A new neutrino mass sum-rule from inverse seesaw.

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[1] L. Dorame, S. Morisi, E. Peinado, J. W. F. Valle and A. D. Rojas, arXiv:1203.0155 [hep-ph].

- Some flavor models based in non-Abelian discrete symmetries, implying the TBM mixing matrix, predict a two-parameter neutrino mass matrix.
- Four mass relations can arise [2]

$$
\begin{aligned}
\chi m_2^\nu + \xi m_3^\nu &= m_1^\nu, \\
\frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} &= \frac{1}{m_1^\nu}, \\
\chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} &= \sqrt{m_1^\nu},
\end{aligned}
$$

$$
\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}.
$$

[2] L. Dorame, D. Meloni, S. Morisi, E. Peinado and J. W. F. Valle, Nucl. Phys. B **861**, 259 (2012) [arXiv:1111.5614 [hep-ph]].

The inverse seesaw mechanism

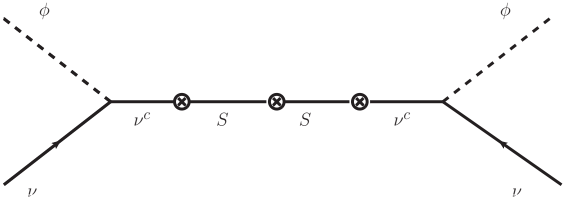
- First example of a low-scale seesaw scheme with naturally light neutrinos.
- Particle content= SM + ν_i^c and S_i.
- In the ν, ν^c, S basis:

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}$$

From diagonalization[3] one obtains the effective light neutrino mass matrix as:

$$m_\nu \sim m_D^T \frac{1}{M^T} \mu \frac{1}{M} m_D,$$

with the entry μ being very small.



3 J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 774 (1982).

Inverse Seesaw schemes

Possible schemes realizing the TBM pattern for the inverse seesaw case[4]

m_D	M	μ
$\mathcal{I}$	$\mathcal{I}$	M_0
$\mathcal{I}$	M_0	$\mathcal{I}$
M_0	$\mathcal{I}$	$\mathcal{I}$

$$\mathcal{I} = \text{Identity matrix}, \qquad M_0 = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}.$$

[4] M. Hirsch, S. Morisi and J. W. F. Valle, Phys. Lett. **B679**, 454 (2009), [0905.3056].

So, if

$$m_D \propto \mathcal{I}, \qquad \mu \propto \mathcal{I}$$

and

$$M \sim M_{TBM} = \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix},$$

then we obtain

$$\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}. \tag{1}$$

Field particles content

In order to obtain the S_4 based inverse seesaw model we assign the quantum numbers of the SM fields (plus the pair of gauge singlets ν_i^c, S_i) under the extra symmetries as

	$\overline{L}$	ν_R	l_R	h	S
$SU(2)$	2	1	1	2	1
S_4	3_1	3_1	3_1	1_1	3_1
$U_l(1)$	-1	1	1	0	-1

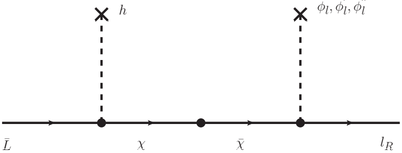
Required couplings

Neutral leptons:

- $\nu_L \nu_R \sim \mathbf{3}_1 \times \mathbf{3}_1 \quad \overbrace{(1,+)}^{(Z_3, Z_2)}$
- $\nu_R S \sim \mathbf{3}_1 \times \mathbf{3}_1 \quad (\omega, -)$
- $SS \sim \mathbf{3}_1 \times \mathbf{3}_1 \quad (1,+)$

Charged Leptons:

- $\bar{L}_L l_R h \sim \mathbf{3}_1 \times \mathbf{3}_1, \ \overbrace{(\omega^2,+)}^{(Z_3, Z_2)}$



We keep renormalizability of the Lagrangian by adding a Frogatt-Nielsen fermion χ and its conjugate χ^c.

Required extra fields

We introduce five flavon fields supplemented by the extra symmetries Z3 and Z2

	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	1	1	1	1	1	1	1	1
S_4	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	0	0	0	0	0	2	1	-1

Fields and their transformation properties under the Z_3, and Z_2 flavor symmetries

	$\bar{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
Z_3	ω^2	ω	1	1	1	ω^2	ω^2	ω	ω	ω	1	ω	ω^2
Z_2	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$

Alignments

Flavon fields alignments:

$$\langle \phi_\nu \rangle = v_\nu (1,0,0),$$
$$\langle \phi_l \rangle = v_l (1,1,1),$$
$$\langle \phi'_l \rangle = v'_l (1,1,1),$$

and also

$$\langle \phi'_\nu \rangle = v'_\nu,$$
$$\langle \phi''_l \rangle = v''_l,$$
$$\langle \sigma \rangle = v_\sigma,$$
$$\langle h \rangle = v.$$

