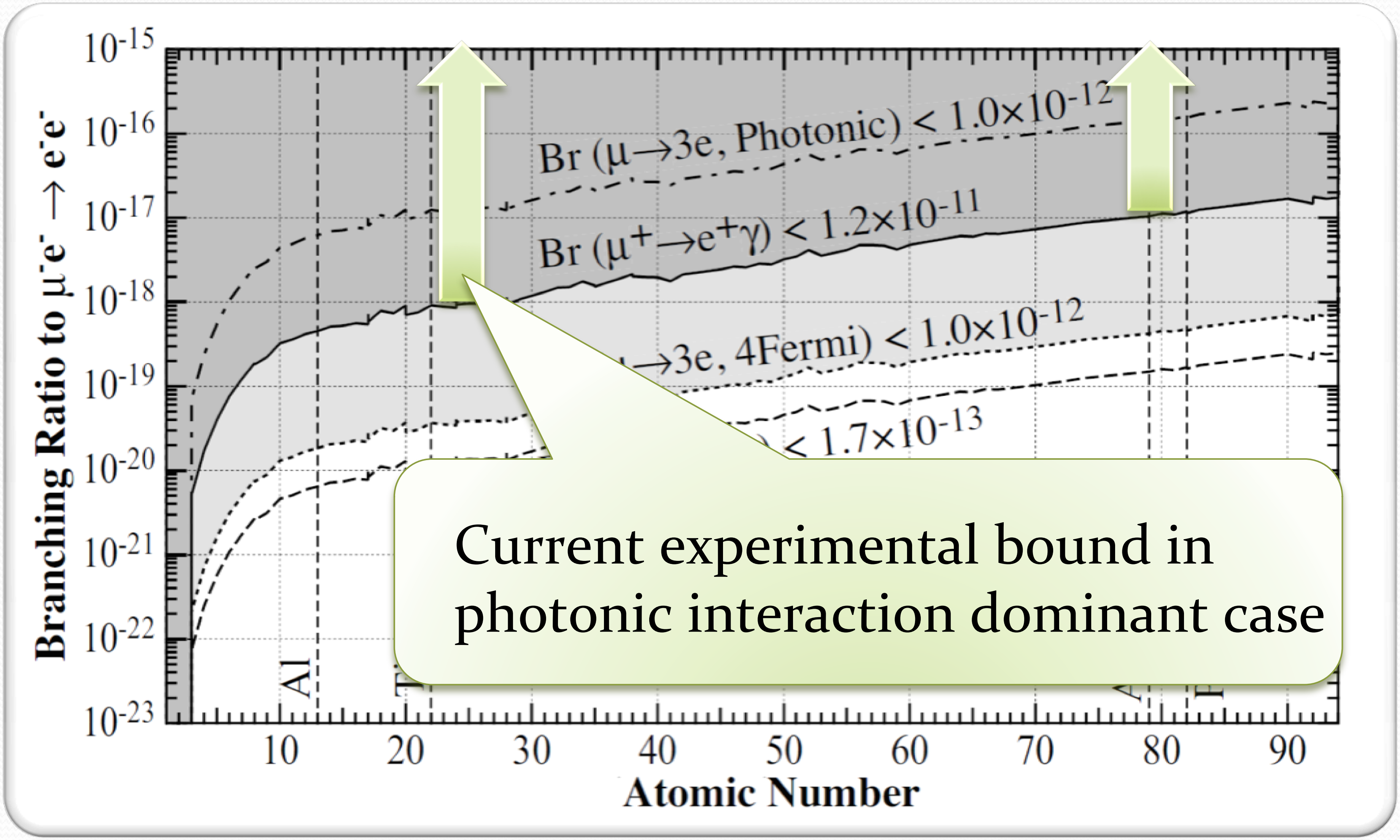
Discovery reach



Discovery reach



Discovery reach



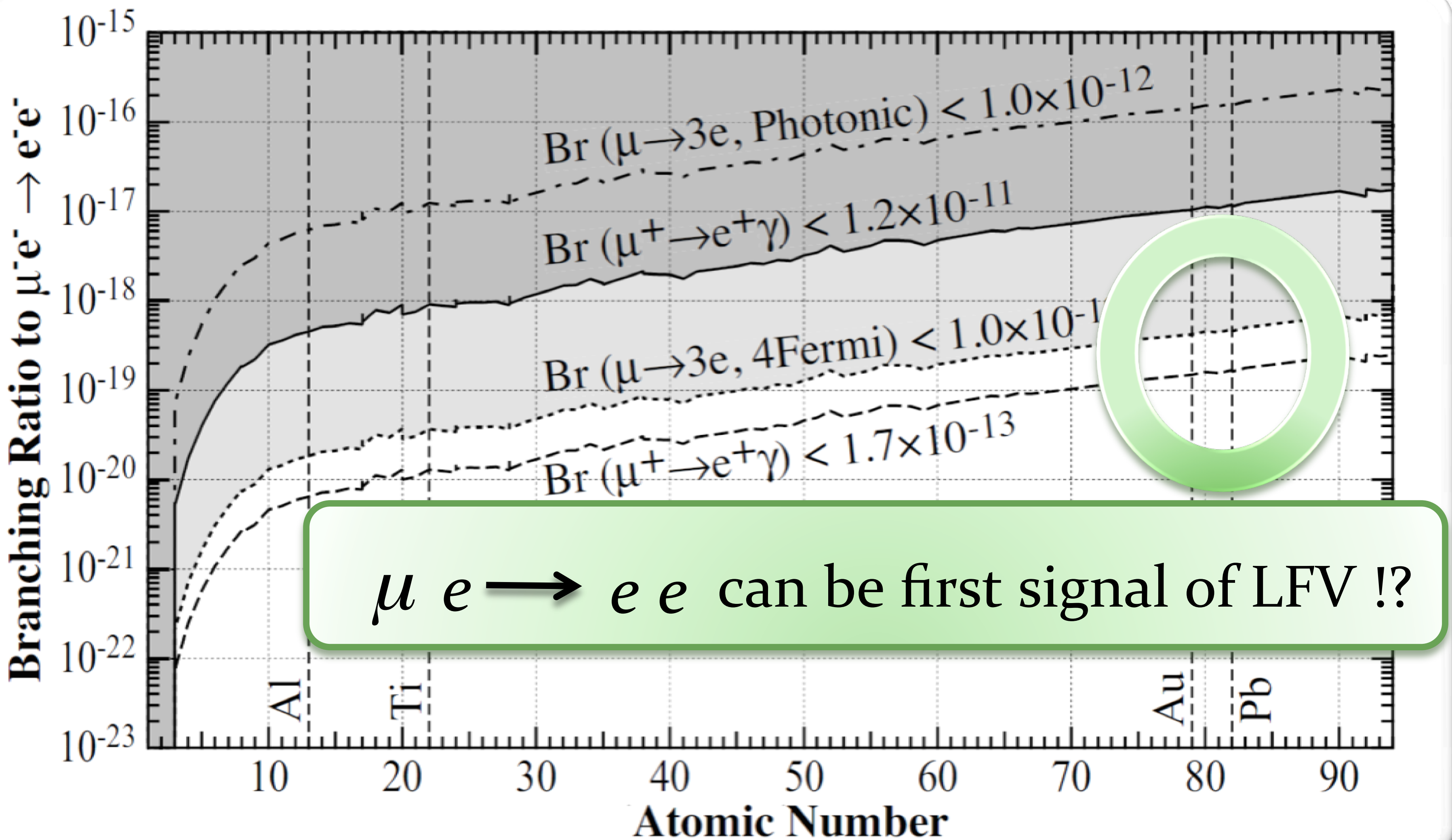
Discovery reach

Collaboration	Searching for	Intensity
MEG	$\mu \to e\gamma$	$10^{7.5}\ \mu/\mathrm{s}$
MUSIC	$\mu \to 3e$	$10^{8}\ \mu/\mathrm{s}$
COMET	$\mu^- N \to e^- N$	$10^{11}\ \mu/\mathrm{s}$
Mu2E (E973)	$\mu^- N \to e^- N$	$10^{11}\ \mu/\mathrm{s}$
PRISM	$\mu^- N \to e^- N$	$10^{12}\ \mu/\mathrm{s}$

