Hunting for de Sitter String Vacua

Gary Shiu

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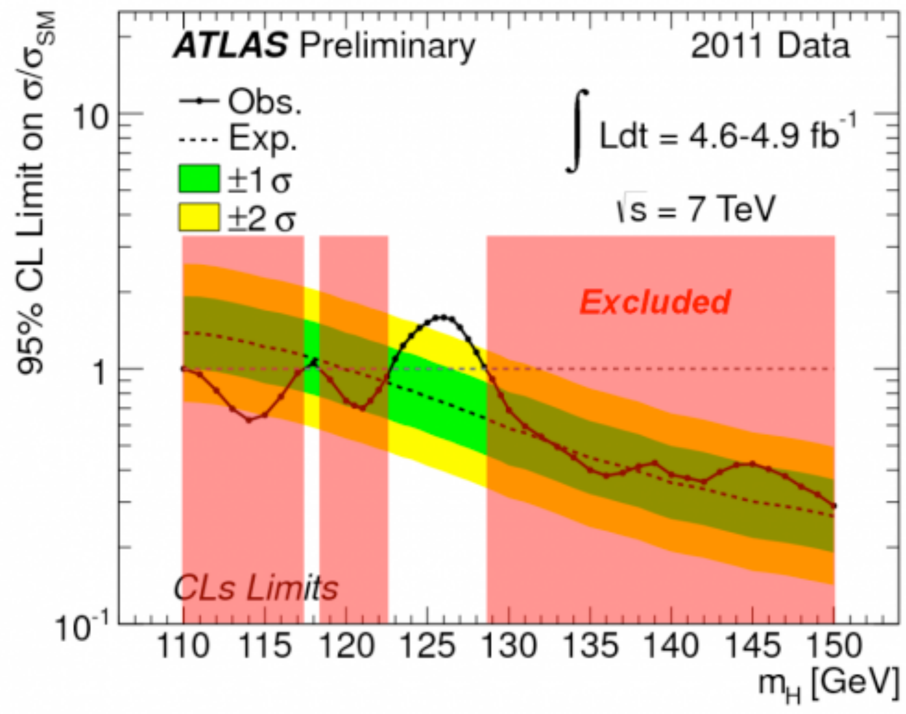
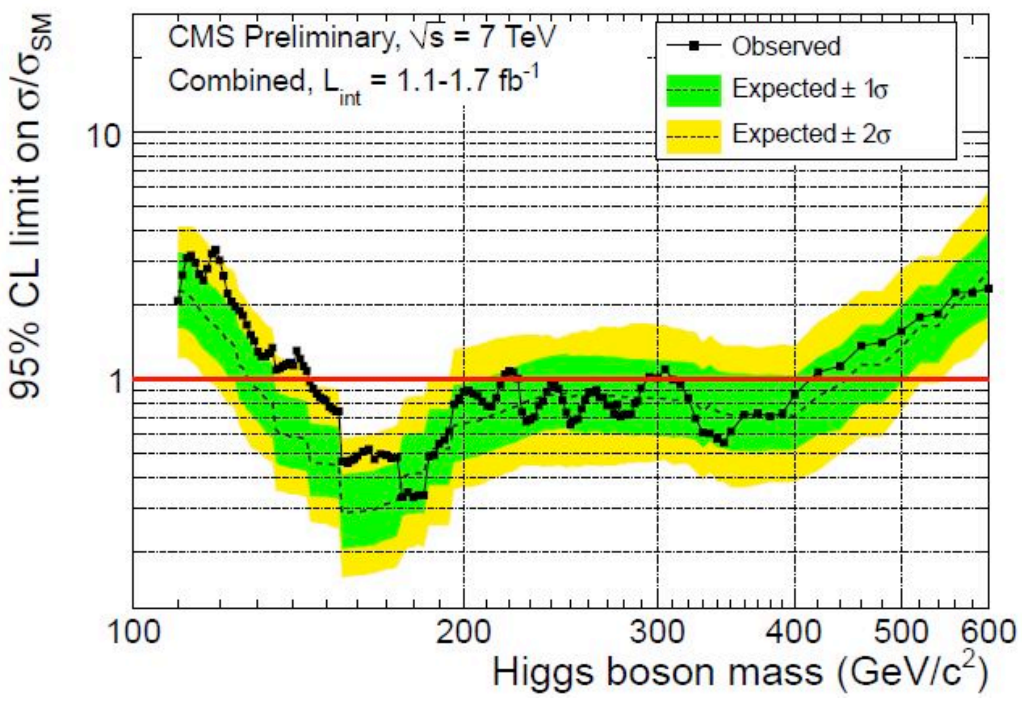
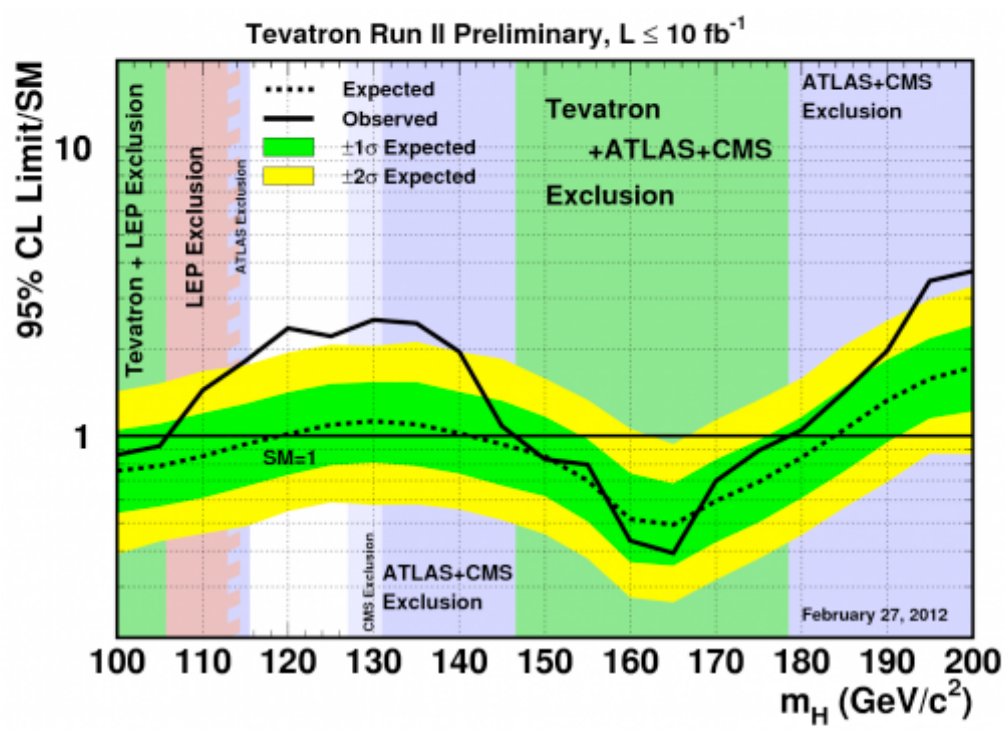
Hunting for the Higgs





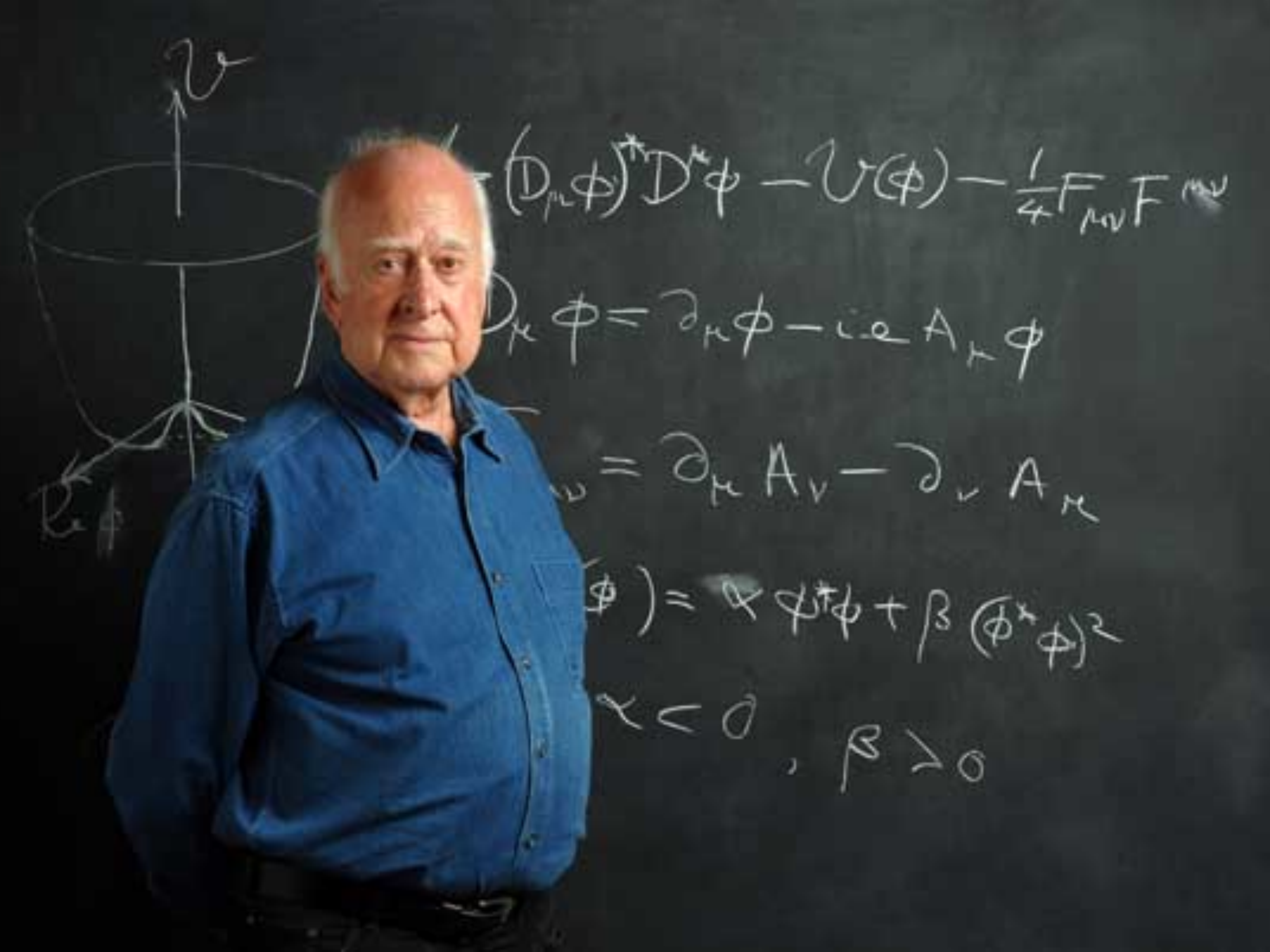


String theory landscape?

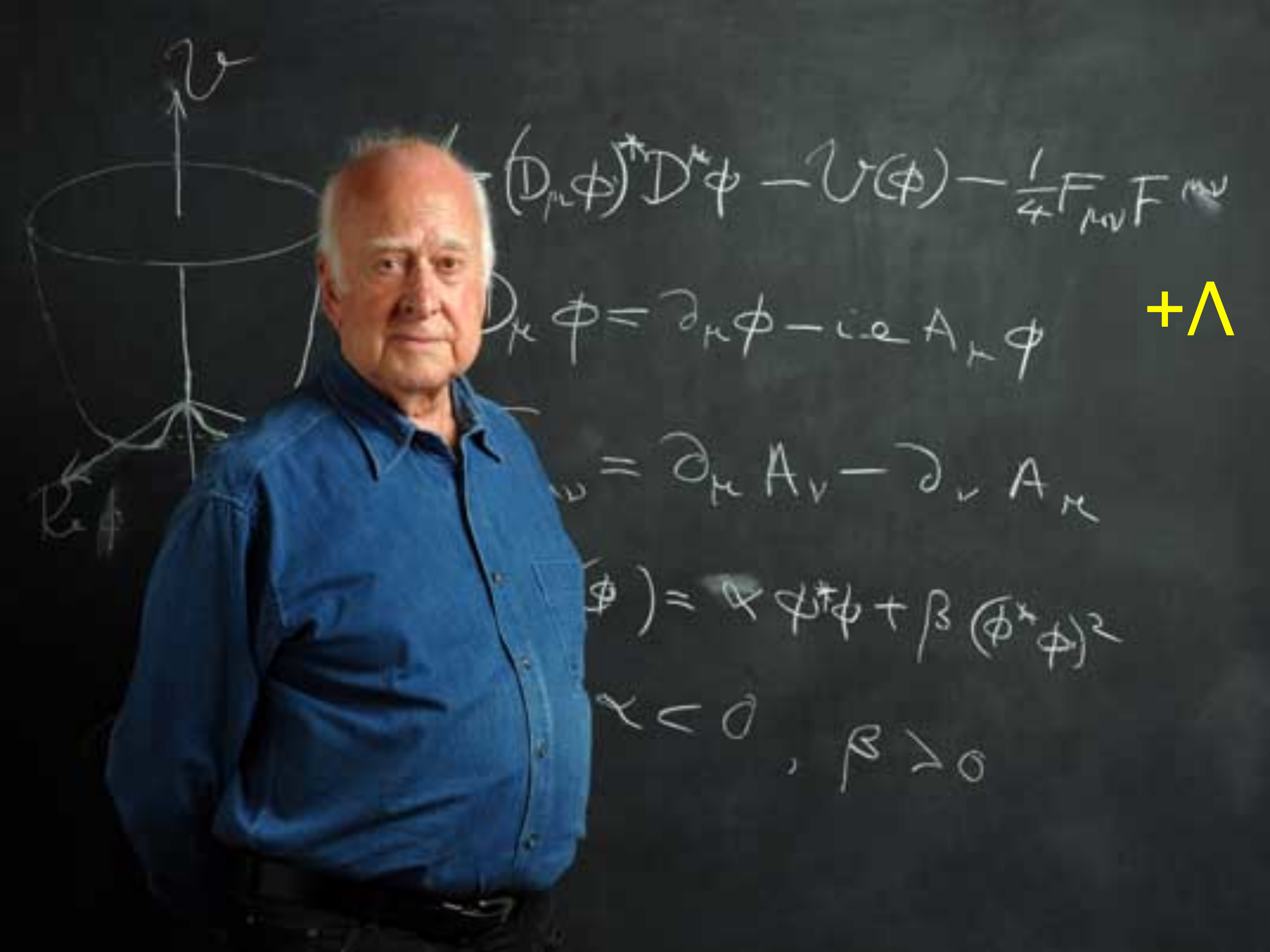
... seems so 5BC

String theory landscape?

... seems so 5BLHC



$(D_\mu \phi)^* D^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$\phi = \partial_\mu \phi - ie A_\mu \phi$
$= \partial_\mu A_\nu - \partial_\nu A_\mu$
$\phi) = \alpha \phi^* \phi + \beta (\phi^* \phi)^2$
$\alpha < 0 , \beta > 0$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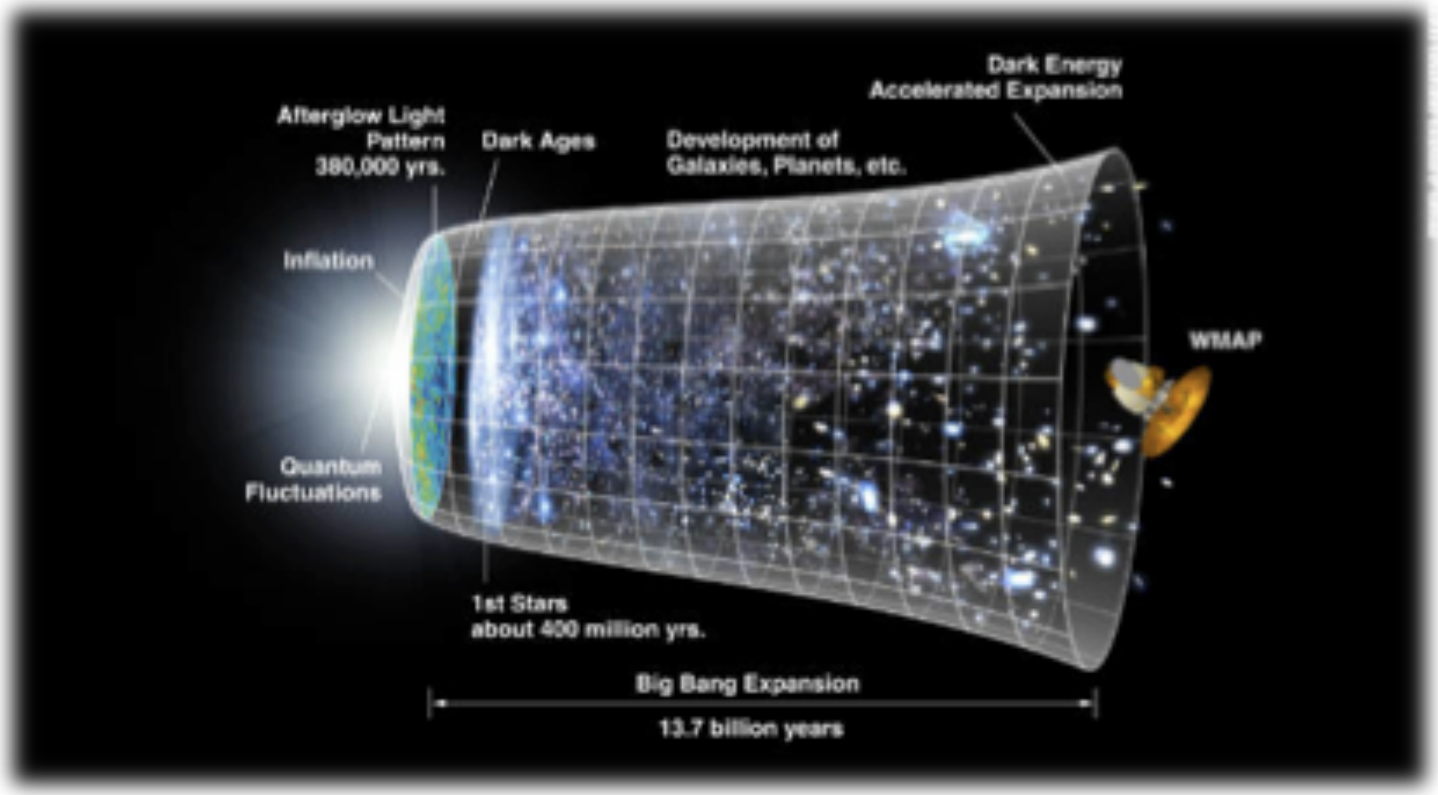
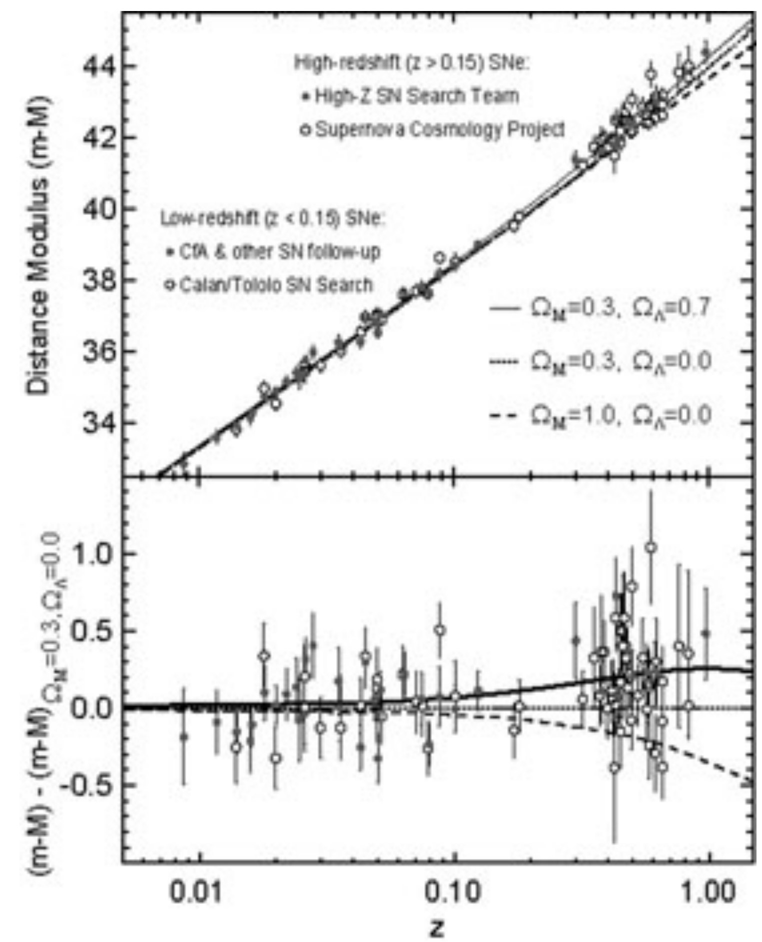
In the year 15AD ...

