Constraining neutrinoless double beta decay

Luis Doramé

In collaboration with D. Meloni, S. Morisi, E. Peinado & J. W. F. Valle

Based on: Nucl.Phys. B861 (2012)

PASCOS, 5 June 2012

Outline

- **Neutrino oscillations**

- **Mass sum rules (MSR) in the context of flavour models**

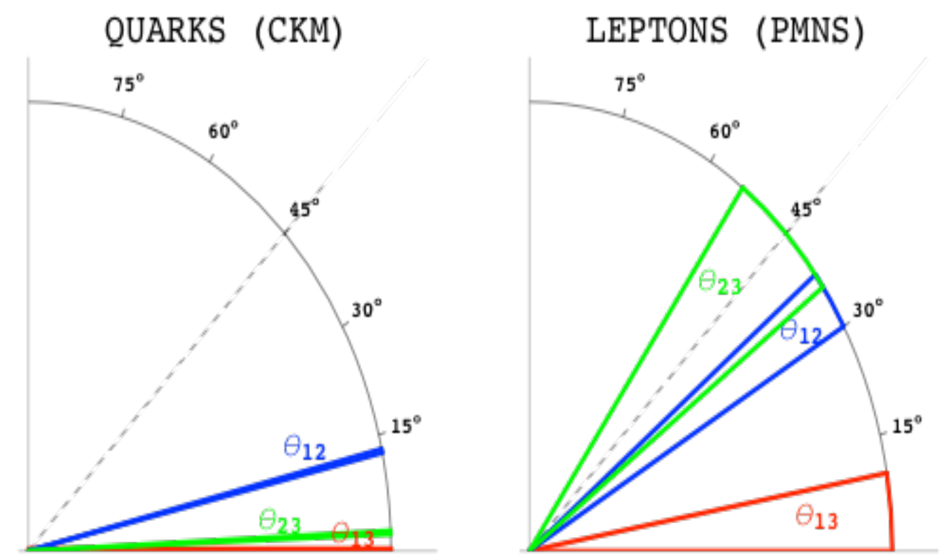
- **Neutrinoless double beta decay (0νββ)**

- **A novel S4 model**

- **Summary**

Lepton mixing matrix

parameter	best fit $\pm 1\sigma$	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
$\Delta m^2_{31}\ [10^{-3}\mathrm{eV}^2]$	$2.50^{+0.09}_{-0.16}$ $-(2.40^{+0.08}_{-0.09})$	$2.25 - 2.68$ $-(2.23 - 2.58)$	$2.14 - 2.76$ $-(2.13 - 2.67)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	$0.52^{+0.06}_{-0.07}$ 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.013^{+0.007}_{-0.005}$ $0.016^{+0.008}_{-0.006}$	0.004–0.028 0.005–0.031	0.001–0.035 0.001–0.039
δ	$\left(-0.61^{+0.75}_{-0.65}\right)\pi$ $\left(-0.41^{+0.65}_{-0.70}\right)\pi$	$0 - 2\pi$	$0 - 2\pi$



T schwetz, M. A. Tortola and J. W. F. Valle, (2011) arxiv:1108.1376v1

Symmetrical parametrization

$$K = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{-i\phi_{12}} & s_{13}e^{-i\phi_{13}} \\ -s_{12}c_{23}e^{i\phi_{12}} - c_{12}s_{13}s_{23}e^{-i(\phi_{23}-\phi_{13})} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i(\phi_{12}+\phi_{23}-\phi_{13})} & c_{13}s_{23}e^{-i\phi_{23}} \\ s_{12}s_{23}e^{i(\phi_{12}+\phi_{23})} - c_{12}s_{13}c_{23}e^{i\phi_{13}} & -c_{12}s_{23}e^{i\phi_{23}} - s_{12}s_{13}c_{23}e^{-i(\phi_{12}-\phi_{13})} & c_{13}c_{23} \end{pmatrix}$$

Schechter, J. W. F. Valle , , PRD22 (1982) 2227
For a recent discussion:
Rodejohann and J. W. F. Valle, (2011)
arXiv:1108.3484

