On Some Fundamental Aspects of Superstring Theory

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Motivation

- Superstring theory is presently the only quantum mechanically consistent theory of gravity which allows perturbative computations.
- Superstring theory is a compelling candidate for unification of the fundamental particles and forces.
- Over the last 40 years, there have been various applications of superstring theory in diverse areas of physics and mathematics.

Some Fundamental Questions

To be able to make experimental predictions from superstring theory, there are some fundamental aspects of the theory which may need to be better understood.

1) What is the appropriate worldsheet description of the superstring?
2) What is the role of spacetime supersymmetry?
3) What is the non-perturbative "definition" of superstring theory?

- The conventional Ramond-Neveu-Schwarz (RNS) worldsheet description of the superstring is inappropriate for describing general backgrounds and is inconvenient for studying the role of supersymmetry.

- The Green-Schwarz (GS) description has certain advantages and recent developments using pure spinor quantization have allowed the efficient computation of scattering amplitudes and a better understanding of supersymmetry.

- Gauge theories can be defined nonperturbatively through the functional integral

$$Z = \int DA \exp[-\frac{1}{g^2} Tr(F^{mn} F_{mn})]$$

- Although a non-perturbative definition of superstring theory is still lacking, there are some clues from AdS-CFT that it is related to the four-dimensional gauge theory N=4 super-Yang-Mills.
- Super-twistors are a very useful tool for describing N=4 super-Yang-Mills, and also appear in pure spinor quantization of the superstring.

Outline

1) RNS description of superstring
2) D=10 superspace and GS description
3) Pure spinor quantization and amplitude computations
4) Reduction to D=4 and super-twistors
5) AdS-CFT conjecture and N=4 super-Yang-Mills

6) New ICTP South American Institute in Sao Paulo

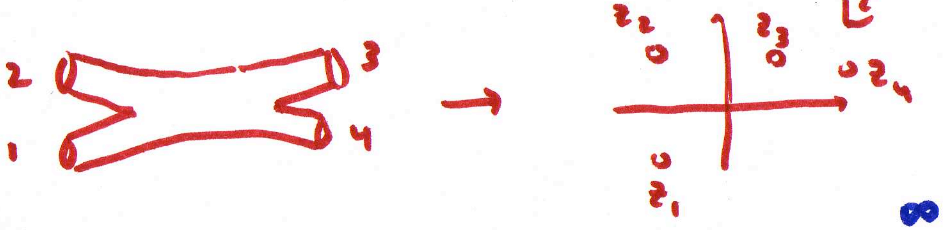
Bosonic string theory is described by worldsheet variables X^m $m=0$ to 25 and ghost variables (b,c) and $(\bar{b},\bar{c})$ from fixing conformal gauge.

$$S = \int d^2z \left(\partial X^m \bar{\partial} X_m + b \bar{\partial} c + \bar{b} \partial \bar{c} \right)$$

Physical states V must preserve conformal inv ⇒ $QV = 0$, $V \cong V + Q\Lambda$

$Q = \int dz \left(c\, \partial X \cdot \partial X + bc\partial c \right)$ is nilpotent BRST operator.

Scattering amplitudes are computed by mapping d=2 worldsheet to complex plane and computing d=2 correlation functions.



$$A_{g=0} = \int DX\, Db\, Dc\; V_1(z_1) V_2(z_2) V_3(z_3) V_4(z_4) e^{-S}$$

Perturbative amplitude: $A = \sum_{g=0}^{\infty} (\lambda_s)^{2g-2} A_g$

λ_s = string coupling constant
g = genus of worldsheet

Bosonic string theory has gravitons and gauge fields, but also has tachyons.

Scattering amplitudes have divergences associated with tachyons and massless tadpoles

⇒ Need to replace bosonic string theory with superstring theory.