D for Dark Energy





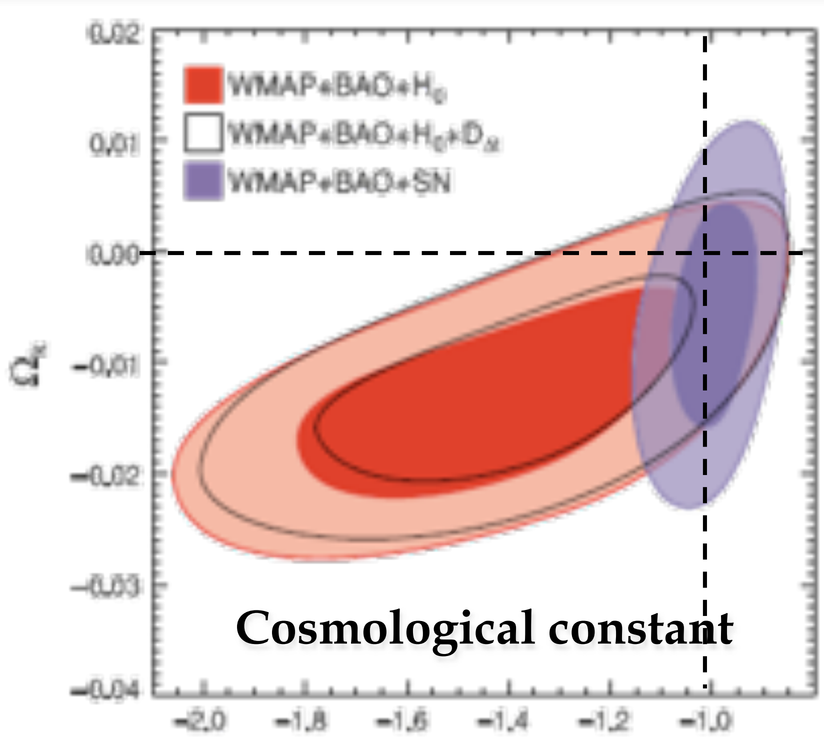



Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory
Saul Perlmutter


Photo: Belinda Pratten, Australian National University
Brian P. Schmidt


Photo: Homewood Photography
Adam G. Riess

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Adam G. Riess

A challenge:

Still, while Riess and his team made a striking discovery, the findings also revealed a new mystery. The universe's acceleration is thought to be driven by an immensely powerful force that since has been labeled "dark energy" — but precisely what that is remains an enigma, "perhaps the greatest in physics today," according to the academy that annually awards Nobel Prizes.

Riess called dark energy the "leading candidate" to explain the acceleration of the universe's expansion, but said he and others in his field have plenty of work to do before they determine how it works.

"You'll win a Nobel Prize if you figure it out," Riess said. "In fact, I'll give you mine."

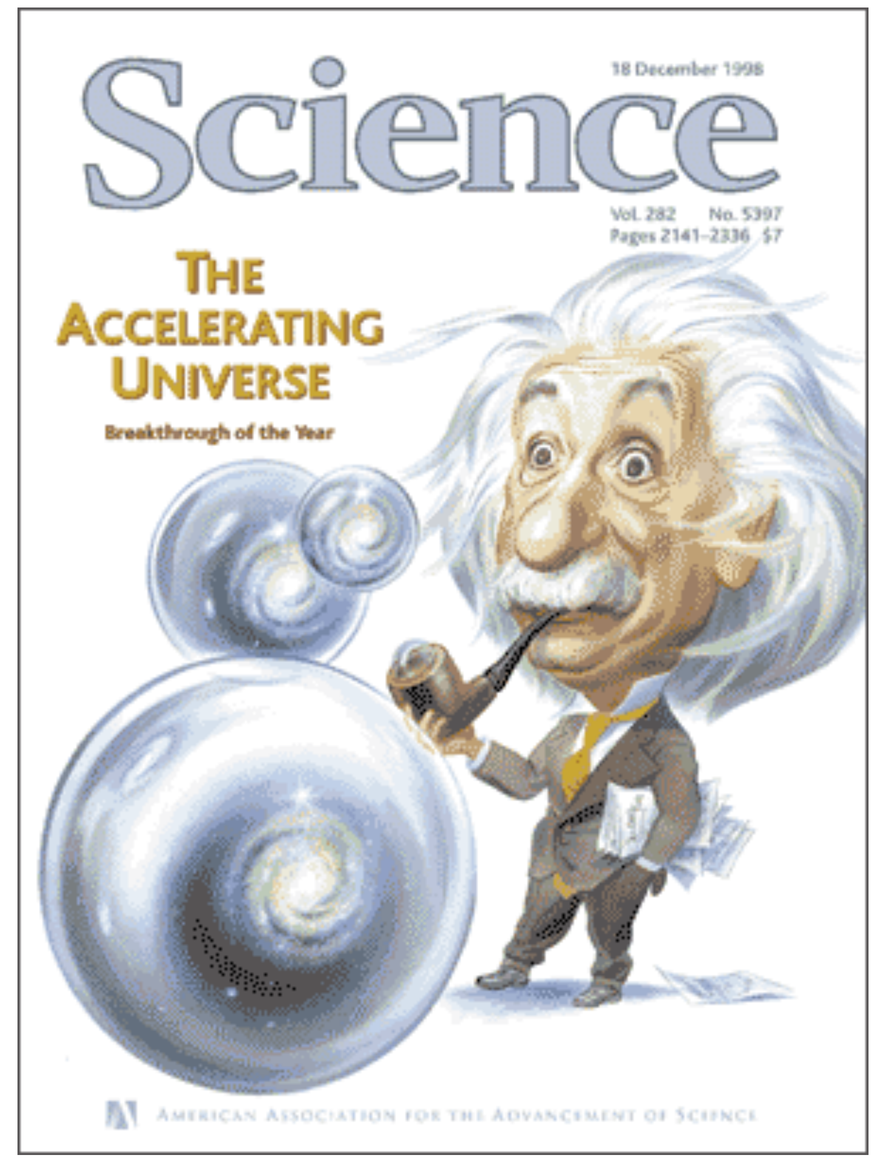
Cosmic Acceleration & String Theory

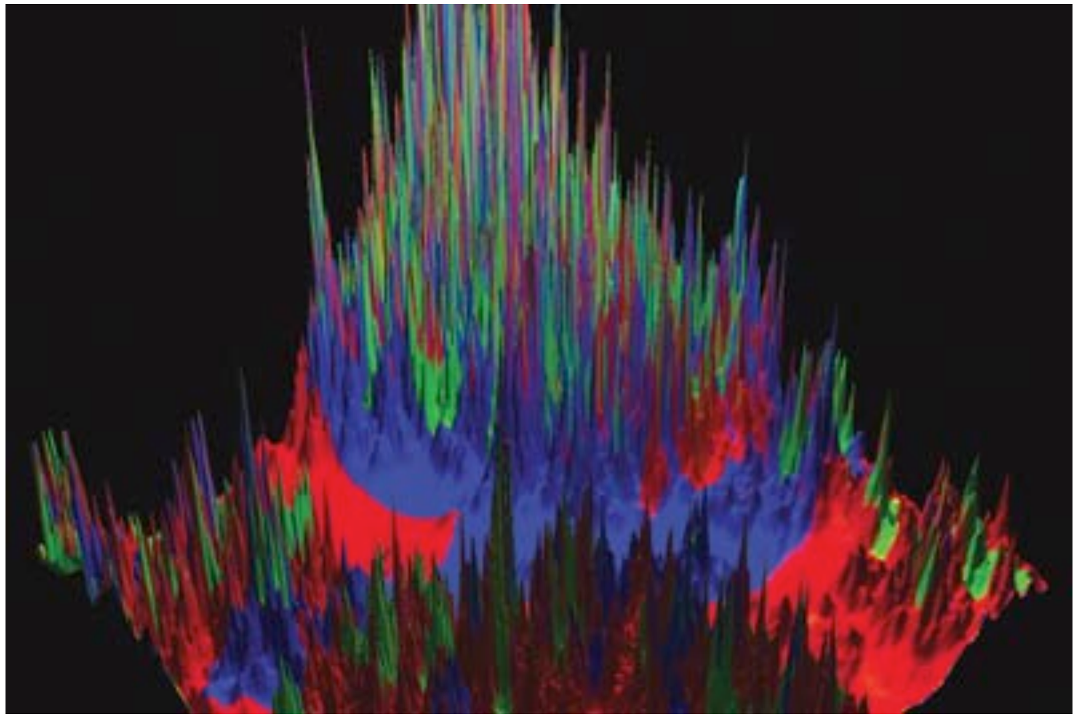
The zero of the vacuum energy:

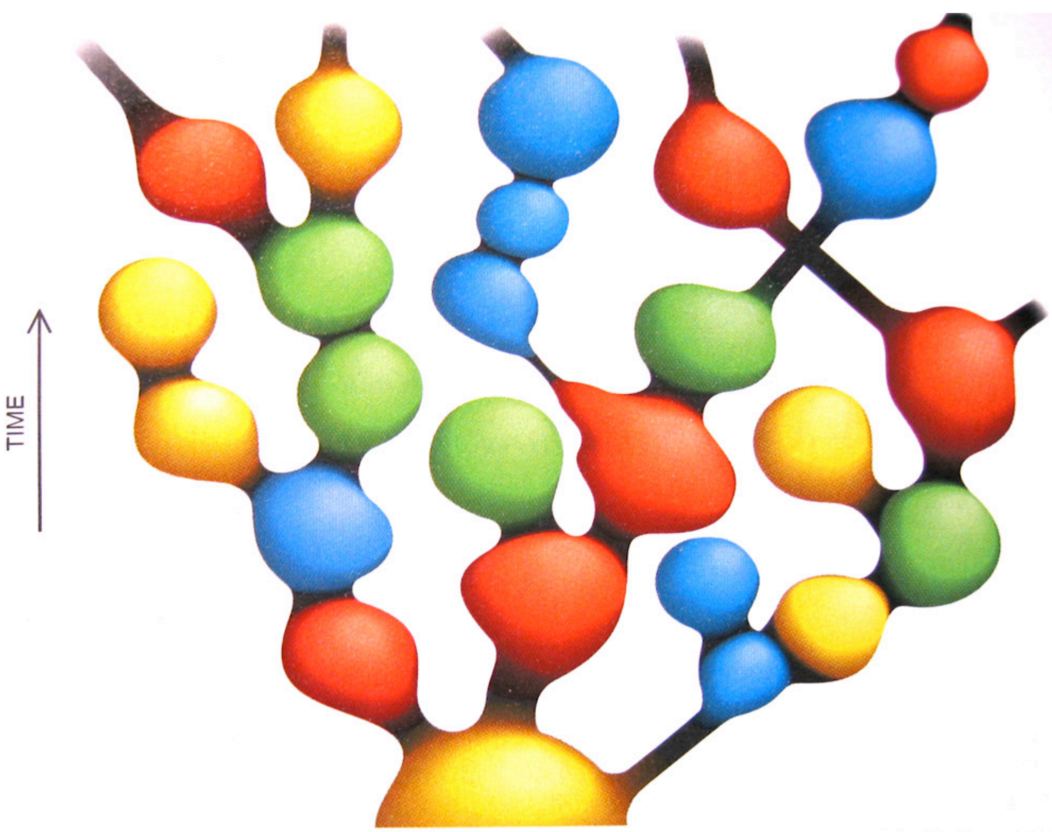
✤ is immaterial in the absence of gravity,

✤ can be tuned at will classically.

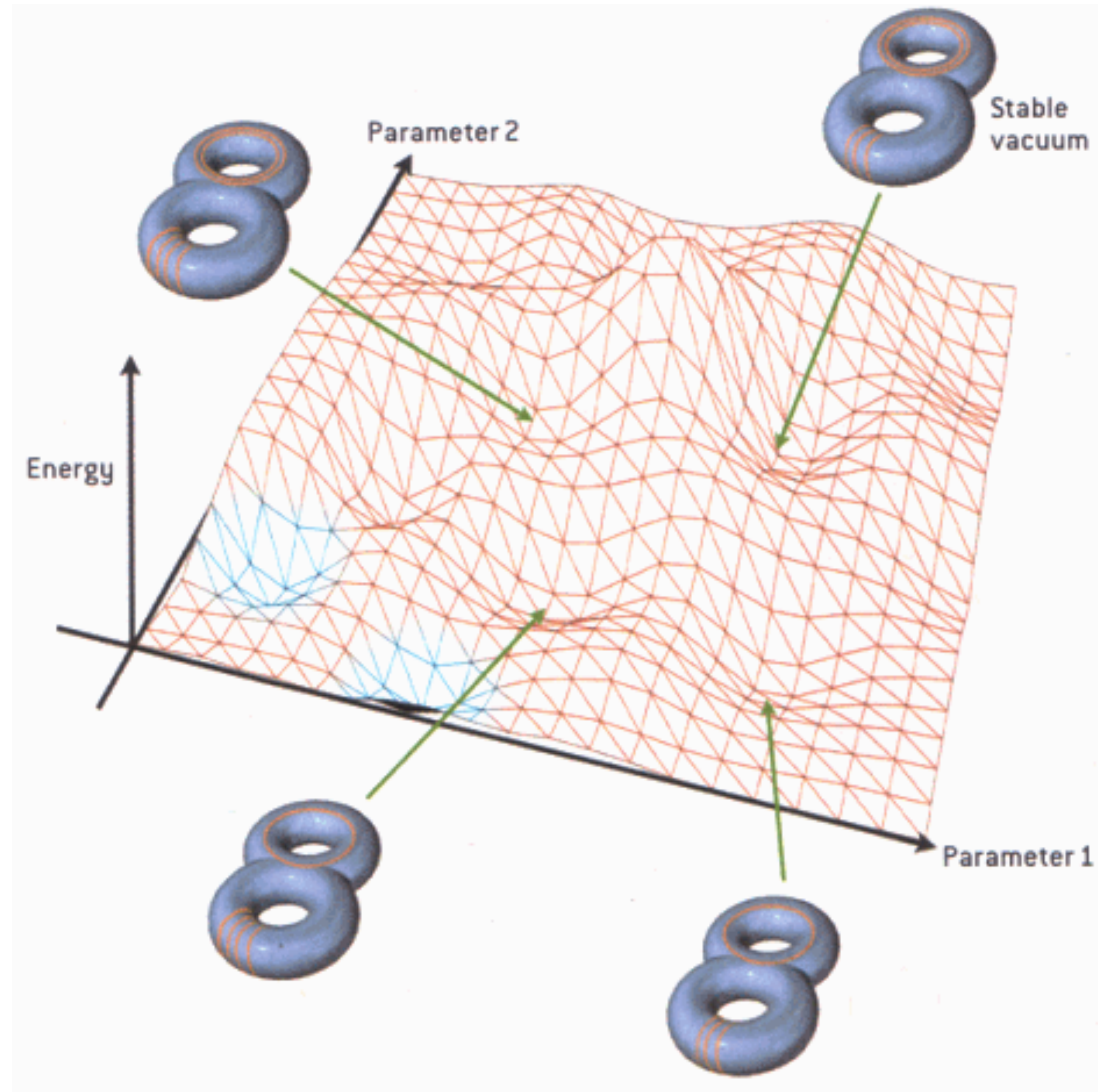
Solution to the dark energy problem likely requires quantum gravity!