With these alignments the three 3×3 blocks are

$$\mu_{ij} S_i S_j \sigma \quad \longrightarrow \quad (\mu) = \begin{pmatrix} \mu v_\sigma & 0 & 0 \\ 0 & \mu v_\sigma & 0 \\ 0 & 0 & \mu v_\sigma \end{pmatrix}$$

$$Y_{D_{ij}} \overline{L}_i \nu_{R_j} h \quad \longrightarrow \quad M_D = \begin{pmatrix} Y_D v & 0 & 0 \\ 0 & Y_D v & 0 \\ 0 & 0 & Y_D v \end{pmatrix},$$

$$Y^k_{\nu ij} \nu_{R_i} S_j \phi_{\nu_k} + Y_{\nu ij} \nu_{R_i} S_j \phi_{\nu'} \quad \longrightarrow \quad M = \begin{pmatrix} Y'_\nu v'_\nu & 0 & 0 \\ 0 & Y'_\nu v'_\nu & Y_\nu v_\nu \\ 0 & Y_\nu v_\nu & Y'_\nu v'_\nu \end{pmatrix},$$

and

$$M_l = \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix}, \qquad \begin{aligned} \alpha &= y_l'' v_l'' \tfrac{v}{\Lambda}, \\ \beta &= y_l v_l - y_l' v_l' \tfrac{v}{\Lambda}, \\ \gamma &= y_l v_l + y_l' v_l' \tfrac{v}{\Lambda}, \end{aligned}$$

which is diagonalized by the "magic" matrix

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \qquad \begin{aligned} &\omega^3 = 1, \\ &1 + \omega + \omega^2 = 0. \end{aligned}$$

After diagonalization, the light neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix}$$

where

$$a = Y'_\nu v'_\nu / (\sqrt{\mu v_\sigma} Y_D v),$$

and

$$b = Y_\nu v_\nu / (\sqrt{\mu v_\sigma} Y_D v).$$

The corresponding eigenvalues are

$$m_1 = \tfrac{1}{(a+b)^2},$$
$$m_2 = \tfrac{1}{(a-b)^2},$$
$$m_3 = \tfrac{1}{a^2}.$$

With these eigenvalues we obtain the desired neutrino mass sum-rule

$$\frac{1}{\sqrt{m_1}} = \frac{2}{\sqrt{m_3}} - \frac{1}{\sqrt{m_2}}.$$

i.e.

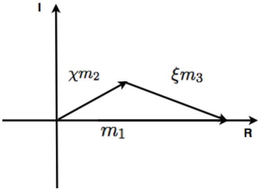
$$(\xi, \chi) = (1, 2)$$

Neutrinoless double beta decay

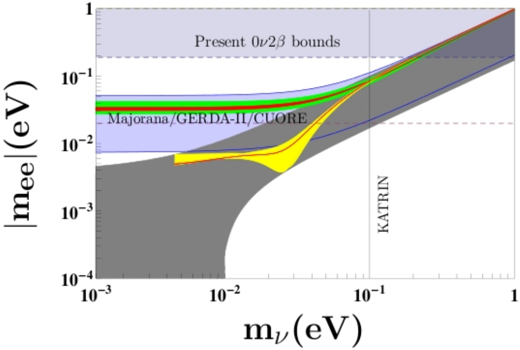
$$|m_{ee}| = \left| c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{2i\phi_{12}} + s_{13}^2\, m_3\, e^{2i\phi_{13}} \right| \quad \text{(symmetrical).}$$

We can interpret geometrically the neutrino mass sum-rule

- $m_1^\nu = m_1^0$
  $m_2^\nu = m_2^0 e^{\imath\phi_{12}}$
  $m_3^\nu = m_3^0 e^{\imath\phi_{13}}$
- $\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}} \Longrightarrow$
- $(\xi, \chi) = (1, 2)$



$|m_{ee}|$ as a function of the lightest neutrino mass corresponding to our mass sum-rule



Quark sector

S_4 charge assignments:

$$Q_D = (Q_1, Q_2) \sim \mathbf{2} \quad \oplus \quad Q_s = Q_3 \sim \mathbf{1}_1$$
$$q_{R_D} = (q_{R_1}, q_{R_2}) \sim \mathbf{2} \quad \oplus \quad q_{R_3} \sim \mathbf{1}_1$$

$$\phi_D \sim \mathbf{2} \quad \oplus \quad \phi_S \sim \mathbf{1}_1$$

Quark sector and their transformation properties under the Z_3, and Z_2 flavor symmetries

	$\overline{Q}_D$	$\overline{Q}_S$	u_{R_D}	u_{R_S}	d_{R_D}	d_{R_S}	ϕ_D	ϕ_S
$SU(2)$	2	2	1	1	1	1	1	1
S_4	2	1_1	2	1_1	2	1_1	2	1_1
Z_3	ω	ω	ω^2	ω^2	ω	ω	ω	ω
Z_2	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$