Ansatz for the lepton mixing matrix values and non-Abelian discrete symmetries

Tri - Bi - maximal

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

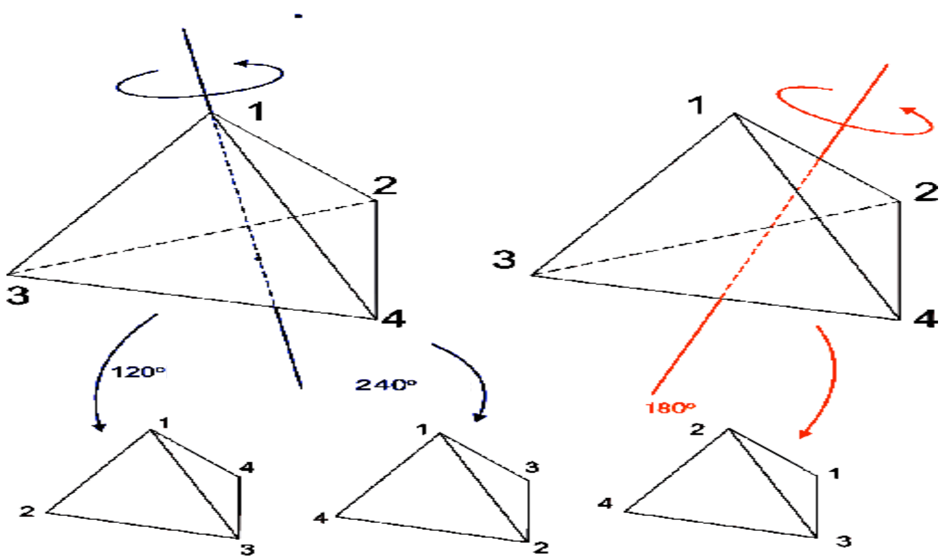
Harrison, Perkins, Scott, 2002

$$\theta_{12} \fallingdotseq 35^\circ, \quad \theta_{23} = \pi/4, \quad \theta_{13} = 0$$

Bi-maximal, Golden ratio

Altarelli, Feruglio, Meloni, Barger, S. Pakvasa, T. J. Weiler, K. Whisnant, W. Grimus , L. Lavoura, ...
For a full discussion on alternatives of TBM see Albrigth, Dueck, Rodejohann, (2010) 1004.2798v1

A_4



S_4, T′, S_3, D_4 ,...

E. Ma, G. Altarelli, F. Feruglio, Babu, S Kaneko, S Morisi, H. Zhang, T Toma, ...

Mass sum rules (MSR)

A)	$\chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$
B)	$\dfrac{\chi}{m_2^\nu} + \dfrac{\xi}{m_3^\nu} = \dfrac{1}{m_1^\nu}$
C)	$\chi\,\sqrt{m_2^\nu} + \xi\,\sqrt{m_3^\nu} = \sqrt{m_1^\nu}$

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C)	$\chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} = \sqrt{m_1^\nu}$
D)	$\dfrac{\chi}{\sqrt{m_2^\nu}} + \dfrac{\xi}{\sqrt{m_3^\nu}} = \dfrac{1}{\sqrt{m_1^\nu}}$

Example

Case B)

An A4 simplest model, Altarelli, Meloni (2009)

$$w_\nu = y_\nu (\nu^c \ell)\, h_u + (M + a\,\xi)\, \nu^c \nu^c + b\, \nu^c \nu^c\, \varphi_S$$

$$m_D = y_\nu\, v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = v_u\, Y_\nu \qquad m_M = \begin{pmatrix} M + a\,u + 2\,b\,v_S & -b\,v_S & -b\,v_S \\ -b\,v_S & 2\,b\,v_S & M + a\,u - b\,v_S \\ -b\,v_S & M + a\,u - b\,v_S & 2\,b\,v_S \end{pmatrix}$$

Seesaw formula

$$m_{light} = -m_D^T\, m_M^{-1}\, m_D$$

All matrices are of the general form

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Are diagonalized by

$$U_{\rm TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

m_{light} eigenvalues

$$\begin{aligned} m_1 &= -\left(\frac{v_u^2\, y_\nu^2}{M + a\,u + 3\,b\,v_S}\right) \\ m_2 &= -\left(\frac{v_u^2\, y_\nu^2}{M + a\,u}\right) \\ m_3 &= \left(\frac{v_u^2\, y_\nu^2}{M + a\,u - 3\,b\,v_S}\right). \end{aligned}$$