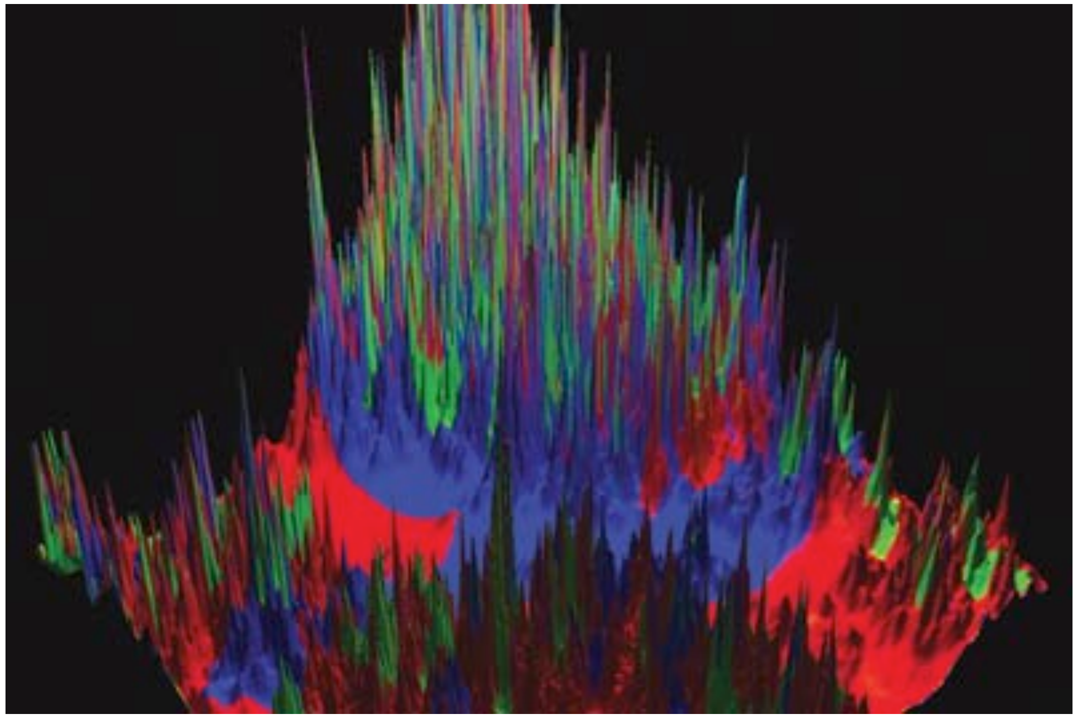


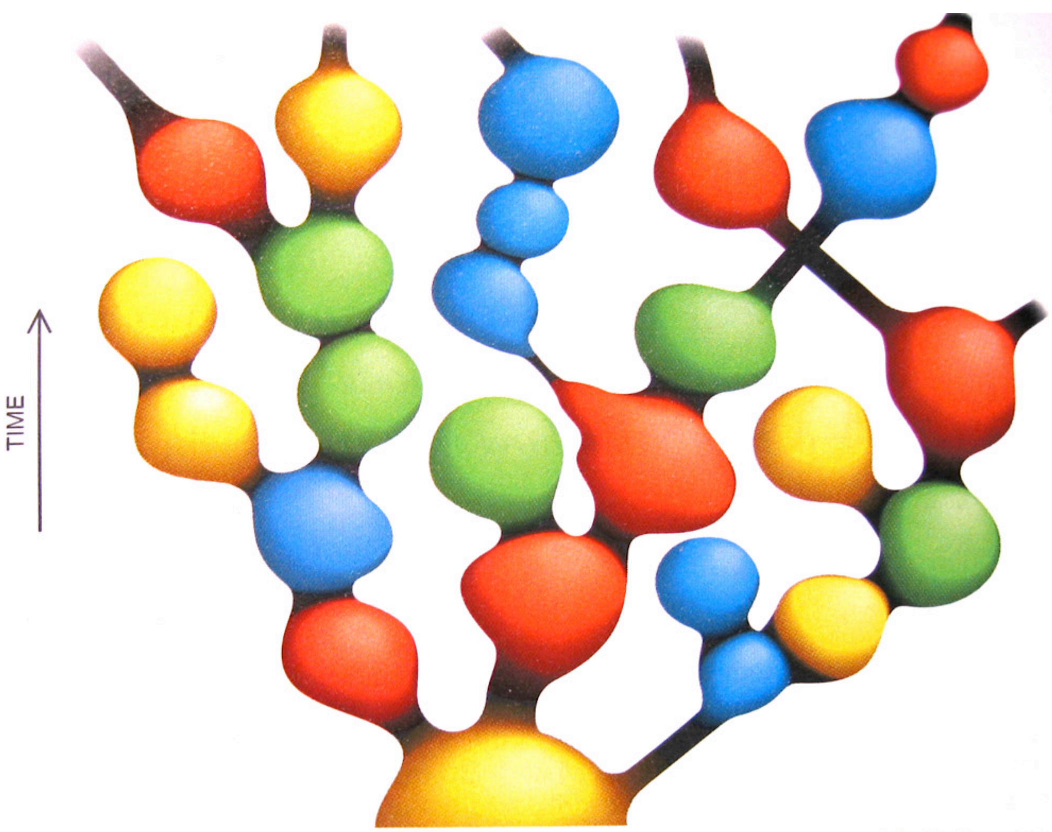


SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.



"The Landscape" (Picture from *Scientific American*)





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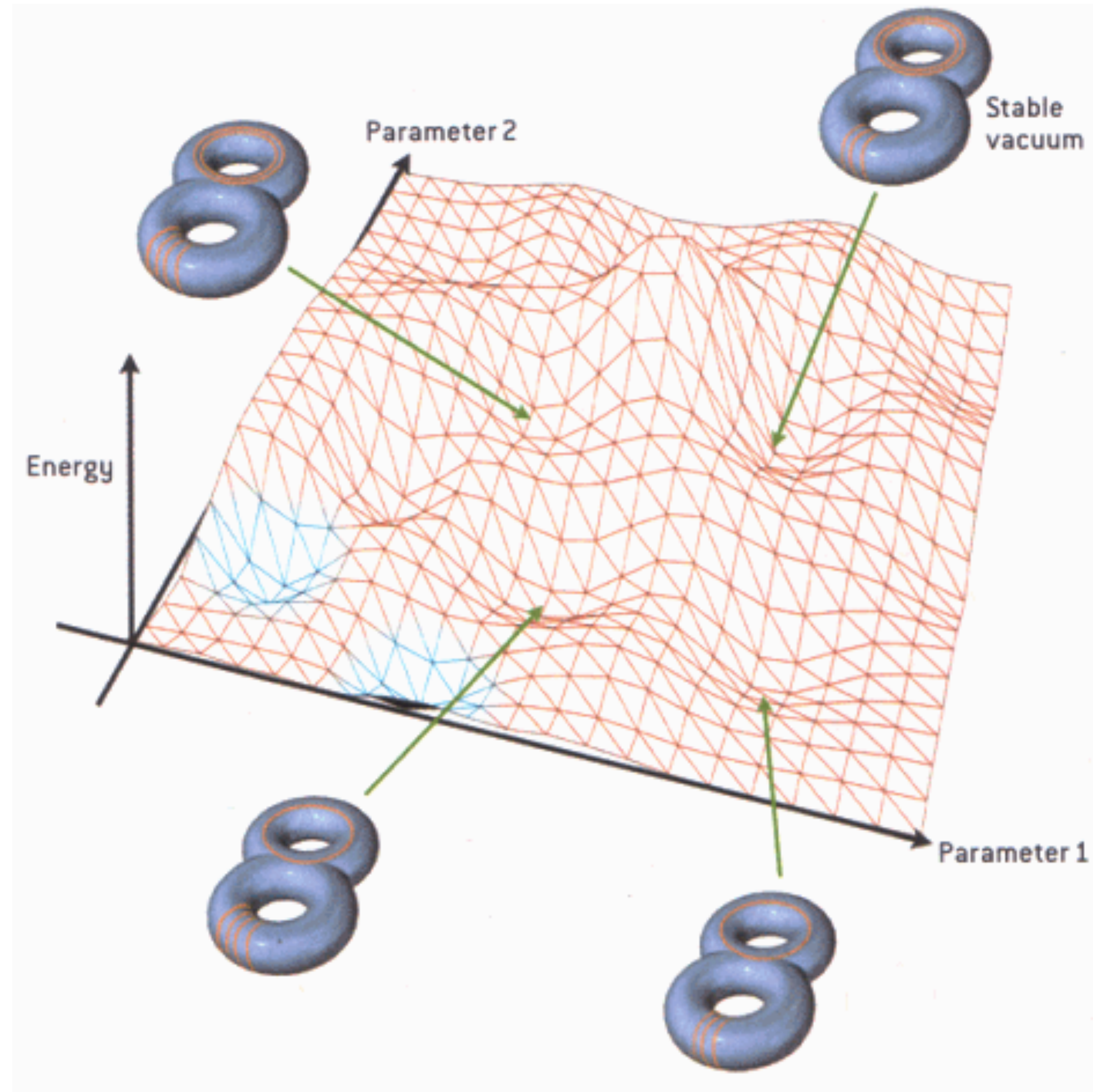


"The Landscape" (Picture from *Scientific American*)

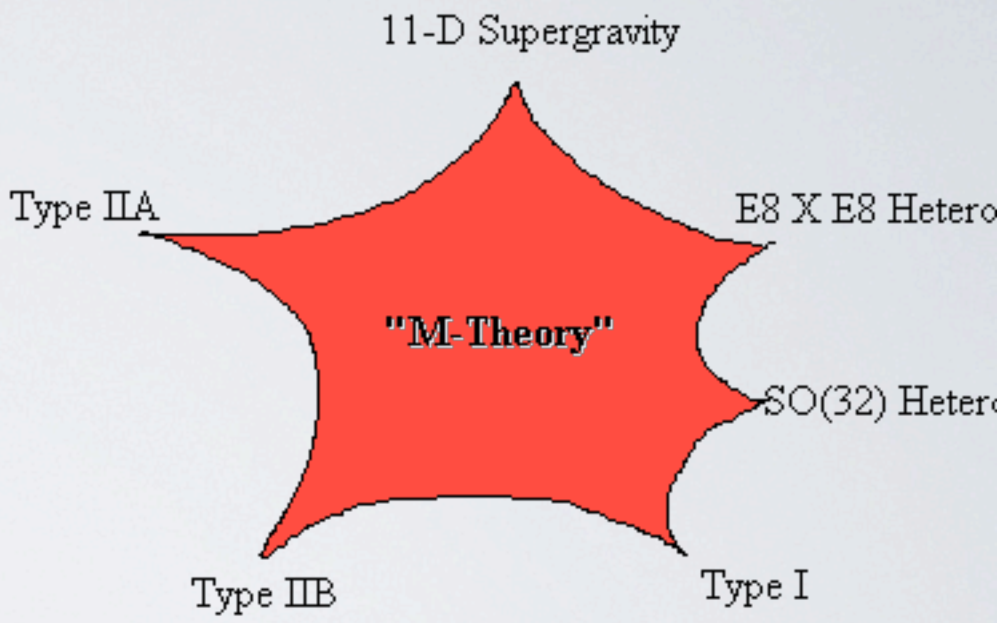
A landscape of string vacua?

Outline of this talk

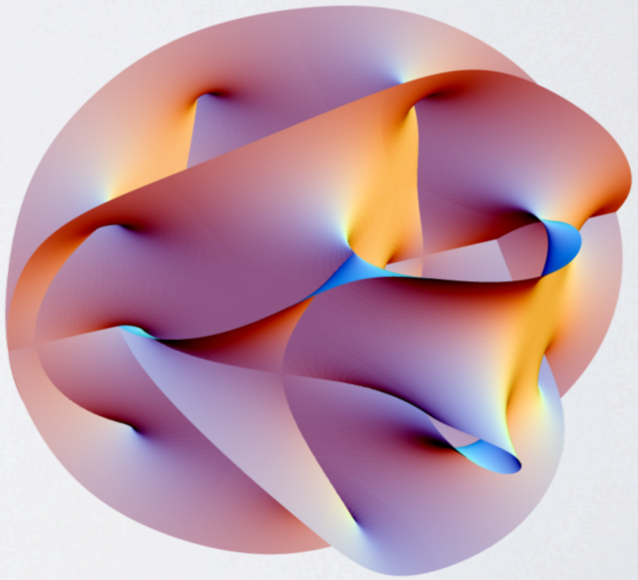
- The case for the landscape
- No-go theorems and attempts to construct explicit models
  - S.S. Haque, GS, B. Underwood, T. Van Riet, Phys. Rev. D79, 086005 (2009).
  - U.H. Danielsson, S.S. Haque, GS, T. Van Riet, JHEP 0909, 114 (2009).
  - U.H. Danielsson, S.S. Haque, P. Koerber, GS, T. Van Riet, T. Wrase, Fortsch. Phys. 59, 897 (2011).
- Stability & Random (Super)gravities
  - GS, Y. Sumitomo, JHEP 1109, 052 (2011).
  - X. Chen, GS, Y. Sumitomo, H. Tye, JHEP 1204, 026 (2012).

STRING THEORY LANDSCAPE

- Many perturbative formulations:



- In each perturbative limit, many topologies:



- For a fixed topology, many choices of fluxes.

STRING THEORY LANDSCAPE

- String theory has many *solutions* ...
- Fluxes contribute to energy density:

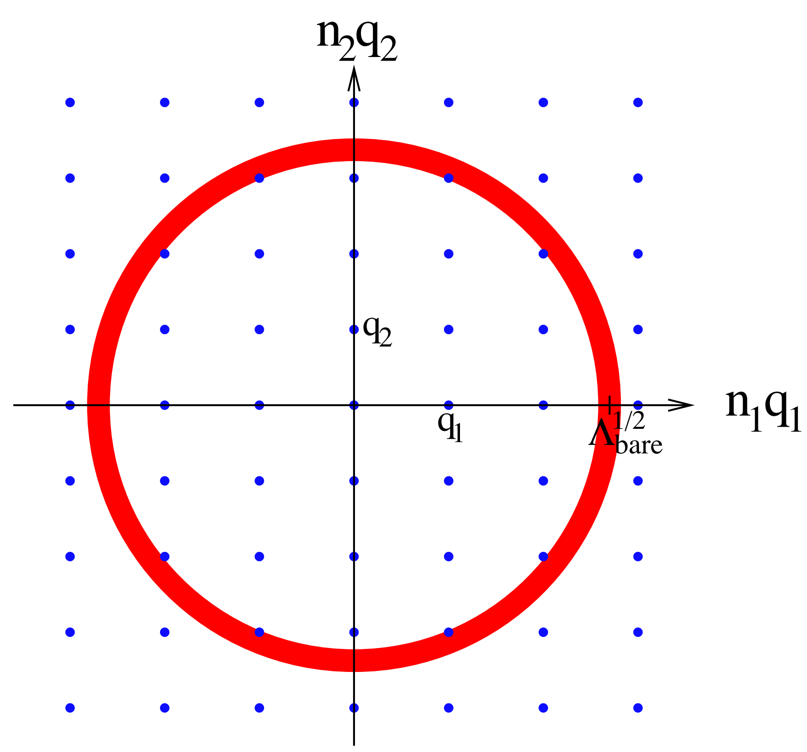
$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2q!} F_q \cdot F_q + \dots \right]$$

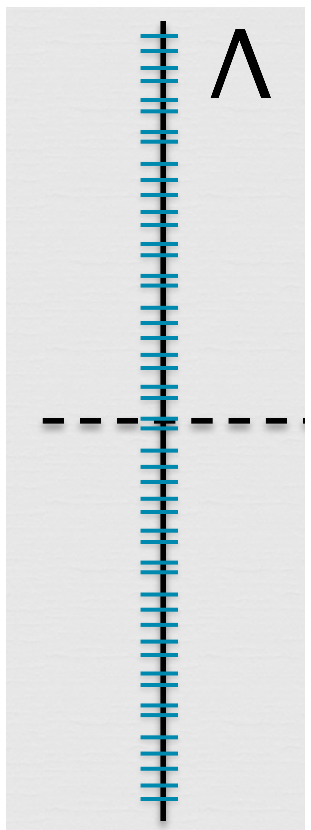
- Quantization of fluxes: $\int_\Sigma F \in \mathbb{Z}$
- A large number of moduli (hence possible fluxes) allows for the fine-tuning of the cc.

$$\Lambda = \Lambda_{\text{bare}} + \frac{1}{2} \sum_i n_i^2 q_i^2$$

Bousso Polchinski

- A large discretuum:





$$\Lambda = \Lambda_{\rm bare} + \frac{1}{2}\sum_i n_i^2 q_i^2$$

- # solutions ~ (# flux quanta)$^{\#\text{moduli}}$ ~m^N~10^{500}

Bousso Polchinski

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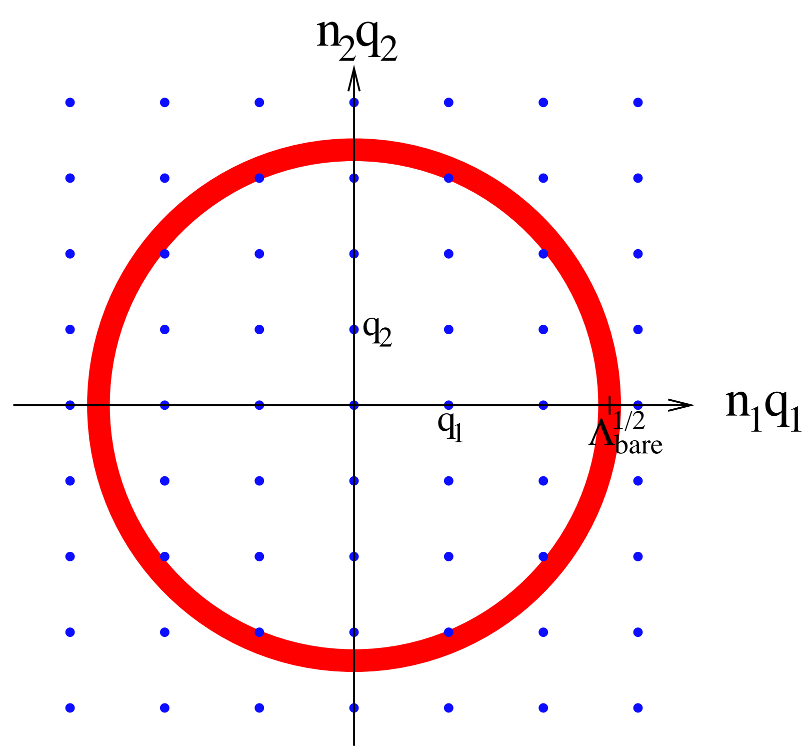