For run-time 1 year $\sim 3\times 10^{7}$ s

10^{18} - 10^{19} muon at COMET experiment

Discovery reach



Discovery reach

Project	Intensity Reach
COMET / PRISM	$10^{18} - 10^{19}$ μ /year
ν factories	10^{21} μ /year

With these number of muons the process will be seen !!

3 Towards a Precise Estimate

- Out-going electron is also attracted

  ⟹ Enhancement of the rate

- Bound electron is more concentrated for relativistic wave fn.

  ⟹ Enhancement of the rate

- Nuclei is not a point charge

  ⟹ Solve Dirac Eq. numerically

For trial, uniform charge density is assumed

$$
\begin{aligned}
V(r) &= -\frac{Z\alpha}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right) \quad \text{for} \quad r < R \\
&= -\frac{Z\alpha}{r} \quad \text{for} \quad r > R
\end{aligned}
$$

$$R = 1.2A^{1/3}\text{fm}$$

Enhancement of the rate for contact interaction

Preliminary

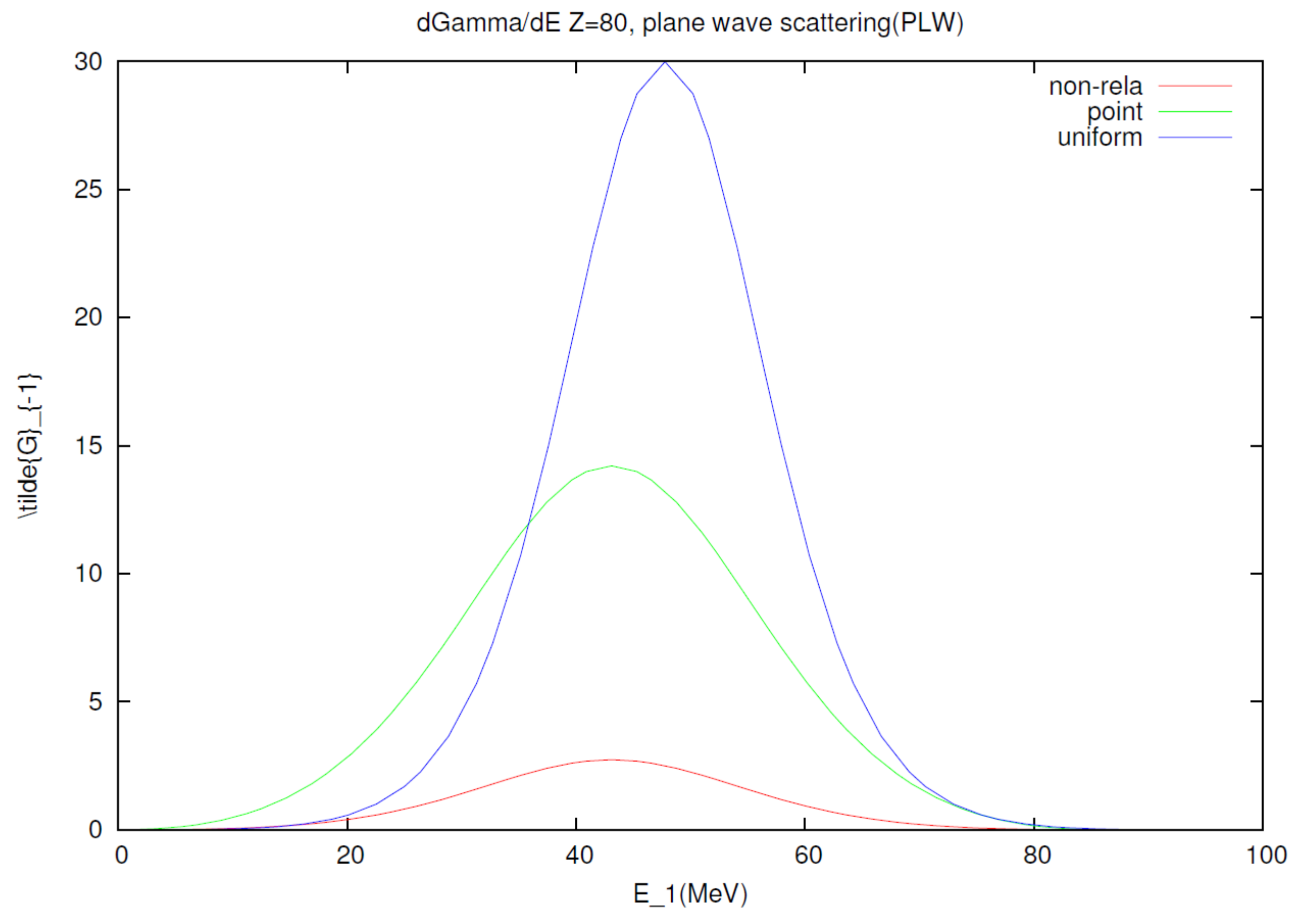
$Z, A = 2Z$	Point colomb	Uniform
40	1.86	1.70
80	16.1	6.62
90	39.1	10.5
100	118	17.4