Sum rule between mass eigenstates $\dfrac{1}{m_3} = \dfrac{1}{m_1} - \dfrac{2}{m_2}$

Case B)

From seesaw type I

$$M^\nu = -m_D M_R^{-1} m_D^T$$

$$M_R \sim M_{TBM} \equiv \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix} \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↓

$$m_i^\nu \propto 1/(\alpha_i a + \beta_i b)$$

↓

$$\frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}$$

J. Barry, W. Rodejohann, G. J. Ding, F. Bazzocchi, L. Merlo, S. Morisi, ...

Inverse Seesaw

Effective neutrino mass matrix

$$\begin{pmatrix} \nu & \nu_R & S \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} \nu \\ \nu_R \\ S \end{pmatrix} \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}$$

R. Mohapatra, J. W. F. Valle (PRD 34 (1986))

Light neutrinos eigenvalues

$$m_\nu = M_D {M^T}^{-1} \mu M^{-1} M_D^T$$

MSR from Inverse Seesaw

Light neutrinos eigenvalues

$$m_\nu = M_D {M^T}^{-1} \mu M^{-1} M_D^T$$

TBM type, an example

$$M_0 \sim TBM$$

cases	1)	2)	3)
M_D	M_0	I	I
M	I	M_0	I
μ	I	I	M_0

M. Hirsch, S. Morisi and J. W. F. Valle, 2008

Case 1) is analogous to Case C)

$$\chi \sqrt{m_2^\nu} + \xi \sqrt{m_3^\nu} = \sqrt{m_1^\nu}$$

Case 3) corresponds to the dimension 5 operator SMR, case A)

$$\chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$$

Case 2) corresponds to CASE D)