Superstring theory is conventionally described by worldsheet variables

$\underbrace{X^m, \psi^m, \bar{\psi}^m}_{\text{fermions}}$ m=0 to 9 and ghosts $(\underbrace{b, c}_{\text{fermions}}, \underbrace{\beta, \gamma}_{\text{bosons}})$ and $(\bar{b}, \bar{c}, \bar{\beta}, \bar{\gamma})$

coming from gauge-fixing superconf. inv (Ramond; Neveu-Schwarz, '71; Friedan-Martinec-Shenker '85)

$$S = \int d^2z \left(\partial x^m \bar{\partial} x_m + \psi^m \bar{\partial} \psi_m + \bar{\psi}^m \partial \bar{\psi}_m + b \bar{\partial} c + \beta \bar{\partial} \gamma + \bar{b} \partial \bar{c} + \bar{\beta} \partial \bar{\gamma} \right)$$

Physical states must preserve superconf. inv ⇒ $QV=0$, $V \simeq V + Q\Lambda$

$$Q = \int dz \left(c(\partial X \cdot \partial X + \psi \cdot \partial \psi) + \gamma \psi \cdot \partial X + \gamma^2 b + c \beta \partial \gamma + b c \partial c \right)$$

$X^m(\sigma+2\pi) = X^m(\sigma)$

$\psi^m(\sigma+2\pi) = \pm \psi^m(\sigma)$

$\bar{\psi}^m(\sigma+2\pi) = \pm \bar{\psi}^m(\sigma)$

⇒ 4 different sectors of superstring

NS-NS $\psi^m(\sigma+2\pi) = -\psi^m(\sigma)$, $\bar{\psi}^m(\sigma+2\pi) = -\bar{\psi}^m(\sigma)$

NS-R " $-$ " " $+$ "

R-NS " $+$ " " $-$ "

R-R " $+$ " " $+$ "

States in NS-NS and R-R sectors are spacetime bosons

States in NS-R and R-NS sectors are spacetime fermions

} After performing projection, four sectors combine into d=10 multiplets of spacetime supersymmetry (Gliozzi, Scherk, Olive, '77)

Although spectrum and amplitudes are $d=10$ supersymmetric, cancellations due to spacetime susy are very hard to study using RNS worldsheet description.

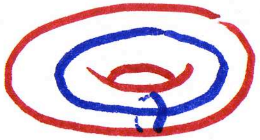
1) States in NS-NS sector (such as the graviton) are easy to describe with (x^m, ψ^m),

e.g. $V = g_{mn} \int d^2z \, (\partial x^m + \psi^m (k\cdot\psi))(\bar\partial x^n + \bar\psi^n (k\cdot\bar\psi)) e^{ik\cdot x}$

But states in other 3 sectors require bosonization of ψ^m and fermionization of (β,γ) ghosts in order to construct spacetime spinors.

$\Rightarrow$ Not known how to describe Ramond-Ramond backgrounds (such as $AdS_5 \times S^5$) using RNS worldsheet description of superstring.

2) Scattering amplitudes require summing over 2^{2g} choices of periodicity conditions for $2g$ non-trivial cycles of genus g surface



Amplitude is free of divergences only after performing sum over 2^{2g} surfaces.

Non-renormalization theorems implied by $d=10$ susy are difficult to verify using RNS description (D'Hoker, Phong '04; Witten '12)

To describe systems with $d=10$ susy, convenient to use variables

(x^m, θ^α) $\alpha = 1$ to 16 for $N=1$ $d=10$ susy $\quad \delta\theta^\alpha = \epsilon^\alpha,\ \delta x^m = i\epsilon\gamma^m\theta$

$(x^m, \theta^\alpha, \bar\theta^\alpha)$ $\alpha = 1$ to 16 for $N=2$ $d=10$ susy. $\quad \delta\theta^\alpha = \epsilon^\alpha,\ \delta\bar\theta^\alpha = \bar\epsilon^\alpha,\ \delta x^m = i\epsilon\gamma^m\theta + i\bar\epsilon\gamma^m\bar\theta$

In the 1980's, Green + Schwarz developed a new worldsheet description of superstring with manifest $d=10$ supersymmetry

$$\mathcal{S} = \int d^2z \left(\Pi^m \bar\Pi_m + i\partial x\cdot(\theta\gamma\bar\partial\theta - \bar\theta\gamma\bar\partial\bar\theta) - i\bar\partial x\cdot(\theta\gamma\partial\theta - \bar\theta\gamma\partial\bar\theta)\right) \quad \Pi^m = \partial x^m + i\theta\gamma^m\partial\theta - i\bar\theta\gamma^m\partial\bar\theta$$

Although $\mathcal{S}$ is not quadratic, it has a local fermionic symmetry which can be gauge-fixed to light-cone gauge where

$$\mathcal{S}_{LC} = \int d^2z \left(\partial x^j \bar\partial x^j + \theta^a \bar\partial\theta^a + \bar\theta^a \partial\bar\theta^a\right) \quad \begin{matrix} j = 1 \text{ to } 8 \\ a = 1 \text{ to } 8 \end{matrix}$$

Spacetime susy $\Rightarrow$ $\theta^\alpha(\sigma+2\pi) = \theta^\alpha(\sigma)$ and $\bar\theta^\alpha(\sigma+2\pi) = \bar\theta^\alpha(\sigma)$ $\Rightarrow$ no need to sum over 2^{2g} surfaces.