Coulomb force does not change chirality and hence same factorization for g's holds

Branching ratio can be

$$10^{-17}$$

Energy distribution



Summary

Summary

- New LFV process $\mu^- e^- \longrightarrow e^- e^-$ in muonic atom
- Clean signal (back to back electron with $E_e \simeq m_\mu/2$)
- Interaction rate $\Gamma(\mu^- e^- \to e^- e^-; Z) \sim (Z-1)^3$

⇒ Advantage : Large nucleus

- Detectable in on-going or future experiments

⇒ We wish to observe LFV in the process

Appendix

Discovery reach

Ratio between BR($\mu^- e \longrightarrow e\ e$) and BR($\mu \longrightarrow e\ e\ e$)

$$\frac{\mathrm{Br}(\mu^- \mathrm{e}^- \to \mathrm{e}^- \mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+ \mathrm{e}^+ \mathrm{e}^-)} = 192\pi (Z-1)^3 \alpha^3 \left(\frac{m_\mathrm{e}}{m_\mu}\right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu}$$

Discovery reach

- Photonic interaction dominant case

Branching ratio ($\mu \longrightarrow e\ e\ e$)

$$\mathrm{Br}(\mu^+ \to \mathrm{e}^+\mathrm{e}^+\mathrm{e}^-) = 128\pi\alpha\left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right)\left[\log\left(\frac{m_\mu}{m_\mathrm{e}}\right)^2 - \frac{11}{4}\right]$$

Ratio between BR($\mu^-\ e \longrightarrow e\ e$) and BR($\mu \longrightarrow e\ e\ e$)

$$\frac{\mathrm{Br}(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+\mathrm{e}^+\mathrm{e}^-)} = 12\pi(Z-1)^3\alpha^3\frac{m_\mathrm{e}}{m_\mu}\frac{\tilde{\tau}_\mu}{\tau_\mu}\left[\log\left(\frac{m_\mu}{m_\mathrm{e}}\right)^2 - \frac{11}{4}\right]^{-1}$$