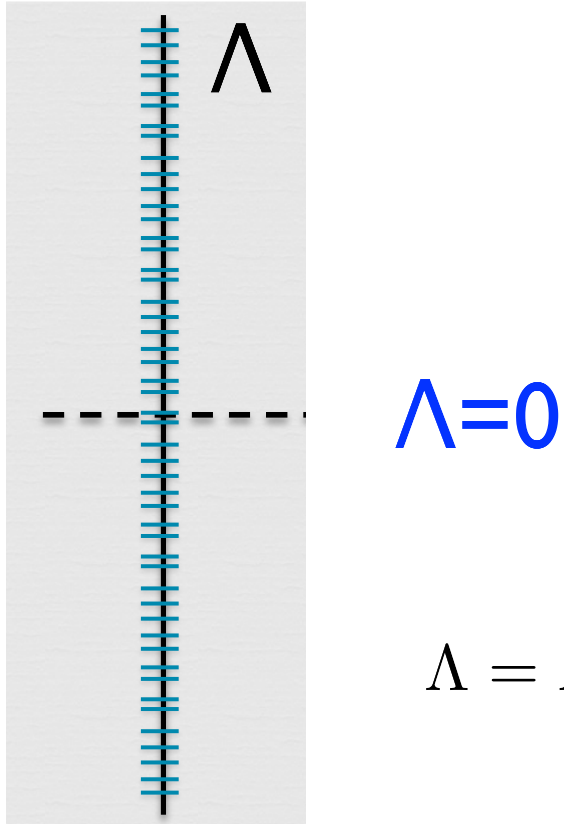

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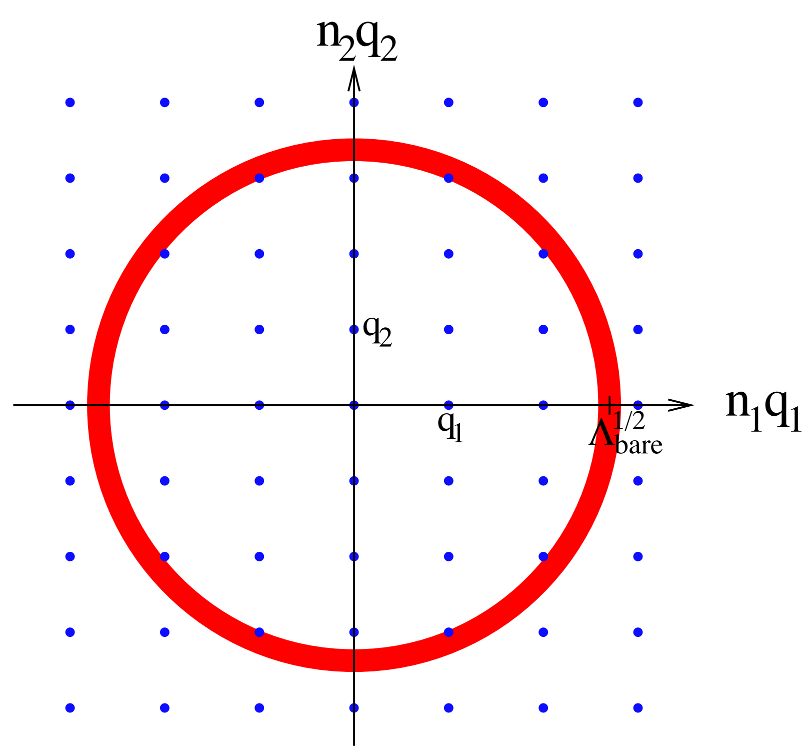

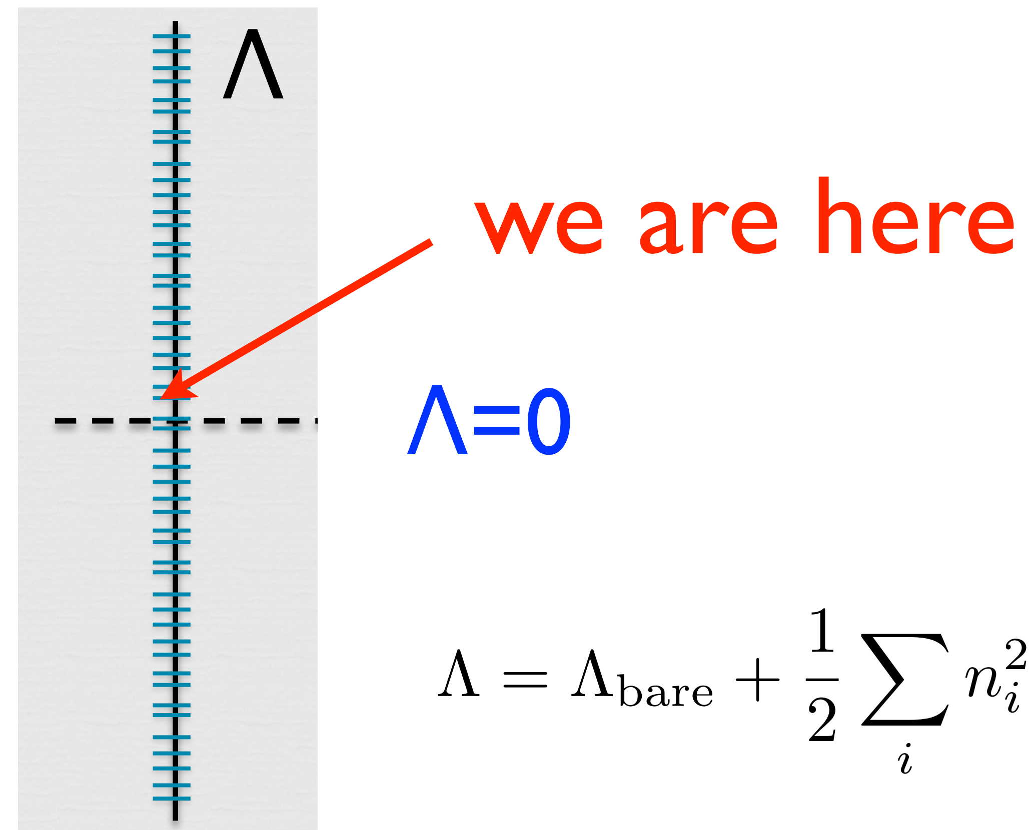

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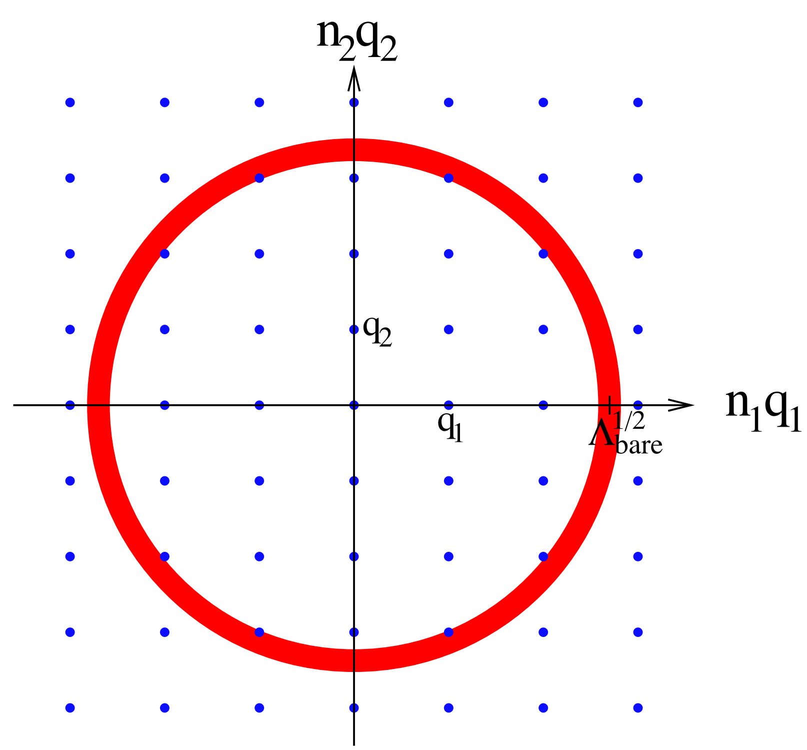

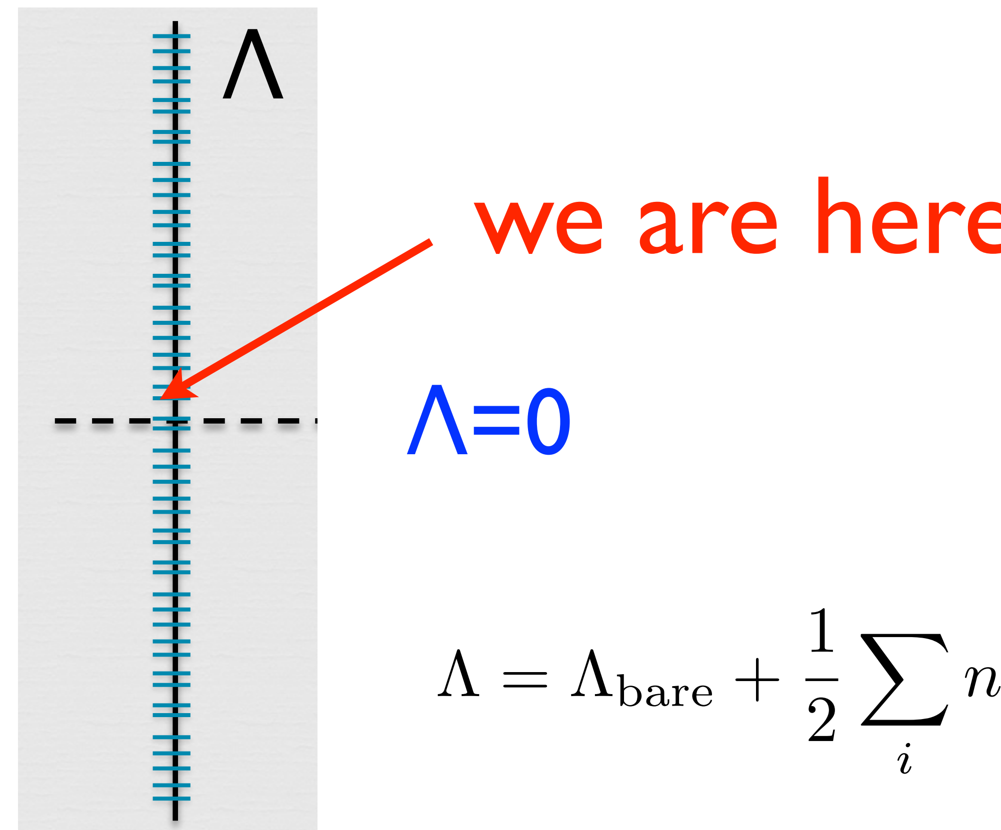

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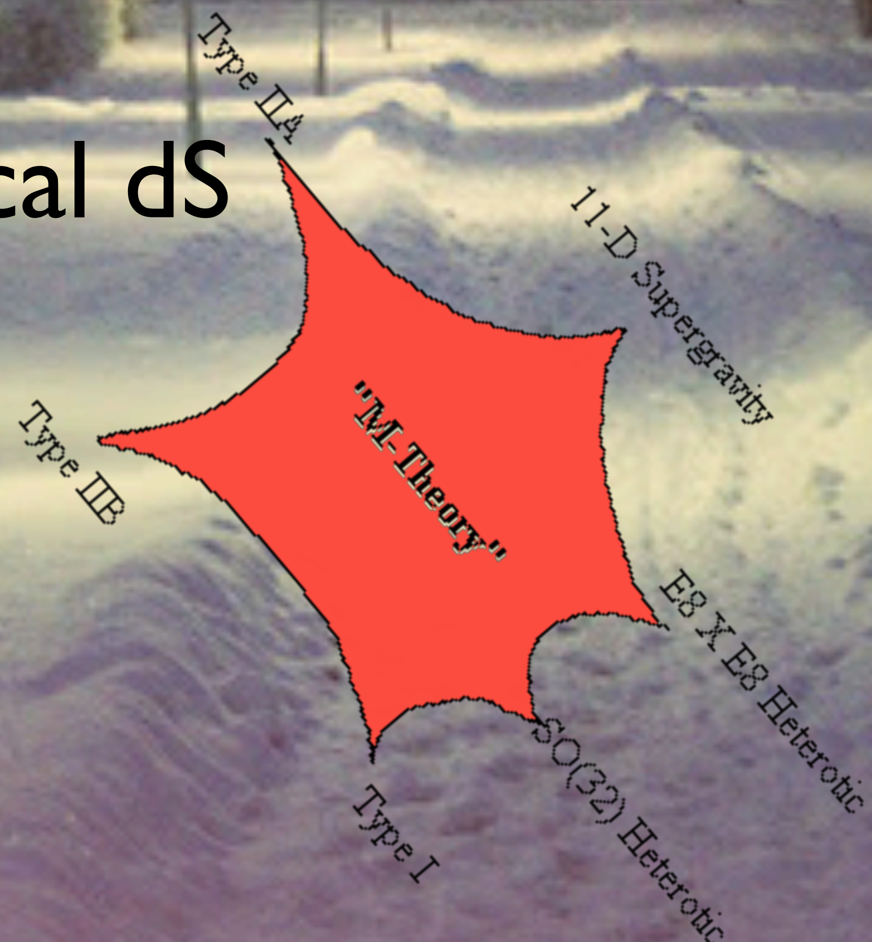
- # solutions ~ (# flux quanta)$^{\#\mathrm{moduli}}$ ~m^N~10^{500}

But how many of them are actually (meta)stable?

Explicit Constructions

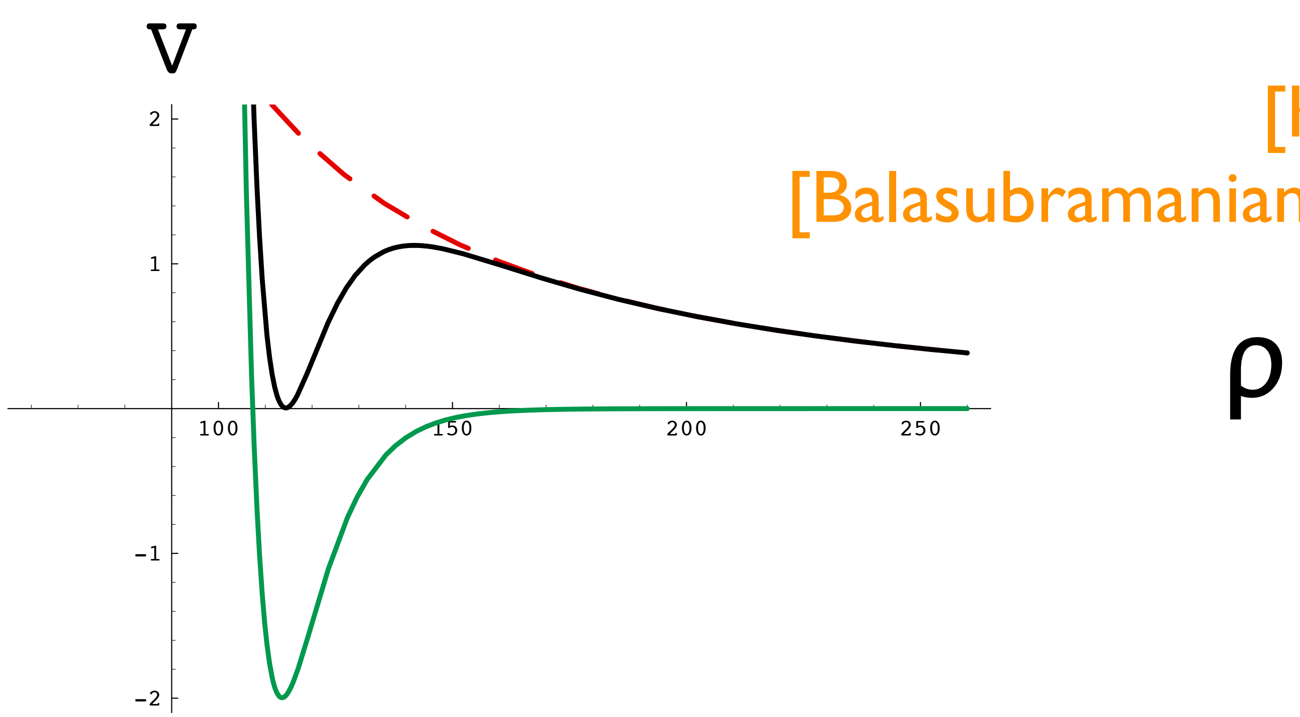
Classical dS

KKLT, LVS, ...



Flux Compactification

- Fluxes stabilize complex structure moduli but Kahler moduli remain unfixed.
- Non-perturbative effects (D7 gauge instantons or ED3 instantons) stabilize the Kahler moduli.
- Anti-branes and/or ΔK_{pert} to "uplift" vacuum energy.



[Kachru, Kallosh, Linde, Trivedi];
[Balasubramanian, Berglund, Conlon, Quevedo];
...

But ....

- **Non-perturbative effects:** difficult to compute *explicitly*. Most work aims to illustrate their existence, rather than to compute the actual contributions:

$$W_{\rm np} = Ae^{-a\rho} \quad \Longrightarrow \quad W_{\rm np} = A(\zeta_i)e^{-a\rho}$$

Moreover, the full moduli dependence is suppressed.

- **Anti D3-branes:** backreaction on the 10D SUGRA proves to be very challenging.

[DeWolfe, Kachru, Mulligan];[McGuirk, GS, Sumitomo];[Bena, Grana, Halmagyi], [Dymarsky], ...

Classical de Sitter solutions

- In Type IIA, fluxes alone can stabilize all moduli; known examples so far are AdS vacua.
- Absence of np effects, and explicit SUSY breaking localized sources, e.g., anti-branes.
- Explicitly computable within classical SUGRA.
- Solve 10D equations of motion (c.f., 4D EFT).
- Readily amenable to statistical studies (later).

Our Ingredients

✤ Fluxes: contribute *positively* to energy and tend to make the internal space *expands:*

$$S = -\frac{1}{2p!}\int_6 \sqrt{g_6}F_{\mu_1\ldots\mu_{p+1}}F^{\mu_1\ldots\mu_{p+1}}$$

✤ Branes: contribute *positively* to energy and tend to *shrink* the internal space (reverse for O-plane which has negative tension):

$$S = -T_{\rm brane}\int_{\rm brane}\sqrt{g_{\rm brane}}$$

✤ Curvature: Positively (negatively) curved spaces tend to shrink (expand) and contribute a negative (positive) energy:

$$\int_{10}\sqrt{|g_{10}|}\mathcal{R}_{10} = \int_4 \sqrt{g_4}\Big(\big(\int_6 \sqrt{g_6}\big)\mathcal{R}_4 + \int_6 \sqrt{g_6}\mathcal{R}_6\Big)$$

Universal Moduli

- Consider metric in 10D string frame and 4d Einstein frame:

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{d}s_4^2 + \rho\,\mathrm{d}s_6^2\,, \qquad\qquad \tau \equiv \rho^{3/2}\mathrm{e}^{-\phi}\,,$$

  ρ, τ are the *universal moduli*.