The effective lagrangian terms for quarks are

$$\begin{aligned}\mathcal{L}_q^d &= (Y_1^d\overline{Q}_S d_{R_S}\phi_S + Y_2^d\overline{Q}_D\phi_D d_{R_D} + Y_3^d\overline{Q}_D d_{R_D}\phi_S + Y_4^d\overline{Q}_D\phi_D d_{R_S} \\ &\quad + Y_5^d\overline{Q}_S\phi_D d_{R_D})h/\Lambda + h.c.,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_q^u &= (Y_1^u\overline{Q}_S u_{R_S}\tilde{\phi}_S + Y_2^u\overline{Q}_D\tilde{\phi}_D u_{R_D} + Y_3^u\overline{Q}_D u_{R_D}\tilde{\phi}_S + Y_4^u\overline{Q}_D\tilde{\phi}_D u_{R_S} \\ &\quad + Y_5^u\overline{Q}_S\tilde{\phi}_D u_{R_D})\tilde{h}/\Lambda + h.c.\end{aligned}$$

The dimension five operators can be given in terms of renormalizable interaction by introducing suitable messenger fields.

Taking the VEV of ϕ_D in the direction

$$\langle \phi_D \rangle \sim (-\sqrt{3}, 1) \ ,$$

the mass matrix for quarks is[5]

$$\mathbf{M}_{u(d)} = \begin{pmatrix} m_1^{u(d)} + m_2^{u(d)} & -\sqrt{3}\ m_2^{u(d)} & -\sqrt{3}\ m_5^{u(d)} \\ -\sqrt{3}\ m_2^{u(d)} & m_1^{u(d)} - m_2^{u(d)} & m_5^{u(d)} \\ -\sqrt{3}\ m_4^{u(d)} & m_4^{u(d)} & m_3^{u(d)} \end{pmatrix} .$$

[5] Similar to the one in reference J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109**, 795 (2003), [hep-ph/0302196]

Finite θ_{13} value

We can obtain corrections from the charged lepton sector by coupling an extra S_4-doublet flavon field inducing nonzero values of θ_{13}.

- For example, consider

$$\phi \sim \mathbf{2}, \text{ with } (\omega, +) \text{ under } Z_3 \times Z_2$$

- then, we must include the term

$$(\bar{L} l_R) h \phi.$$

Assuming

$$\langle\phi\rangle = (u_1, u_2),$$

a natural vacuum alignment is

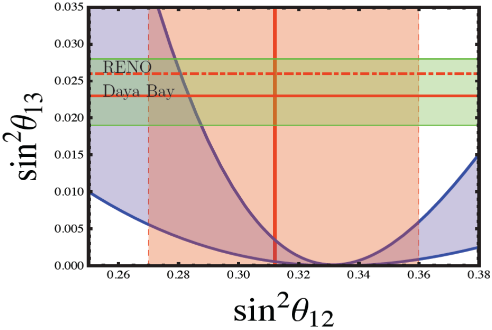
$$u_1 = -\sqrt{3}u_2.$$

The contribution from term $(\bar{L}l_R)h\phi$ to the charged lepton mass matrix is

$$\delta M_l = \begin{pmatrix} -\sqrt{\frac{2}{3}}vu_2 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}}vu_1 + \sqrt{\frac{1}{6}}vu_2 & 0 \\ 0 & 0 & -\sqrt{\frac{1}{2}}vu_1 + \sqrt{\frac{1}{6}}vu_2 \end{pmatrix}$$

Then, the total $M_l + \delta M_l$ is no longer diagonalized by U_ω.

Correlations between reactor and solar neutrino mixing angles.



Conclusions

- We have constructed an S_4 based model, implementing the inverse seesaw mechanism, which predicts the TBM mixing matrix and renders a new mass sum-rule.
- Corrections to the θ_{13} value can be obtained from the charged sector introducing a flavon doublet with appropriate alignmnet.
- The deviation of θ_{13} from zero can be substantial provided the departure of θ_{12} from its TBM value is also large.
- The model is consistent with the measurements of the two recent reactor experiments, only if the solar angle lies substantially below the TBM prediction (at 2σ).

Lepton sector Lagrangians

The renormalizable Lagrangian relevant for neutrinos is

$$\mathcal{L}_\nu = Y_{D_{ij}} \overline{L}_i \nu_{R_j} h + Y^k_{\nu ij} \nu_{R_i} S_j \phi_{\nu_k} + Y'_{\nu ij} \nu_{R_i} S_j \phi'_\nu + \mu_{ij} S_i S_j \sigma,$$

while the renormalizable Yukawa terms involving the messenger fields are

$$\mathcal{L}_\chi = M_\chi \chi \chi^c + \bar{L} h \chi + \chi^c l_R \phi_l + \chi^c l_R \phi'_l + \chi^c l_R \phi''_l .$$

After integrating out the messenger fields χ, the effective Lagrangian for charged leptons takes the form

$$\mathcal{L}_l = \frac{y_l}{\Lambda}(\bar{L} l_R) h \phi_l + \frac{y_l'}{\Lambda}(\bar{L} l_R) h \phi_l' + \frac{y_l''}{\Lambda}(\bar{L} l_R) h \phi_l'',$$

where Λ is the effective scale.