Discovery reach

- Photonic interaction dominant case

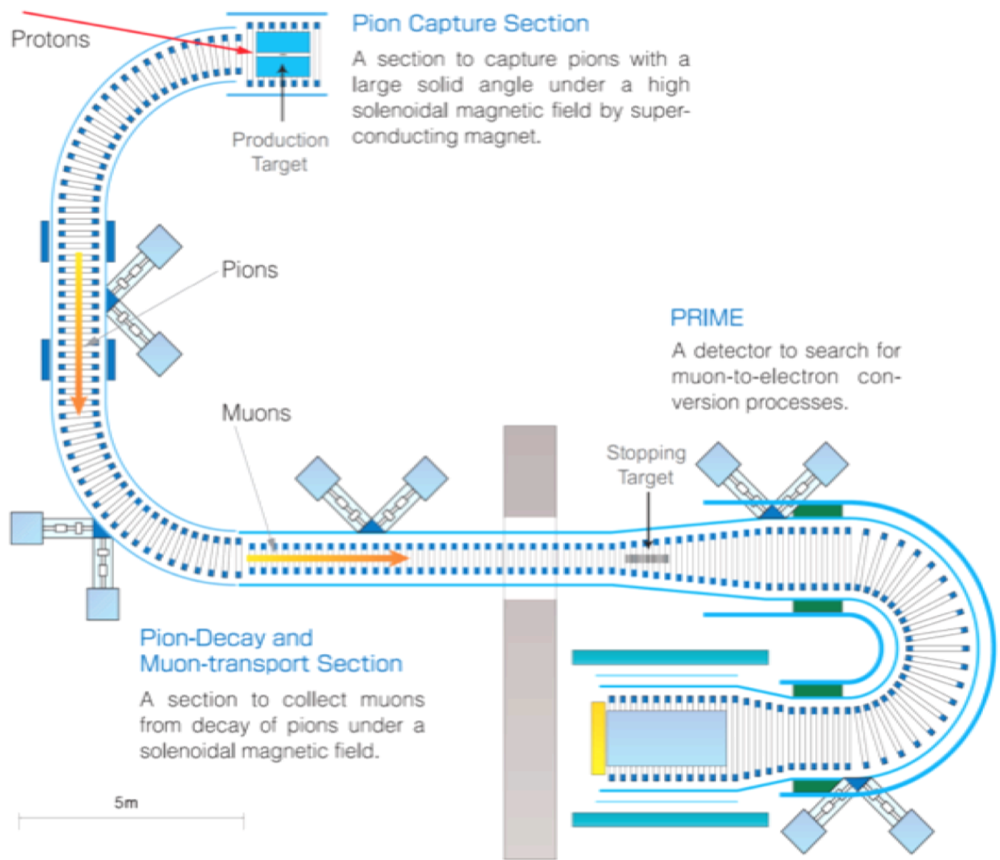
Branching ratio ($\mu \longrightarrow e\,\gamma$)

$$\mathrm{Br}(\mu^+ \to \mathrm{e}^+\gamma) = \frac{\Gamma(\mu^+ \to \mathrm{e}^+\gamma)}{\Gamma(\mu^+ \to \mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\mu;\mathrm{Free})}$$
$$= 384\pi^2\left(|A_\mathrm{R}|^2 + |A_\mathrm{L}|^2\right)$$

Ratio between BR($\mu^- e \longrightarrow e\, e$) and BR($\mu \longrightarrow e\,\gamma$)

$$\frac{\mathrm{Br}(\mu^-\mathrm{e}^- \to \mathrm{e}^-\mathrm{e}^-)}{\mathrm{Br}(\mu^+ \to \mathrm{e}^+\gamma)} = 4(Z-1)^3\alpha^4\frac{m_\mathrm{e}}{m_\mu}\frac{\tilde{\tau}_\mu}{\tau_\mu}$$
$$= 5.49\times 10^{-11}(Z-1)^3\frac{\tilde{\tau}_\mu}{\tau_\mu}$$

COMET (COherent Muon Electron Transition)



$$B(\mu^- + Al \rightarrow e^- + Al) < 10^{-16}$$