- The various ingredients contribute to V in some specific way:

$$V_R = U_R\rho^{-1}\tau^{-2}, \quad U_R(\varphi) \sim \int \sqrt{g_6}\,(-R_6),$$

$$V_H = U_H\rho^{-3}\tau^{-2}, \quad U_H(\varphi) \sim \int \sqrt{g_6}\,H^2,$$

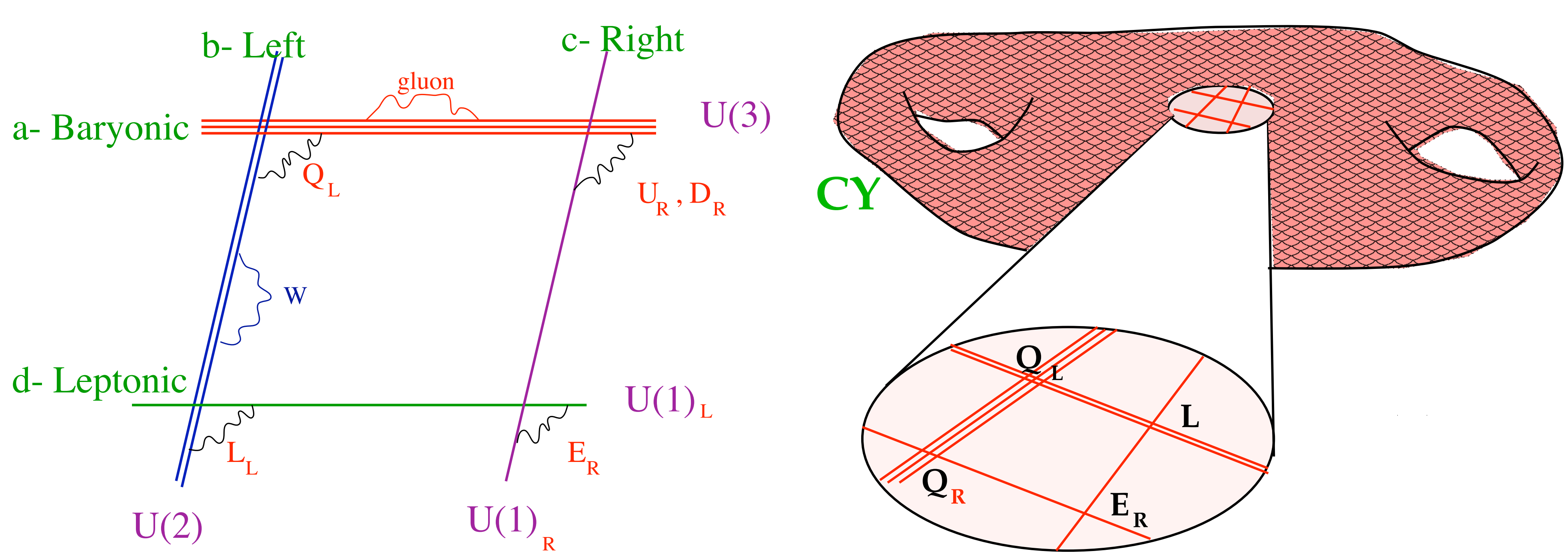
$$V_q = U_q\rho^{3-q}\tau^{-4}, \quad U_q(\varphi) \sim \int \sqrt{g_6}F_q^2 > 0$$

$$V_p = U_p\rho^{\frac{p-6}{2}}\tau^{-3}, \quad U_p(\varphi) = \mu_p\,\mathrm{Vol}(M_{p-3}).$$

- The full 4D potential $V(\rho,\tau,\phi_i) = V_R + V_H + V_q + V_p$.

Intersecting Brane Models

- Consider Type IIA string theory with intersecting D6-branes/ O6-planes in a Calabi-Yau space:



a popular framework for building the Standard Model (and beyond) from string theory. See [Blumenhagen, Cvetic, Langacker, GS]; [Blumenhagen, Kors, Lust, Stieberger];[Marchesano]; ... for reviews.

No-go Theorem(s)

- For Calabi-Yau, $V_R = 0$, we have: $V = V_H + \sum_q V_q + V_{D6} + V_{O6}$
- The universal moduli dependence leads to an inequality:

$$-\rho \frac{\partial V}{\partial \rho} - 3\tau \frac{\partial V}{\partial \tau} = 9V + \sum_q q V_q \geq 9V$$

- This <u>excludes</u> a **de Sitter vacuum**:

$$\frac{\partial V}{\partial \rho} = \frac{\partial V}{\partial \tau} = 0 \text{ and } V > 0$$

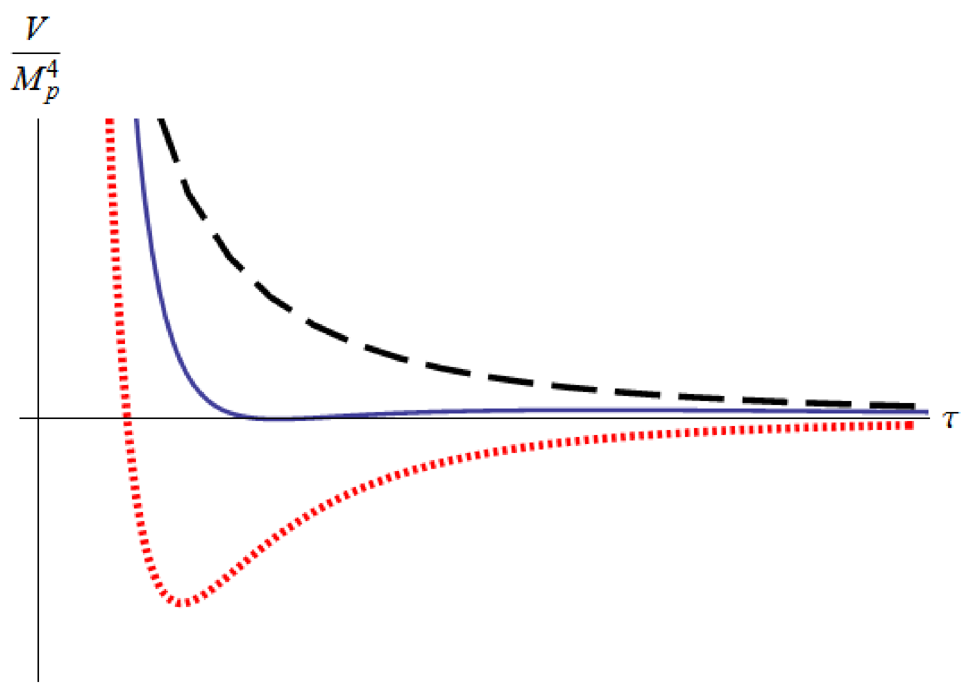
Hertzberg, Kachru, Taylor, Tegmark

as well as **slow-roll inflation** since $\epsilon \geq \mathcal{O}(1)$.

- More general no-goes were found for Type IIA/B theories with various D-branes/O-planes. [Haque, GS, Underwood, Van Riet, 08]; [Danielsson, Haque, GS, van Riet, 09];[Wrase, Zagermann, 10].

No-go Theorem(s)

✤ Evading these no-goes: O-planes [introduced in any case because of [Gibbons; de Wit, Smit, Hari Dass; Maldacena,Nunez]], fluxes, often also *negative* curvature. [Silverstein + above cited papers]



Heuristically: negative internal scalar curvature acts as an uplifting term.

✤ *Classical AdS vacua* from IIA flux compactifications with D6/O6 were found [Derendinger et al; Villadoro et al; De Wolfe et al; Camara et al].

✤ *Minimal ingredients* needed for dS [Haque, GS, Underwood, Van Riet]:

1) O6-planes 2) Romans mass 3) H-flux 4) Negatively curved internal space.

Minimal Constraints for Stability

[GS, Sumitomo, 11]

- Sylvester's Criterion: An N x N Hermitian matrix is positive definite iff **all** upper-left n x n submatrices (n≤N) are positive definite.

- Mass matrix M of 2D universal moduli subspace must satisfy:

$$\det M > 0, \quad \mathrm{tr} M > 0$$

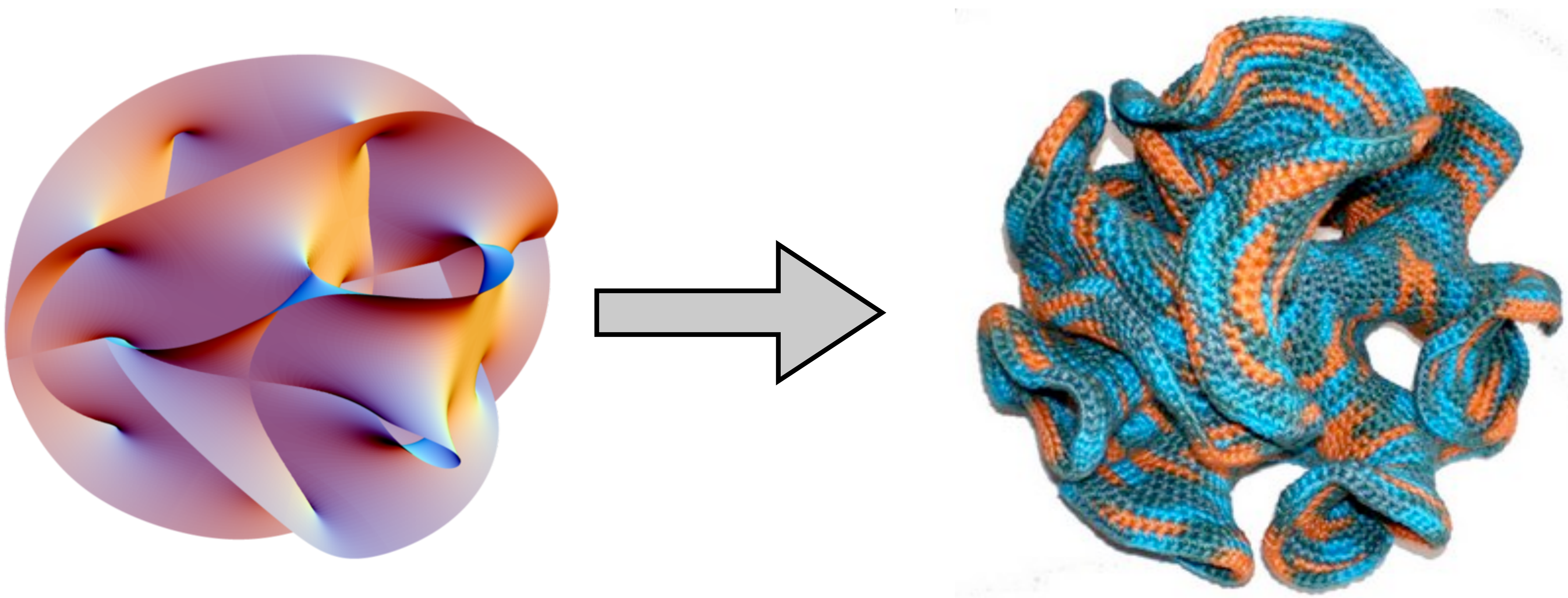
- The *minimal* ingredients for classical dS extrema tabulated in [Danielsson, Haque, GS, van Riet, 09];[Wrase, Zagermann, 10]:

Curvature	No-go, if	No no-go in IIA with	No no-go in IIB with
$V_{R_6} \sim -R_6 \leq 0$	$q+p-6 \geq 0, \ \forall p, q,$ $\epsilon \geq \frac{(3+q)^2}{3+q^2} \geq \frac{12}{7}$	O4-planes and H, F_0-flux	O3-planes and H, F_1-flux
$V_{R_6} \sim -R_6 > 0$	$q+p-8 \geq 0, \ \forall p, q,$ (except $q=3, p=5$) $\epsilon \geq \frac{(q-3)^2}{q^2-8q+19} \geq \frac{1}{3}$	O4-planes and F_0-flux O4-planes and F_2-flux O6-planes and F_0-flux	O3-planes and F_1-flux O3-planes and F_3-flux O3-planes and F_5-flux O5-planes and F_1-flux

all turn out to have an unstable mode!

Minimal Ingredients

✤ *A negatively curved internal space:*



Ricci-flat → Negatively curved

✤ *Backreaction of NS-NS & RR fluxes including the Romans mass.*

✤ *Orientifold planes*

Generalized Complex Geometry

- Interestingly, such extensions were considered before in the context of generalized complex geometry (GCG).
- Among these GCG, many are negatively curved (e.g., twisted tori), at least in some region of the moduli space [Lust et al; Grana et al; Kachru et al; ...].
- Attempts to construct explicit dS models were made soon after no-goes [Haque,GS,Underwood,Van Riet];[Flauger,Paban,Robbins, Wrase]; [Caviezel,Koerber,Lust,Wrase,Zagermann];[Danielsson,Haque,GS,van Riet]; [de Carlos,Guarino,Moreno];[Caviezel,Wrase,Zagermann];[Danielsson, Koerber,Van Riet]; ....
- A systematic search within a broad class of such manifolds [Danielsson, Haque, Koerber, GS, van Riet, Wrase].

Two Approaches

SUSY broken
@ or above
KK scale

SUSY broken
below
KK scale

Do not lead to an effective SUGRA in dim. reduced theory

[Silverstein, 07];
[Andriot, Goi, Minasian, Petrini, 10];
[Dong, Horn, Silverstein, Torroba, 10];
...

Lead to a 4d SUGRA (N=1):

[This talk]

➡ Spontaneous ~~SUSY~~ state

➡ Potentially lower ~~SUSY~~ scale

➡ Much more control on the EFT

➡ c.f. dS searches within SUGRA

[Roest et al];[de Roo et al]

Search Strategy

- GCG: natural framework for N=1 SUSY compactifications when backreaction from fluxes are taken into account.
- Type IIA SUSY AdS vacua arise from specific SU(3) structure manifolds [Lust, Tsimpis];[Caviezel et al];[Koerber, Lust, Tsimpsis]; ...
- Modify the AdS ansatz for the fluxes (which solves the flux eoms from the outset) and search for dS solutions.
- Spontaneously SUSY breaking state in a 4D SUGRA: powerful results & tools from SUSY, GCG.

SU(3) Structure

- SUSY implies the existence of a nowhere vanishing internal 6d spinor η_+ (and complex conjugate η_-).

- Characterized by a real 2-form J and a complex 3-form Ω:

$$J = \frac{i}{2||\eta||^2}\eta_+^\dagger \gamma_{i_1 i_2}\eta_+ \mathrm{d}x^{i_1} \wedge \mathrm{d}x^{i_2}$$
$$\Omega = \frac{1}{3!||\eta||^2}\eta_-^\dagger \gamma_{i_1 i_2 i_3}\eta_+ \mathrm{d}x^{i_1} \wedge \mathrm{d}x^{i_2} \wedge \mathrm{d}x^{i_3}$$

satisfying $\Omega \wedge J = 0\,, \qquad \Omega \wedge \Omega^* = (4i/3)\, J \wedge J \wedge J = 8i\, \mathrm{vol}_6\,.$

- J, Ω define SU(3) structure, not SU(3) holonomy: generically dJ≠0 and dΩ≠0.

SU(3) Torsion Classes

The non-closure of the exterior derivatives characterized by:

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$
$$\mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega$$

Torsion classes	Name
$\mathcal{W}_1 = \mathcal{W}_2 = 0$	Complex
$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Symplectic
$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Kähler
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Kähler
$\mathrm{Im}\,\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Half-flat
$\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 0, (1/2)\mathcal{W}_4 = (1/3)\mathcal{W}_5 = -\mathrm{d}A$	Conformal Calabi-Yau

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$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Symplectic
$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Kähler
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Kähler
$\mathrm{Im}\,\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Half-flat
$\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 0, (1/2)\mathcal{W}_4 = (1/3)\mathcal{W}_5 = -\mathrm{d}A$	Conformal Calabi-Yau

compatible with the orbifold/orientifold symmetries considered

Universal Ansatz

- Ricci tensor can be expressed explicitly in terms of J, Ω and the torsion forms [Bedulli, Vezzoni].
- In terms of the universal forms: $\Big\{J, \Omega, W_1, W_2, W_3\Big\}$

  one finds a natural ansatz for the fluxes:

$$
\begin{aligned}
e^{\Phi}\hat{F}_0 &= f_1\,,\\
e^{\Phi}\hat{F}_2 &= f_2 J + f_3\hat{W}_2\,,\\
e^{\Phi}\hat{F}_4 &= f_4 J\wedge J + f_5\hat{W}_2\wedge J\,,\\
e^{\Phi}\hat{F}_6 &= f_6\mathrm{vol}_6\,,\\
H &= f_7\Omega_R + f_8\hat{W}_3\,,\\
j &= j_1\Omega_R + j_2\hat{W}_3\,.
\end{aligned}
$$

same ansatz in finding SUSY AdS vacua [Lust, Tsimpis]

- Universal ansatz: forms appear in *all* SU(3) structure (in this case, half flat) manifolds.

O-planes

✤ To simplify, we take the *smeared* approximation:

$$\delta \to \text{constant}$$

i.e., we solve the eoms in an “average sense”. If backreaction is ignored, eoms are not satisfied pointwise [Douglas, Kallosh].

✤ Finding backeacted solutions with localized sources proves to be challenging (more later) [Blaback, Danielsson, Junghans, Van Riet, Wrase, Zagermann].

✤ The Bianchi identity becomes:

$$\mathrm{d}\hat{F}_2 + H\hat{F}_0 = -j\,, \qquad\qquad e^{\Phi} j = j_1 \Omega_R + j_2 \hat{W}_3\,.$$

✤ The source terms of smeared O-planes in dilaton/Einstein eoms can be found in [Koerber, Tsimpis, 07].