Green + Schwarz easily proved that one-loop amplitudes have no divergences, but computations at higher loops was complicated by lack of $SO(9,1)$ Lorentz inv.

Covariant quantization of GS description was open problem until 2000.

Local fermionic GS symmetry generated by $d_\alpha = p_\alpha - (\gamma^m \theta)_\alpha \partial X_m$ $\quad p_\alpha = \frac{\partial L}{\partial(\dot{\theta}^\alpha)}$

Since $\{\partial_\alpha, \partial_\beta\} = \gamma^m_{\alpha\beta} \Pi_m$, have both first and second-class constraints.

Covariant quantization achieved by introducing bosonic ghosts λ^α (NB, '00) such that $\lambda \gamma^m \lambda = 0$ "pure spinor". λ^α has 11 independent components.

$$S = \int d^2z \, (\partial x^m \bar{\partial} X_m + \underbrace{p_\alpha \bar{\partial} \theta^\alpha + \bar{p}_\alpha \partial \bar{\theta}^\alpha}_{\text{fermions}} + \underbrace{w_\alpha \bar{\partial} \lambda^\alpha + \bar{w}_\alpha \partial \bar{\lambda}^\alpha}_{\text{bosons}})$$

No conformal anomaly since $c_x + c_\theta + c_\lambda = 10 - 32 + 22 = 0$.

Physical states V must preserve fermionic symmetry $\Rightarrow QV = 0$, $V \cong V + Q\Lambda$ where $Q = \int dz \, \lambda^\alpha d_\alpha$ is the nilpotent BRST operator.

Massless open: $V = \lambda^\alpha e^{ik\cdot x} A_\alpha(\theta) = \lambda^\alpha e^{ik\cdot x} (\underset{\substack{\uparrow \\ \text{gluon}}}{a_m} (\gamma^m \theta)_\alpha + (\underset{\substack{\uparrow \\ \text{gluino}}}{\chi} \gamma_m \theta)(\gamma^m \theta)_\alpha + \ldots)$

Massless closed: $V = \lambda^\alpha \bar{\lambda}^\beta e^{ik\cdot x} A_{\alpha\beta}(\theta, \bar{\theta}) = \lambda^\alpha \bar{\lambda}^\beta e^{ik\cdot x} (\underset{\substack{\uparrow \\ \text{graviton}}}{g_{mn}} (\gamma^m \theta)_\alpha (\gamma^n \bar{\theta})_\beta + \underset{\substack{\uparrow \\ \text{R-R field strength}}}{F^{\gamma\delta}} (\gamma_m \theta)_\alpha (\gamma^m \theta)_\gamma (\gamma_n \bar{\theta})_\beta (\gamma^n \bar{\theta})_\delta + \ldots)$

Scattering amplitudes are computed by the functional integral

$$a_g = \int d\tau^{6g-6} \int DX \int D\theta \int D\bar{\theta} \int Dp \int D\bar{p} \int D\lambda \int D\bar{\lambda} \int Dw \int D\bar{w} \; e^{-S} \int V_1 \int V_2 \int V_3 \int V_4 \left(\int b \int \bar{b}\right)^{3g-3}$$

All variables are periodic $\Rightarrow$ no sum over surfaces with different structures

On genus g surface, θ^α and $\bar{\theta}^\alpha$ each have 16 zero modes and

p_α and $\bar{p}_\alpha$ each have $16g$ zero modes.

$\int d\tau^{6g-6}$ parametrizes moduli space of genus g surfaces.

$\left(\int b \int \bar{b}\right)^{3g-3}$ provides correct measure factor where $\{Q, b\} = T$ is stress-tensor.

Using that a_g vanishes unless all fermionic zero modes are absorbed by vertex operators or $\left(\int b \int \bar{b}\right)^{3g-3}$, can easily prove following non-renorm. theorems: (NB, '06)

1) $a_g = 0$ if there are 3 or fewer external massless states.

2) Terms in effective action $\partial^L R^4$ for $L < 8$ only get contributions when genus $g \le L/2$ (e.g. $\partial^4 R^4$ does not get contributions for $g > 2$).