Finding Solutions

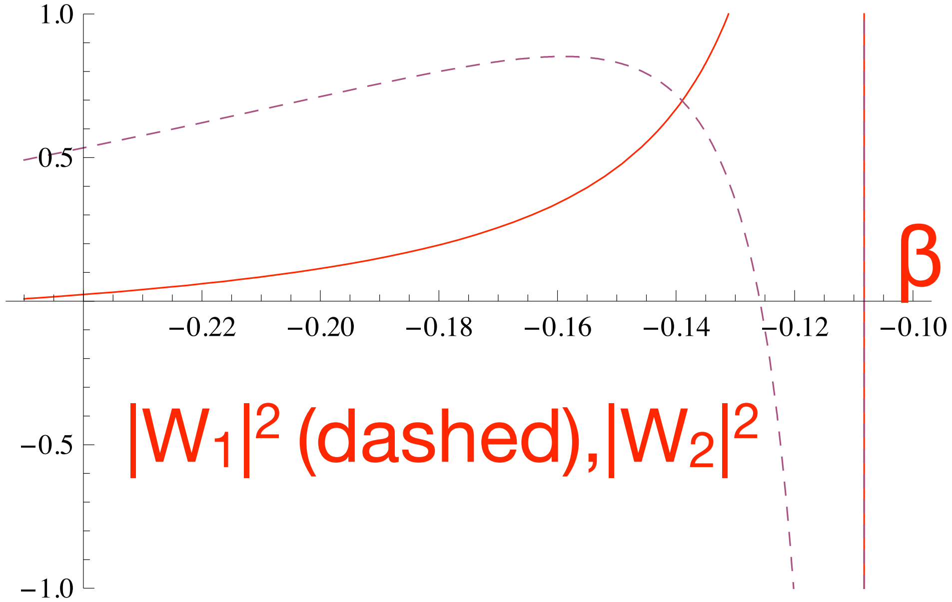
- The dilaton/Einstein/flux eoms and Bianchi identities can be expressed as algebraic equations (skip details).
- To find solutions other than the SUSY AdS, impose constraints:

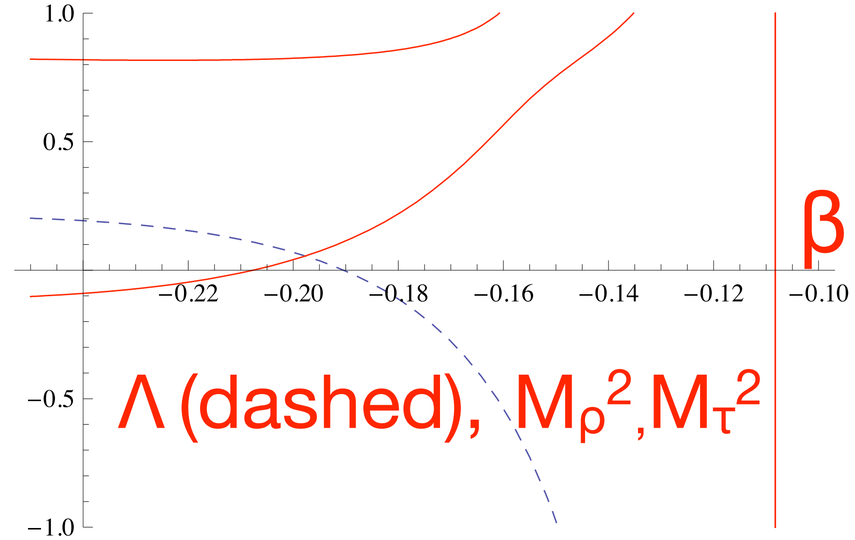
$$
\begin{aligned}
\mathrm{d}\hat{W}_2 &= c_1 \Omega_R + d_1 \hat{W}_3 \,, \\
\hat{W}_2 \wedge \hat{W}_2 &= c_2 J \wedge J + d_2 \hat{W}_2 \wedge J \,, \\
\mathrm{d} \star_6 \hat{W}_3 &= c_5 J \wedge J + c_3 \hat{W}_2 \wedge J \,, \\
\frac{1}{2} (\hat{W}_{3\,ikl} \hat{W}_{3\,j}{}^{kl})^{+} &= d_4 J_{ik} \hat{W}_2{}^{k}{}_{j} \,.
\end{aligned}
$$

for some c's and d's.

Finding Solutions

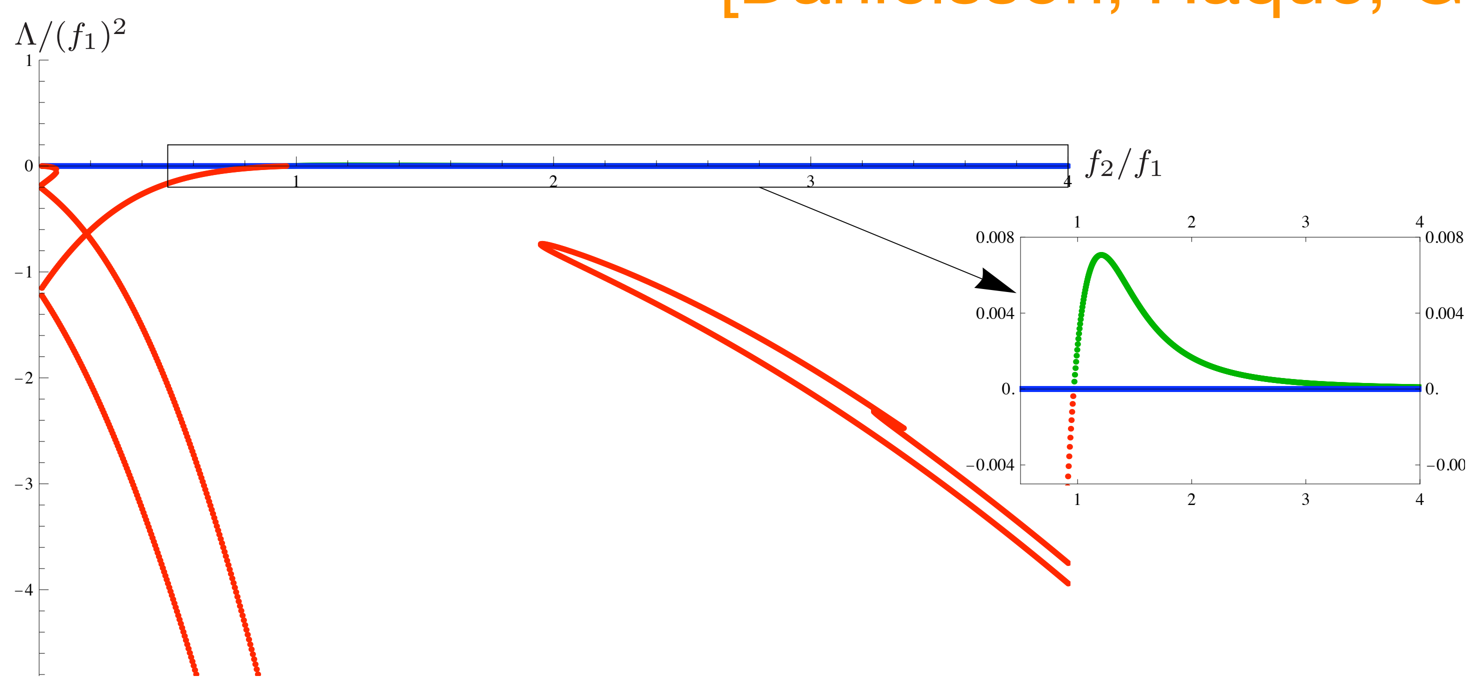
$W_3 = 0$





[Danielsson, Haque, GS, Van Riet]

$W_2 = 0$



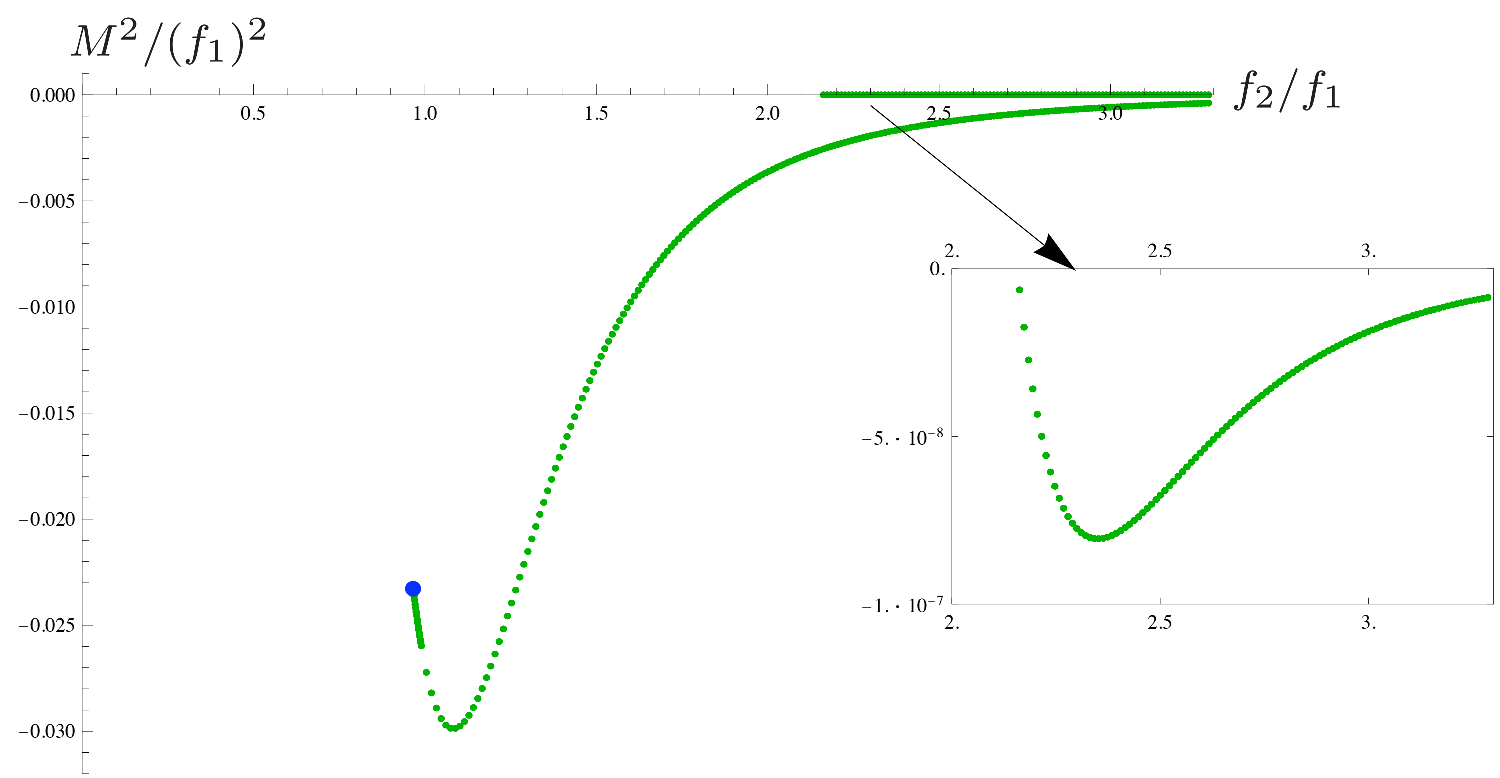
[Danielsson, Koerber, Van Riet]

Explicit Model Building

- ✤ Bottom-up approach: we found *necessary* constraints on fluxes & torsion classes for *universal* dS solutions, a useful first step.

- ✤ Bottom-up constraints (with W2=0) can be satisfied with an explicit model: an SU(2) x SU(2) group manifold.

Unfortunately, out of 14 scalars, one is tachyonic !

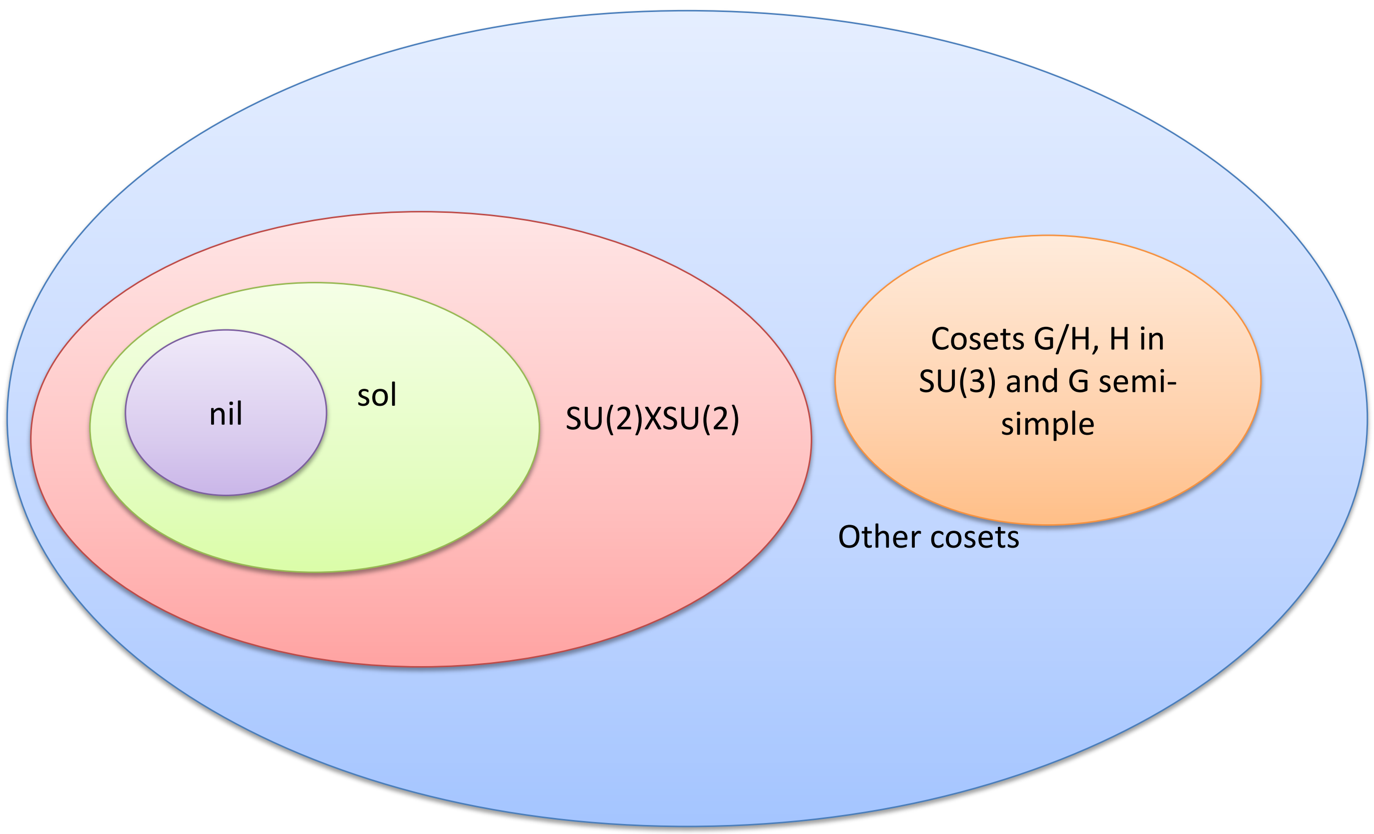


[Danielsson, Koerber, Van Riet]

A Systematic Search

[Danielsson, Haque, Koerber, GS, Van Riet, Wrase]

✤ Focus on homogenous spaces (G/H, H ⊆ SU(3)) where we can explicitly construct the SU(3) structure.



G=Semi-simple
[Caviezel,Koerber,Lust,Tsimpis, Zagermann]; ...

Nilmanifold
Silverstein, ...

Solmanifold
[Grana, Minasian, Petrini, Tomasiello]; [Andriot, Goi, Minasian, Petrini]; ...

Unexplored!

➡ We cover ***all*** group manifolds, by classifying 6d groups.

Group Manifolds

- A coframe of left-invariant forms: $g^{-1}\mathrm{d}g = e^a T_a$

  that obeys the Maurer-Cartan relations: $\mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c$

- From these MC forms, we can construct J, Ω, and the metric:

$$\mathrm{d}s^2 = \mathcal{M}_{ab} e^a \otimes e^b$$

- Levi's theorem: $\mathfrak{g} = \mathfrak{s} \ltimes \mathfrak{r}$

  semi-simple $\mathfrak{s}$; radical $\mathfrak{r}$ = largest solvable ideal

  Ideal: $[\mathfrak{g}, \mathfrak{i}] \subseteq \mathfrak{i}$.

  Solvable: $\mathfrak{g}^n = [\mathfrak{g}^{n-1}, \mathfrak{g}^{n-1}]$ vanishes at some point

Group Manifolds

- Semi-simple:

Case
$\mathfrak{so}(3) \times \mathfrak{so}(3)$
$\mathfrak{so}(3) \times \mathfrak{so}(2,1)$
$\mathfrak{so}(2,1) \times \mathfrak{so}(2,1)$
$\mathfrak{so}(3,1)$

- Semi-direct product of semi-simple algebra & radical: $\mathfrak{g} = \mathfrak{s} \ltimes \mathfrak{r}$

Unimodular algebra:

$$f^a{}_{ab} = 0\,, \quad \text{for all } b$$

necessary condition for non-compact group space to be made compact.

Case	Representations
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{u}(1)^3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$ and $\rho = \mathbf{3}$
$\mathfrak{so}(3) \ltimes_\rho \mathrm{Heis}_3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{iso}(2)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{iso}(1,1)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{u}(1)^3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$, $\rho = \mathbf{1} \oplus \mathbf{2}$ and $\rho = \mathbf{3}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathrm{Heis}_3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$ and $\rho = \mathbf{1} \oplus \mathbf{2}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{iso}(2)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{iso}(1,1)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$

[Danielsson, Haque, Koerber, GS, Van Riet, Wrase]

Group Manifolds

- Solvable groups:

Name	Algebra	O5	O6	Sp
$\mathfrak{g}_{3.4}^{-1} \oplus \mathbb{R}^3$	$(q_1 23, q_2 13, 0, 0, 0, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	123, 145, 146, 156, 245, 246, 256, 345, 346, 356	✓
$\mathfrak{g}_{3.5}^{0} \oplus \mathbb{R}^3$	$(-23, 13, 0, 0, 0, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	123, 145, 146, 156, 245, 246, 256, 345, 346, 356	✓
$\mathfrak{g}_{3.1} \oplus \mathfrak{g}_{3.4}^{-1}$	$(-23, 0, 0, q_1 56, q_2 46, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.1} \oplus \mathfrak{g}_{3.5}^{0}$	$(-23, 0, 0, -56, 46, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.4}^{-1} \oplus \mathfrak{g}_{3.5}^{0}$	$(q_1 23, q_2 13, 0, -56, 46, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.4}^{-1} \oplus \mathfrak{g}_{3.4}^{-1}$	$(q_1 23, q_2 13, 0, q_3 56, q_4 46, 0) \quad q_1, q_2, q_3, q_4 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.5}^{0} \oplus \mathfrak{g}_{3.5}^{0}$	$(-23, 13, 0, -56, 46, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{4.5}^{p,-p-1} \oplus \mathbb{R}^2$	?			-
$\mathfrak{g}_{4.6}^{-2p,p} \oplus \mathbb{R}^2$	?			-
$\mathfrak{g}_{4.8}^{-1} \oplus \mathbb{R}^2$	$(-23, q_1 34, q_2 24, 0, 0, 0) \quad q_1, q_2 > 0$	14, 25, 26, 35, 36	145, 146, 256, 356	-
$\mathfrak{g}_{4.9}^{0} \oplus \mathbb{R}^2$	$(-23, -34, 24, 0, 0, 0)$	14, 25, 26, 35, 36	145, 146, 256, 356	-
$\mathfrak{g}_{5.7}^{1,-1,-1} \oplus \mathbb{R}$	$(q_1 25, q_2 15, q_2 45, q_1 35, 0, 0) \quad q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.8}^{-1} \oplus \mathbb{R}$	$(25, 0, q_1 45, q_2 35, 0, 0) \quad q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.13}^{-1,0,r} \oplus \mathbb{R}$	$(q_1 25, q_2 15, -q_2 r45, q_1 r35, 0, 0) \; r \neq 0, \; q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.14}^{0} \oplus \mathbb{R}$	$(-25, 0, -45, 35, 0, 0)$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.15}^{-1} \oplus \mathbb{R}$	$(q_1(25-35), q_2(15-45), q_2 45, q_1 35, 0, 0) \quad q_1, q_2 > 0$	14, 23, 56	146, 236	✓
$\mathfrak{g}_{5.17}^{p,-p,r} \oplus \mathbb{R}$	$(q_1(p25+35), q_2(p15+45), q_2(p45-15), q_1(p35-25), 0, 0)$ $r^2 = 1, \; q_1, q_2 > 0$	14, 23, 56 $p=0$: 12, 34	146, 236 $p=0$: 126, 135, 245, 346	✓
$\mathfrak{g}_{5.18}^{0} \oplus \mathbb{R}$	$(-25-35, 15-45, -45, 35, 0, 0)$	14, 23, 56	146, 236	✓
$\mathfrak{g}_{6.3}^{0,-1}$	$(-26, -36, 0, q_1 56, q_2 46, 0) \quad q_1, q_2 > 0$	24, 25	134, 135, 456	✓
$\mathfrak{g}_{6.10}^{0,0}$	$(-26, -36, 0, -56, 46, 0)$	24, 25	134, 135, 456	✓

[Turkowski];[Andriot,Goi,Petrini,Minasian];
[Grana,Minasian,Petrini,Tomasiello]

Orientifolding

- dS critical point of effective N=1 SUGRA from group manifolds.
- Orbifolding further by discrete $\Gamma \subset SU(3)$.
- Among the Abelian orbifolds of (twisted) T^6, only two Z_2 x Z_2 orientifolds can evade $\epsilon \geq O(1)$ [Flauger, Paban, Robbins, Wrase]
- Consider Z_2 x Z_2 orientifolds of the group spaces we classified.

$$\theta_1 : \begin{cases} e^1 & \rightarrow & -e^1 \\ e^2 & \rightarrow & -e^2 \\ e^3 & \rightarrow & e^3 \\ e^4 & \rightarrow & -e^4 \\ e^5 & \rightarrow & e^5 \\ e^6 & \rightarrow & -e^6 \end{cases}, \qquad \theta_2 : \begin{cases} e^1 & \rightarrow & -e^1 \\ e^2 & \rightarrow & e^2 \\ e^3 & \rightarrow & -e^3 \\ e^4 & \rightarrow & e^4 \\ e^5 & \rightarrow & -e^5 \\ e^6 & \rightarrow & -e^6 \end{cases} \qquad \sigma : \begin{cases} e^1 & \rightarrow & e^1 \\ e^2 & \rightarrow & e^2 \\ e^3 & \rightarrow & e^3 \\ e^4 & \rightarrow & -e^4 \\ e^5 & \rightarrow & -e^5 \\ e^6 & \rightarrow & -e^6 \end{cases}$$

[Other Z_2 x Z_2 orientifold has a different σ]

Constructing SU(3) Structure

e^1	e^2	e^3	e^4	e^5	e^6
$\otimes$	$\otimes$	$\otimes$	$-$	$-$	$-$
$\otimes$	$-$	$-$	$\otimes$	$\otimes$	$-$
$-$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$
$-$	$-$	$\otimes$	$\otimes$	$-$	$\otimes$

- O-planes: $j_6 = j_A e^{456} + j_B e^{236} + j_C e^{134} + j_D e^{125}$
- J and Ω_R are odd under orientifolding:

$$J = a e^{16} + b e^{24} + c e^{35} \,,$$
$$\Omega_R = v_1 e^{456} + v_2 e^{236} + v_3 e^{134} + v_4 e^{125} \,,$$

- The metric fluxes are even:

$$\mathrm{d}e^1 = f^1{}_{23} e^{23} + f^1{}_{45} e^{45} \,, \qquad \mathrm{d}e^2 = f^2{}_{13} e^{13} + f^2{}_{56} e^{56} \,,$$
$$\mathrm{d}e^3 = f^3{}_{12} e^{12} + f^3{}_{46} e^{46} \,, \qquad \mathrm{d}e^4 = f^4{}_{36} e^{36} + f^4{}_{15} e^{15} \,,$$
$$\mathrm{d}e^5 = f^5{}_{14} e^{14} + f^5{}_{26} e^{26} \,, \qquad \mathrm{d}e^6 = f^6{}_{34} e^{34} + f^6{}_{25} e^{25} \,.$$

- Metric g and Ω_I can be expressed in terms of the "moduli":

$$g = \frac{1}{\sqrt{v_1 v_2 v_3 v_4}} \Big(a v_3 v_4 \,,\ -b v_2 v_4 \,,\ c v_2 v_3 \,,\ -b v_1 v_3 \,,\ c v_1 v_4 \,,\ a v_1 v_2 \Big) \qquad \sqrt{v_1 v_2 v_3 v_4} = -abc$$

$$\Omega_I = \sqrt{v_1 v_2 v_3 v_4} \Big(v_1^{-1} e^{123} + v_2^{-1} e^{145} - v_3^{-1} e^{256} - v_4^{-1} e^{346} \Big)$$

Constructing SU(3) Structure

- Parity under orientifolding implies Im W_1= Im W_2= W_4 = W_5=0

  ➡ Half-flat SU(3) Structure Manifold

- Construct the remaining torsion classes:

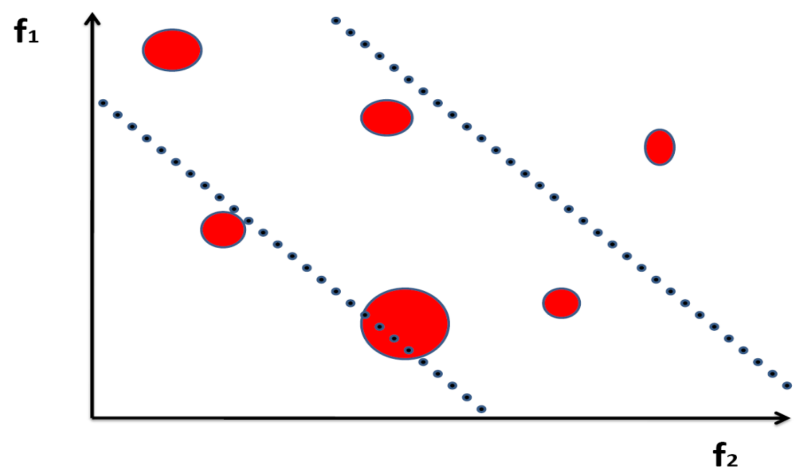
$$
\begin{aligned}
W_1 &= -\tfrac{1}{6} \star_6 (\mathrm{d}J \wedge \Omega_I)\,, \\
W_2 &= - \star \mathrm{d}\Omega_I + 2W_1 J\,, \\
W_3 &= \mathrm{d}J - \tfrac{3}{2} W_1 \Omega_R\,.
\end{aligned}
$$

- Search for dS solutions satisfying constraints obtained earlier.

A Mini Landscape

- # of unipotent 6D group spaces ~ $O(50)$. Among them, only a handful have de Sitter critical points that are compatible with orbifold/orientifold symmetries.
- Each of these group spaces has $O(10)$ left-invariant modes. Tadpole constraints restrict flux quanta on each cycle ≤ O(10).
- A sample space of $O(10^{10})$ solutions, no dS that is tachyon free.
- Flux quantization:

Pictorially



For SU(2)xSU(2) examples, can explicitly check flux quantization demands solutions outside SUGRA.

Probability Estimate

- Consider $V(\phi) = \sum_{j=1}^{N} V_j(\phi_j)$
- Then $V_{min}(\phi) = \sum_j V_{j,min}(\phi_j)$. If V_j has n_j minima, then there are $\prod n_j$ classical minima. For $n_j \sim n$, # minima = n^N [Susskind]. This is implicit in BP.
- Say V_j has $2n_j$ extrema, roughly half of which are minima.
- Probability for an extremum to be a minimum is

$$\mathcal{P} = 1/2^N = e^{-N \ln 2}$$

- Still, there are $\mathcal{P}$ x (# extrema) = $e^{N \ln n}$ minima.

Probability for de Sitter Vacua

- We are interested in dS vacua from string theory.
- The various Φ_j interact with each other. It is difficult to estimate how many minima there are.
- Explicit form of V is typically very complicated, e.g., in IIA:

$$V = e^K \left(K^{ij} D_{t^i} W \overline{D_{t^j} W} + K^{K\overline{L}} D_{N^K} W \overline{D_{N^L} W} - 3|W|^2 \right) + \frac{1}{2} (\mathrm{Re} f)^{-1^{\alpha\beta}} D_\alpha D_\beta$$

$$K = -2 \ln \left(-i \int e^{-2\phi} \Omega \wedge \Omega^* \right) - \ln \left(\frac{4}{3} \int J \wedge J \wedge J \right)$$

$$\sqrt{2} W = \int \left(\Omega_c \wedge (-iH + dJ_c) + e^{iJ_c} \wedge \hat{F} \right)$$

$$f_{\alpha\beta} = -\hat{\kappa}_{i\alpha\beta} t^i, \qquad \hat{\kappa}_{i\alpha\beta} = \int Y_i^{(2-)} \wedge Y_\alpha^{(2+)} \wedge Y_\beta^{(2+)},$$

$$D_\alpha = -\frac{e^{\phi_4}}{\sqrt{2\mathrm{vol}_6}} \hat{r}_\alpha^K \mathcal{F}_K, \qquad dY_\alpha^{(2+)} = \hat{r}_\alpha{}^K Y_K^{(3+)}.$$

$$J = k^i Y_i^{(2-)}$$

$$\Omega = \mathcal{F}_K Y_K^{(3-)} + i \mathcal{Z}^K Y_K^{(3+)}$$

$$J_c = J - iB = t^i Y_i^{(2-)}$$

$$\Omega_c = e^{-\phi} \mathrm{Im}(\Omega) + i C_3 = N^K Y_K^{(3+)}$$

Random Matrices

- The Hessian mass matrix $\boldsymbol{H} = V_{ij}$ at an extremum $V_i = 0$ must be positive definite for (meta)stability.
- We can use Sylvester's criterion to check whether there are tachyons, but time-consuming for a large Hessian $\boldsymbol{H}$.
- If the Hessian is large and complicated, how do we estimate the probability of an extremum to be a min.?
- Random matrix Theory (RMT) provides an estimate.

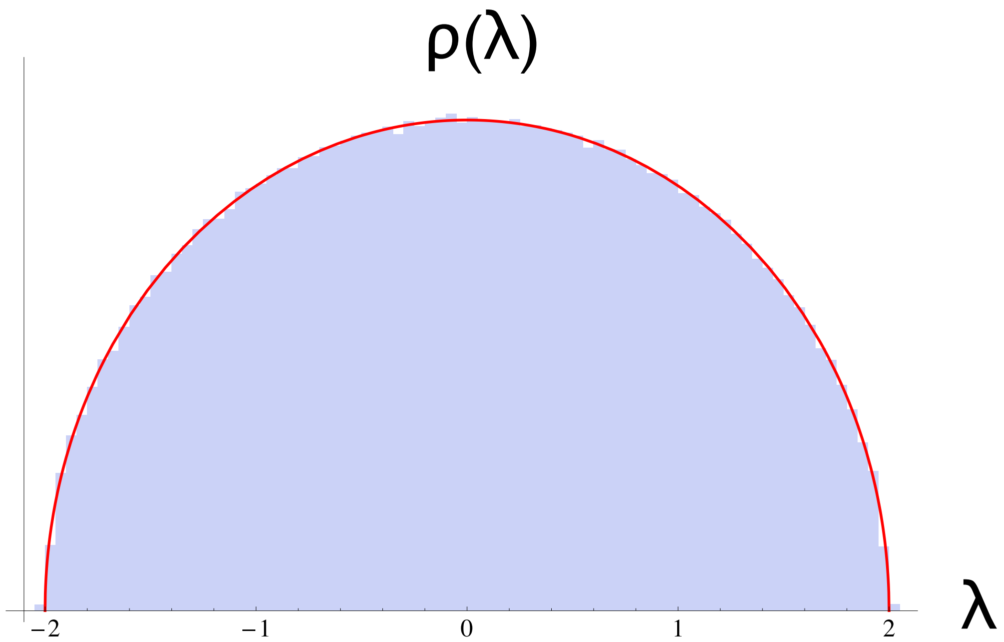
Random Matrix Theory

- A tool to study a large complicated matrix statistically [Wigner, Tracy-Widom, ...]
- Given a random ***H***, the theory of fluctuation of extreme eigenvalues allows one to compute the probability of drawing a *positive definite* matrix from the ensemble.
- Eigenvalue repulsion: probability for ***H*** to have no negative eigenvalue is *Gaussianly suppressed*.
- Some initial foray in applying these RMT results to cosmology was made [Aazami, Easther (2005)].

Wigner Ensemble

$$M = A + A^{\dagger}$$

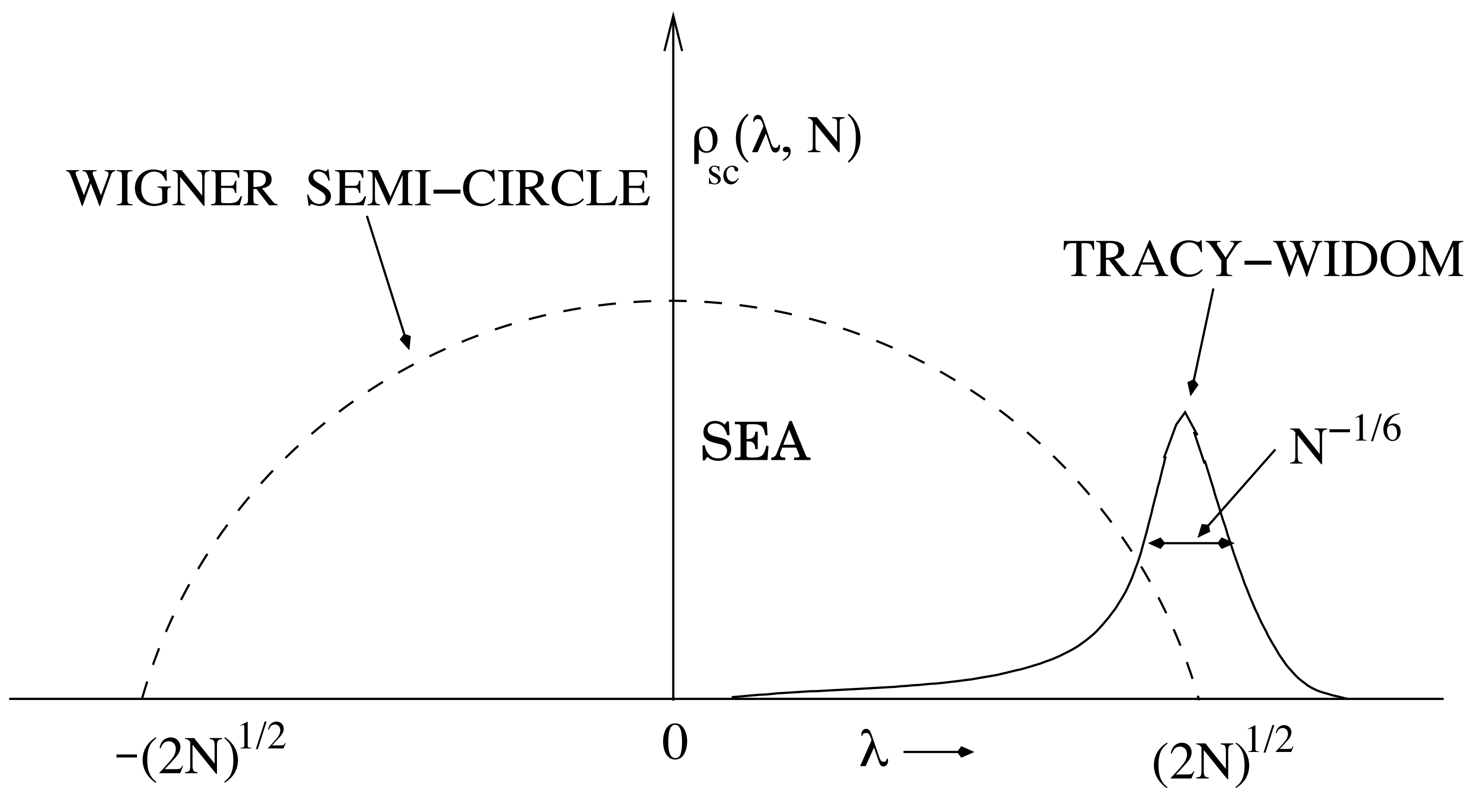
Dyson



Wigner's semi-circle

Elements of A are independent identically distributed variables drawn from some statistical distribution.

Tracy-Widom & Beyond



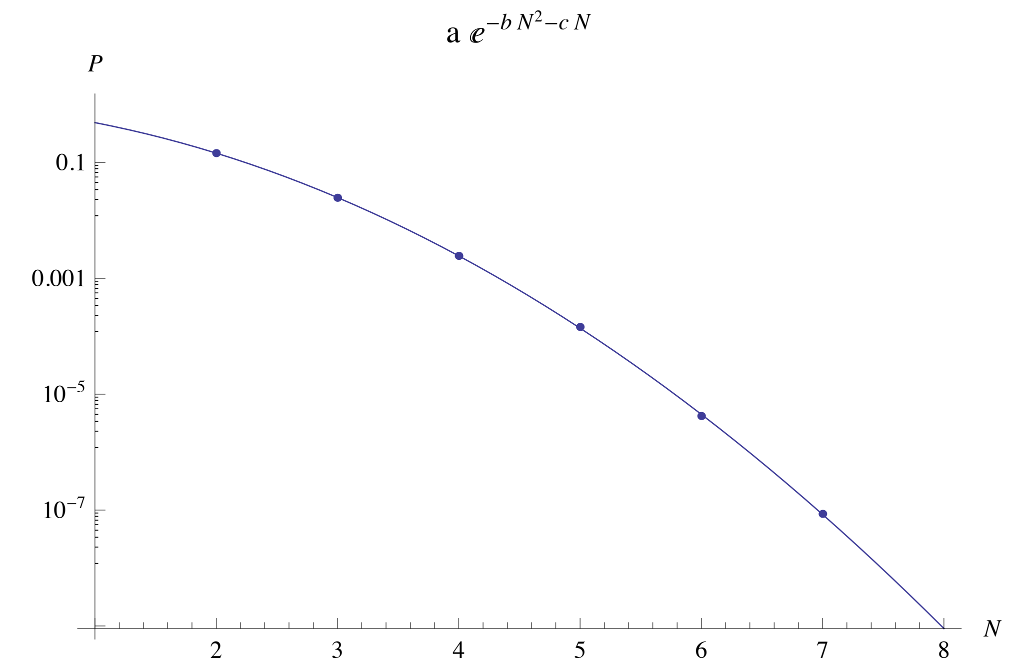
Study of the fluctuations of the smallest (largest) eigenvalue was initiated by Tracy-Widom, and generalized to large fluctuations by Dean and Majumdar (cond-mat/0609651).