Coefficient of 2-loop 4-pt amplitude has been explicitly computed (Gomez+Mafra, '10)

After compactification on T^6 to four dimensions, the massless states of the open superstring describe $\mathcal{N}=4$ super-Yang-Mills which is a consistent $\mathcal{N}=4$ $d=4$ superconformally invariant theory, with 1 gluon, 4 gluinos and 6 scalars.

(The massless states of the closed superstring describe $\mathcal{N}=8$ supergravity which is <u>not</u> expected to be a consistent theory.)

Although one can use $\mathcal{N}=4$ $d=4$ superspace variables $(x^m, \theta^{\alpha j}, \bar\theta^{\dot\alpha}_j)$ $\begin{matrix} m=0 \text{ to } 3 \\ \alpha,\dot\alpha = 1 \text{ to } 2 \\ j = 1 \text{ to } 4 \end{matrix}$

to describe $\mathcal{N}=4$ super-YM as a superfield $V = \lambda^{\alpha j} A_{\alpha j}(x,\theta,\bar\theta) + \bar\lambda^{\dot\alpha}_j A^j_{\dot\alpha}(x,\theta,\bar\theta)$

satisfying $QV=0$, $V \simeq V + Q\Lambda$, there exists a simpler description using "supertwistor" variables $(\underbrace{\lambda^\alpha, \mu^{\dot\alpha}}_{\text{bosons}}, \underset{\text{fermions}}{\underset{\uparrow}{\eta^j}}) \equiv Z^J$ $J = (\alpha, \dot\alpha, j)$. (Ferber, '77)

Supertwistor variables transform <u>linearly</u> under the superconformal group $PSU(2,2|4)$ as $Z^J \to M^J_K Z^K$ and are related to $\mathcal{N}=4$ superspace variables by

$$\mu^{\dot\alpha} = \sigma_m^{\alpha\dot\alpha} x^m \lambda_\alpha, \quad \eta^j = \theta^{\alpha j} \lambda_\alpha$$

Note that $(x^m, \theta^{\alpha j})$ is unchanged under rescaling $Z^J \to c Z^J$.

Onshell super-YM states are described by superfields $\Phi(Z^J)$

$$\Phi(\mu,\lambda,\eta) = \underset{\substack{\uparrow \\ \text{antiselfdual} \\ \text{gluon}}}{a_+}(\mu,\lambda) + \eta^j \underset{\substack{\uparrow \\ \text{4 antichiral} \\ \text{gluinos}}}{\bar{\chi}_j}(\mu,\lambda) + \eta^j \eta^k \underset{\substack{\uparrow \\ \text{6 scalars}}}{\varphi_{jk}}(\mu,\lambda) + \epsilon_{jklm} \eta^j \eta^k \eta^l \underset{\substack{\uparrow \\ \text{4 chiral gluinos}}}{\chi^m}(\mu,\lambda) + (\eta)^4 \underset{\substack{\uparrow \\ \text{selfdual gluon}}}{a_-}(\mu,\lambda)$$

Scattering amplitudes are much easier to express using supertwistors, e.g.

$$a = \int d^8\theta \prod_{r=1}^{N} \frac{\Phi_r(\eta_r)}{\epsilon_{\alpha\beta} \lambda^\alpha_r \lambda^\beta_{r+1}}$$

describes "MHV" tree amplitudes involving 2 antiselfdual and N selfdual gluons.

Although can use standard Feynman diagrams to compute sYM amplitudes, there now exist twistor methods which are much more efficient (Britto et al '05 - '12, Arkani-Hamed et al, Cachazo et al)

Tree-level amplitudes can be computed using "twistor-string theory"

$$S = \int d^2z \left(Y_J \bar{\partial} Z^J + \bar{Y}_J \partial \bar{Z}^J \right)$$

Witten '04
NB '04

Is twistor string theory related to superstring theory?

In 1997, Maldacena conjectured equivalence of the superstring in an $AdS_5 \times S^5$ Ramond-Ramond background and $\mathcal{N}=4$ $d=4$ super-YM with gauge group $U(N)$

$$g_{string} = g^2_{YM} \quad , \quad r_{AdS} = (g^2_{YM} N)^{1/4}$$

If correct, conjecture implies that nonperturbative string theory (with asymptotic AdS_5 boundary) can be <u>defined</u> by $\mathcal{N}=4$ $d=4$ super-YM.