Probability of Stability

Consider a Gaussian orthogonal ensemble

Probability of the form:

$$\mathcal{P} = a\, e^{-bN^2 - cN}$$ [Chen, GS, Sumitomo, Tye]



seems to work well, and agrees with:

The large N analytic result of Dean & Mujumdar $\mathcal{P} \approx e^{-\frac{\ln 3}{4} N^2}$
and further refinement by Borot et al:

$$\mathcal{P} = \exp\left[-\frac{\ln 3}{4} N^2 + \frac{\ln(2\sqrt{3}-3)}{2} N - \frac{1}{24} \ln N - 0.0172\right]$$

If the probability is Gaussianly suppressed, while # extrema goes like e^{cN} (recall 10^{500}), unlikely to find metastable vacua.

Random Supergravities

- Consider the SUGRA potential:

$$V = e^K \left(D^A W D_A W - 3|W|^2 \right)$$

and its Hessian, which is a function of $D_A W$, $D_A D_B W$, and $D_A D_B D_C W$, as well as W.

- Instead of randomizing elements of ***H***, one can randomize K, W, and its covariant derivatives [Denef, Douglas];[Marsh, McAllister, Wrase]

- This approach is applicable to F-term breaking, but not to D-term breaking, and models with explicit SUSY breaking.

- Also a different ansatz $\mathcal{P} = a e^{-bN^c}$ was used. Quantitative details differ, but $\mathcal{P}$ less likely than exponential also found.

Random Supergravities

The Hessian is well approximated by a sum of a Wigner matrix and two Wishart matrices.

$$M = A + A^\dagger \qquad\qquad M = AA^\dagger$$

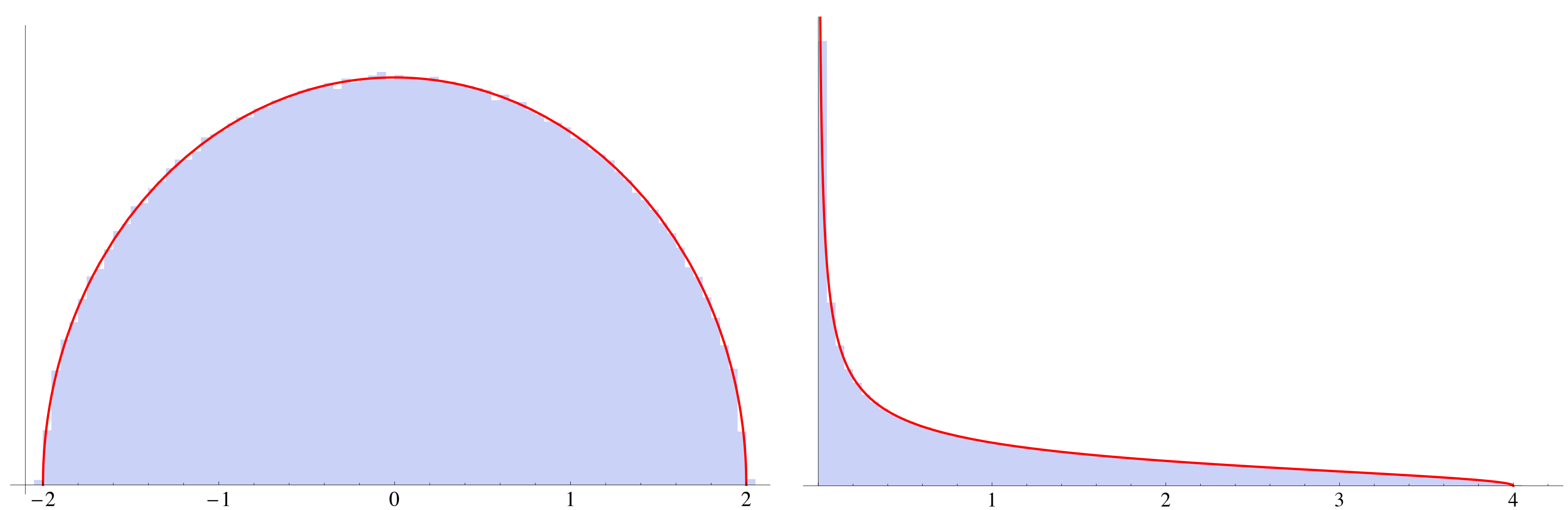


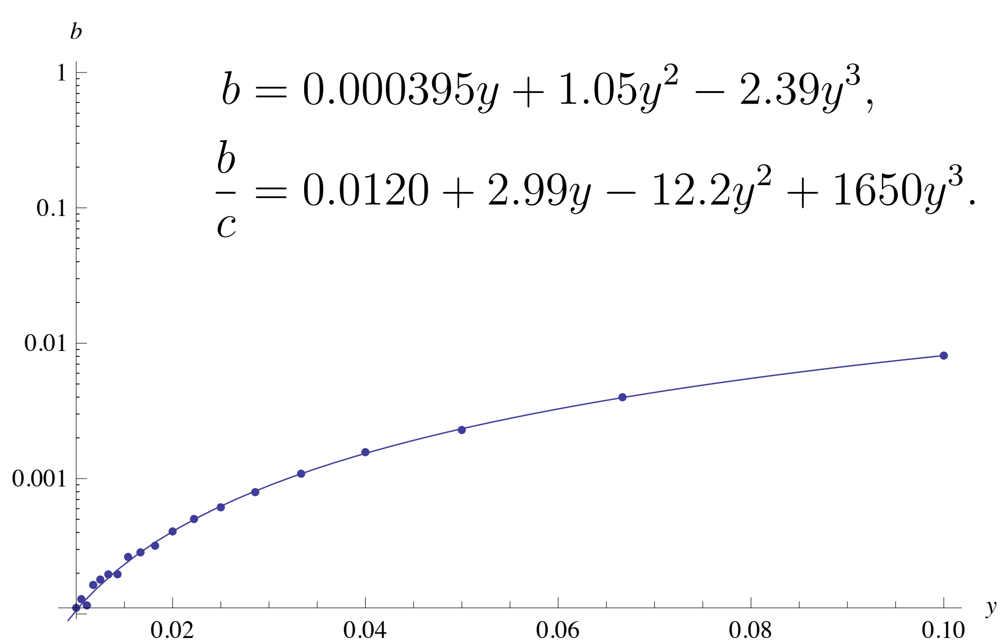
Figure 1: The eigenvalue spectra for the Wigner ensemble (left panel), and the Wishart ensemble with $N = Q$ (right panel), from 10^3 trials with $N = 200$.

IIA Flux Vacua

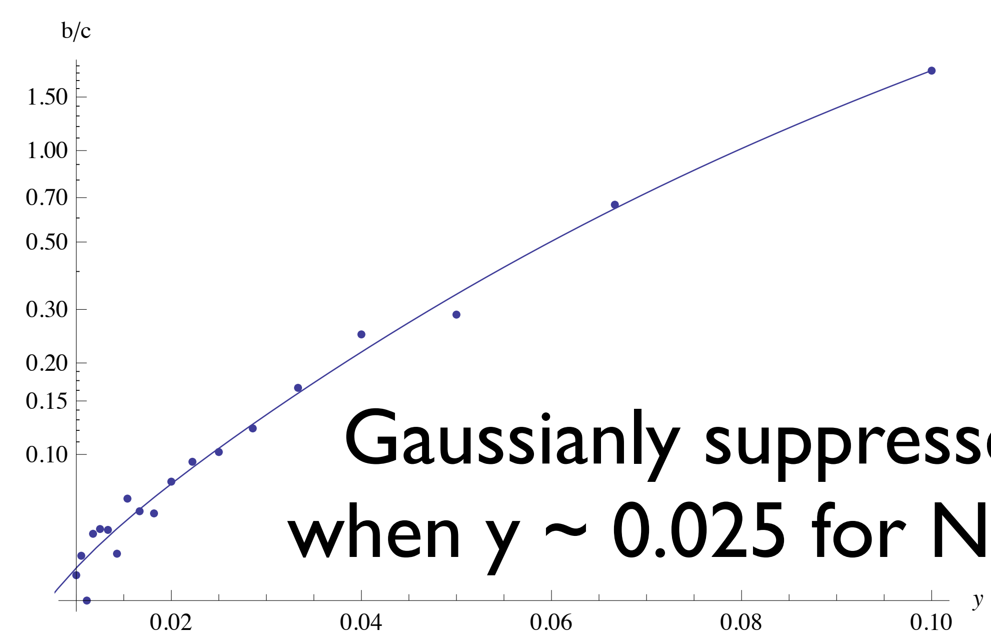
- An infinite family of AdS vacua are known to arise from flux compactifications of IIA SUGRA [Derendinger et al; Villadoro et al; De Wolfe et al; Camara et al].
- Attempts to construct IIA dS flux vacua often start with similar setups as SUSY AdS ones and then introduce new ingredients to uplift (e.g., negative curvature of internal space).
- We can model the Hessian as ***H*** = A + B where A= diagonal mass matrix at AdS min., B is uplift contribution.
- A does not have to be positive definite for stability, as long as the BF bound is satisfied. To play it safe, we start with a SUSY AdS vacuum with A=*positive definite* diagonal matrix.

IIA Flux Vacua

- We take B to be a randomized real symmetric matrix.
- A and B have variances σ_A and σ_B. The relative ratio $y=\sigma_B/\sigma_A$ determines the amount of uplift.
- The ansatz $\mathcal{P} = a\, e^{-bN^2-cN}$ works well when the mass matrix is not completely random, but has a hierarchy:



$$b = 0.000395y + 1.05y^2 - 2.39y^3,$$
$$\frac{b}{c} = 0.0120 + 2.99y - 12.2y^2 + 1650y^3.$$



Gaussianly suppressed when y ~ 0.025 for N=10

Chen, GS, Sumitomo, Tye

A Type IIA Example

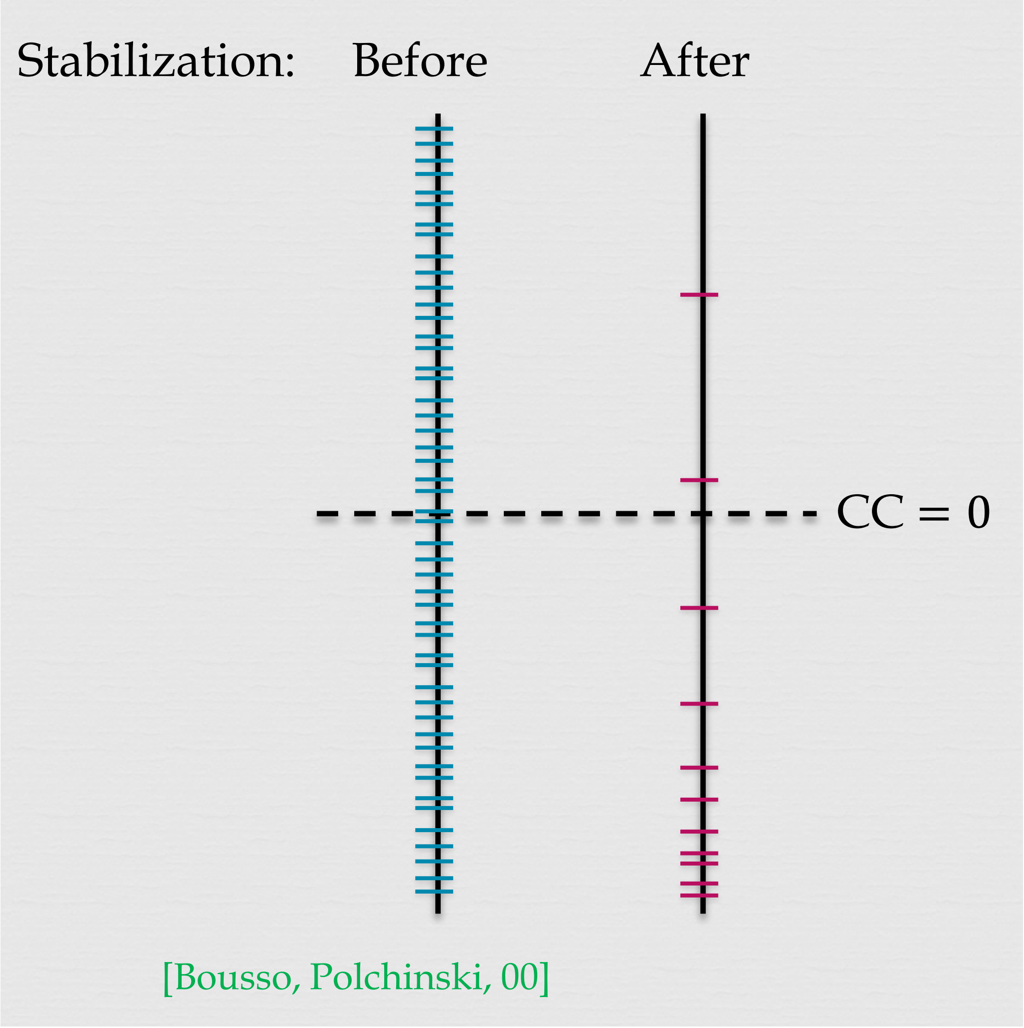
- Return to the SU(2)xSU(2) group manifold studied earlier in the systematic search of [Danielsson, Haque, Koerber, GS, Van Riet, Wrase]
- This model evades the no-goes for dS extrema and stability in the universal moduli subspace. There are 14 moduli.
- Evaluating the variance: $y \sim \left(\frac{\frac{1}{14\times 13/2}\sum_{A<B} M_{AB}^2}{\frac{1}{14}\sum_{A=1}^{14} M_{AA}^2} \right)^{1/2} = 0.274.$ >> 0.025.
- There is no surprise that tachyon appears.
- Tachyon appears in a 3x3 sub-Hessian. Chen, GS, Sumitomo, Tye
- In this model, η = V''/V ≲ -2.4 at the extremum, so the tachyon becomes more tachyonic as the CC increases.

CC and Stability

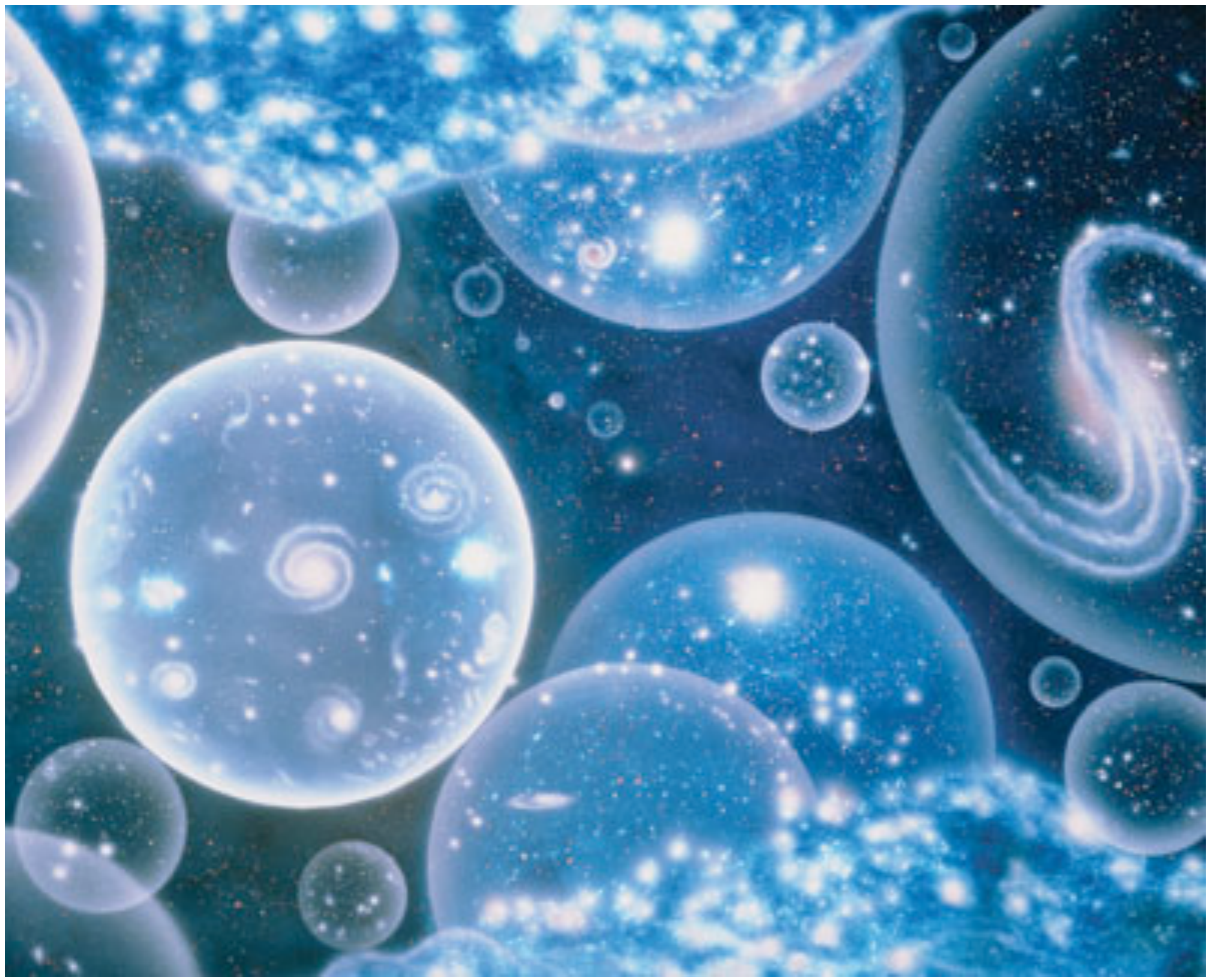
- As we lift the CC, the off-diagonal terms become bigger and the extremum becomes unstable.
- In general, we expect some moduli to be very heavy and essentially decouple from the light sector, so $N = N_H + N_L$.
- The # of extrema is controlled by N, while the fraction of stable critical points is controlled by N_L.
- Example: a 2-sector SUGR where some moduli have very large SUSY masses while SUSY is broken in a decoupled sector involving only the light moduli.
- As we go to higher energies, more moduli come into play (larger eff. N) ⇨ probability more Gaussianly suppressed.

Less Democratic Landscape

Raising the CC destabilizes the classically stable vacua.



Implications to the Landscape?



Summary

- No-go theorems for de Sitter vacua from string theory, and the *minimal* ingredients to evade them.
- Finding de Sitter solutions is hard.
- A systematic search for IIA dS vacua within a broad class of SU(3) structure manifolds has so far come up empty.
- Finding de Sitter vacua is even harder.
- In some situations, the probability of finding de Sitter vacua is Gaussianly suppressed.
- This may point to a different picture of the landscape.

THANKS