When $g_{string} << 1$ and $r_{AdS} >> 1$, R-R background is weakly curved and superstring can be replaced by supergravity approximation.

New information about strongly coupled super-YM ($g^2_{YM} N >> 1$) has been obtained from conjecture (e.g. integrability, Wilson line/amplitude duality, quark-antiquark potential, ...).

But to compare superstring and twistor string, need to understand regime $g_{string} << 1$ and $r_{AdS} << 1$ where R-R background is strongly curved and super-YM is weakly coupled ($g^2_{YM} N << 1$).

In this regime, R-R background is strong ⇒ cannot use RNS description.

Using pure spinor quantization of GS $AdS_5 \times S^5$ action,

$$\mathcal{S} = \int d^2z \; r^2_{AdS} \left[J^m \bar{J}^m + (\gamma^{01234})_{\alpha\hat{\beta}} \left(J^\alpha \bar{J}^{\hat{\beta}} - \frac{1}{4} \bar{J}^\alpha J^{\hat{\beta}} \right) + w_\alpha \bar{\nabla} \lambda^\alpha + \bar{w}_\alpha \nabla \bar{\lambda}^\alpha \right]$$

is a complicated non-linear action constructed from PSU(2,2|4)-invariant Metsaev-Tseytlin currents $J^m = (g^{-1}\partial g)^m$, $J^\alpha = (g^{-1}\partial g)^\alpha$, $J^{\hat{\alpha}} = (g^{-1}\partial g)^{\hat{\alpha}}$.

where $g(x, \theta, \bar{\theta})$ takes values in $PSU(2,2|4) / SO(4,1) \times SO(5)$.

Some indications that $\mathcal{S}$ can be expressed as "topological" action in limit that $r^2_{AdS} \to 0$ which is related to twistor-string action.

(NB + C. Vafa, '08)

If understood, this might lead to a perturbative proof of Maldacena's conjecture.

<u>More ambitious goal</u>: Understand how $AdS_5 \times S^5$ / super-YM string theory is related to d=11 M-theory and other non-perturbative regimes.

ICTP South American Institute for Fundamental Research (ICTP-SAIFR)

Location: IFT-UNESP, Sao Paulo

Acting director: Nathan Berkovits
Acting vice-director: Rogerio Rosenfeld
Executive Secretary: Nadia Roque
Computer Systems Manager: Danilo Ramos
Financial Manager: Lilia Faria

Motivation

- Theoretical physics institutes play an important role in bringing researchers together and reducing the amount of bureaucracy involved in inviting visitors and organizing activities.
- There are dozens of such institutes in North America, Europe, and Asia in both developed and developing countries.
- South America has a few small theoretical physics institutes (CECS in Valdivia, CBPF in Rio de Janeiro, CEFIMAS in Buenos Aires, IIP in Natal), but each of them have disadvantages.

Why IFT-UNESP in Sao Paulo?

- IFT-UNESP is one of the oldest and most prestigious graduate schools in theoretical physics in South America.
- The Sao Paulo state research funding agency FAPESP is the most stable and well-funded agency in South America.
- IFT-UNESP is centrally located next to a major bus/train station near the largest South American airport and close to several large universities.

Connection with ICTP

- The ICTP in Trieste has supported theoretical physics in developing countries for over 45 years.
- With the rapid growth of countries like Brasil, the ICTP has decided to create regional ICTP centers which can more efficiently support less-developed countries of the region.
- In addition to providing organizational expertise, the ICTP in Trieste will also provide funding for visitors to the center from other South American countries.
- The ICTP is planning to open soon a Central American regional center (ICTP-MAIS) in Chiapas, Mexico with Director Arnulfo Zepeda.

Short history of ICTP-SAIFR

March 2010 - ICTP management suggests opening of South American regional center at IFT-UNESP in Sao Paulo.

August 2010 - Rector of UNESP gives enthusiastic suport.

November 2010 - Memorandum of Understanding is signed between ICTP and UNESP.

June 2011 - First meeting of Steering Committee is held. Acting director and scientific council are appointed and 2012 activities are decided.

August 2011 - Three ICTP-SAIFR secretaries are hired.

September 2011 - International seach committee for first 2 permanent positions is formed.

November 2011 - 5-year budget is appoved by FAPESP.

January 2012 - First 2-week school is organized in Mathematical Biology

February 2012 - Joint Steering Committee/Scientific Council meeting and Opening Ceremony are held. 2013 activities are decided.

ICTP SAIFR International Centre for Theoretical Physics South American Institute for Fundamental Research

Ceremony of Inauguration of ICTP-SAIFR on February 6, 2012

(left to right)

Carlos Brito Cruz (Fapesp scientific director)
Juan Maldacena (representing South America)
Fernando Quevedo (steering council chair and ICTP director)
Julio Cezar Durigan (UNESP rector)
Juan Montero (IFT director)
Jacob Palis (Brazil Academy of Science president)
Peter Goddard (scientific council chair and IAS director)

Contributions to Budget

UNESP

5 permanent research professor positions

3 secretaries

Infrastructure of IFT-UNESP

FAPESP (5-year projeto tematico)

9 postdoctoral positions

180 months of visitors

2-week schools for PhD students

ICTP

Visitors from other South American countries

Councils

Steering committee

Fernando Quevedo (chair)-ICTP director
Julio Cezar Durigan - UNESP acting rector
Carlos Brito Cruz - FAPESP scientific director
Jacob Palis - Brazilian Academy of Science president
Juan Maldacena – representing South America

Scientific council

Peter Goddard (chair) - IAS Princeton director
Seifallah Randjbar-Daemi - ICTP vice-director
Juan Montero - IFT-UNESP director
Marcela Carena - Fermilab, Batavia
Marcel Clerc - Univ. de Chile, Santiago
Luiz Davidovich - UFRJ, Rio de Janeiro
Daniel Sudarsky - UNAM, Mexico City
Matias Zaldarriaga - IAS, Princeton
Anthony Zee - Univ. of California, Santa Barbara
Barton Zwiebach - MIT, Cambridge

2012 ICTP-SAIFR activities

São Paulo International Schools on Theoretical Physics
(2-week PhD schools on specific research topic)

- Southern-Summer School on Mathematical Biology
  January 16-29, 2012 – Organizers: M. Clerc, R. Kraenkel

- Advanced School in General Relativity: Relativistic Astrophysics and Cosmology
  July 16-27, 2012 – Organizers: G. Matsas, D. Vanzella

- Symbolic Computation in Theoretical Physics: Integrability and super-Yang-Mills
  November 5-16, 2012 – Organizers: N. Berkovits, N. Gromov, P. Vieira

Workshops

Mini-workshop Perspectives in Non-Perturbative QCD, May 7-8, 2012

Joint IIP-ICTP Workshop on Gravity and String Theory, May 8-9, 2012

IX Simposio Latinoamericano de Fisica de Alta Energia (SILAFAE), Dec. 10-14, 2012

2013 ICTP-SAIFR activities

São Paulo International Schools on Theoretical Physics
(2-week PhD schools on specific research topic)

- January 2013: 2nd Mathematical Biology school
- March 2013: LHC physics school
- May 2013: Nonperturbative QCD school
- August 2013: Quantum gravity school
- October 2013: Astrophysics school

Workshops

Quantum Gravity in the Southern Cone, September 2013

Workshop on Higher-Spin Theories, November 2013

Participation in Activities

- All applications are online and are judged by the selection committee of the activity.
- Sao Paulo International Schools are typically two weeks and involve 4-5 lecturers and 60 students. Travel and local expenses are paid for students from cities outside Sao Paulo.
- Workshops are typically 2-5 days and local expenses are paid for participants.
- Proposals for 2014 activites can be submitted online and will be judged in February 2013.

Permanent Job Applications

Applications for permanent research professor positions are online and are evaluated by an international search committee. There is no application deadline but the committee is already evaluating applications for the first two positions.

International Search Committee:

Peter Goddard (IAS) – committee chairman

Marcela Carena (Fermilab)

David Gross (KITP)

Leo Kadanoff (Chicago)

Martin Rees (Cambridge)

Robert Wald (Chicago)

Edward Witten (IAS)

Matias Zaldarriaga (IAS)

Postdoctoral and visitor applications

- Applications for postdocs and for short-term and long-term visits are also online. There is no application deadline, and several postdoctoral positions are currently available.
- The ICTP-SAIFR has signed visiting agreements with Perimeter Institute and the CERN Theory Group which allows ICTP-SAIFR members to visit these institutions for up to 4 weeks/year.

- All online applications are done through the webpage

www.ictp-saifr.org

and any questions can be sent to

secretary@ictp-saifr.org