$$\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}$$

Neutrinoless double beta decay

$0\nu\beta\beta$

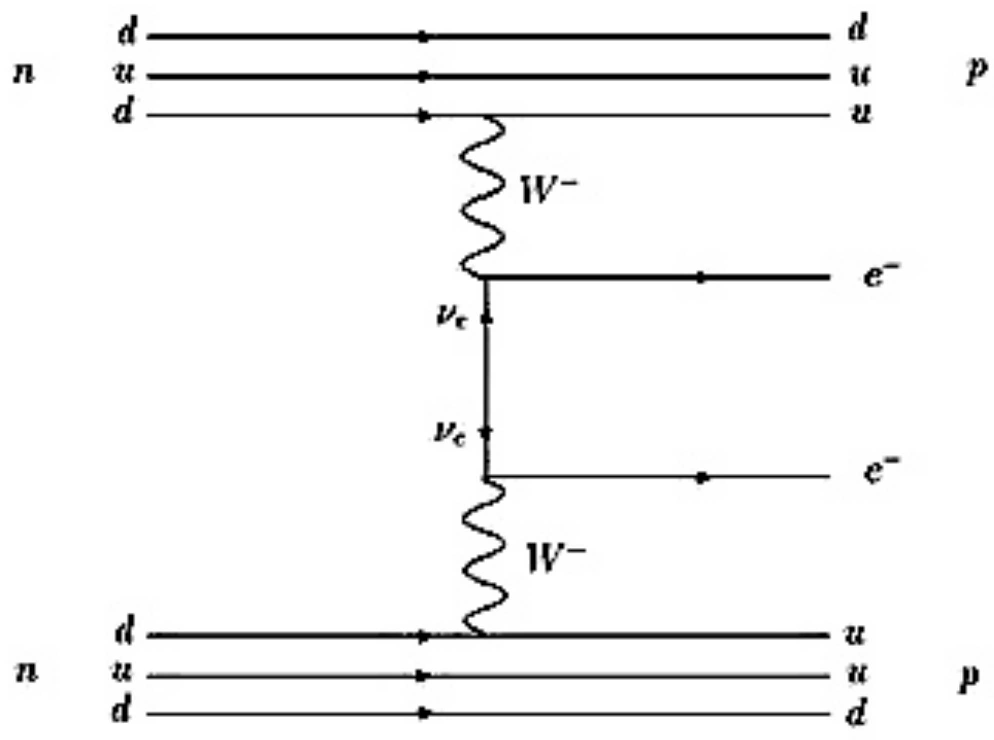
- Violates lepton number by 2 units

- Experimentally not observed

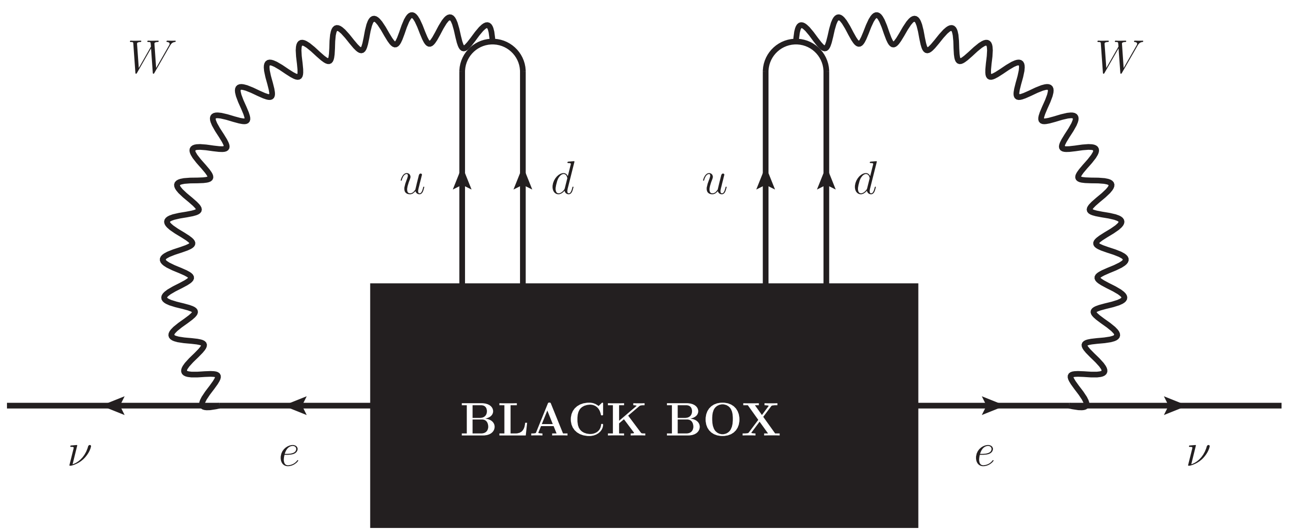
- $T^{0\nu\beta\beta}_{1/2}(^{76}Ge) \geq 10^{25}$ years

-Observation implies physics beyond the standard model

$(A,Z) \rightarrow (A, Z+2) + 2e^-$



Black Box theorem



Schechter and J. W. F. Valle, PRD22 (1982) 2227
For recent discussion see Duerr, Lindner, Merle (2011)1105.0901

Neutrinoless double beta decay

$0\nu\beta\beta$

Isotope	$T_{1/2}^{0\nu}$ [yrs]	Experiment	$\langle m_{ee}\rangle_{\rm min}^{\rm lim}$ [eV]	$\langle m_{ee}\rangle_{\rm max}^{\rm lim}$ [eV]
^{48}Ca	5.8×10^{22}	CANDLES[18]	3.55	9.91
^{76}Ge	1.9×10^{25}	HDM[19]	0.21	0.53
	1.6×10^{25}	IGEX[20]	0.25	0.63
^{82}Se	3.2×10^{23}	NEMO-3[21]	0.85	2.08
^{96}Zr	9.2×10^{21}	NEMO-3[22]	3.97	14.39
^{100}Mo	1.0×10^{24}	NEMO-3[21]	0.31	0.79
^{116}Cd	1.7×10^{23}	SOLOTVINO[23]	1.22	2.30
^{130}Te	2.8×10^{24}	CUORICINO[24]	0.27	0.57
^{136}Xe	5.0×10^{23}	DAMA[25]	0.83	2.04
^{150}Nd	1.8×10^{22}	NEMO-3[26]	2.35	5.08

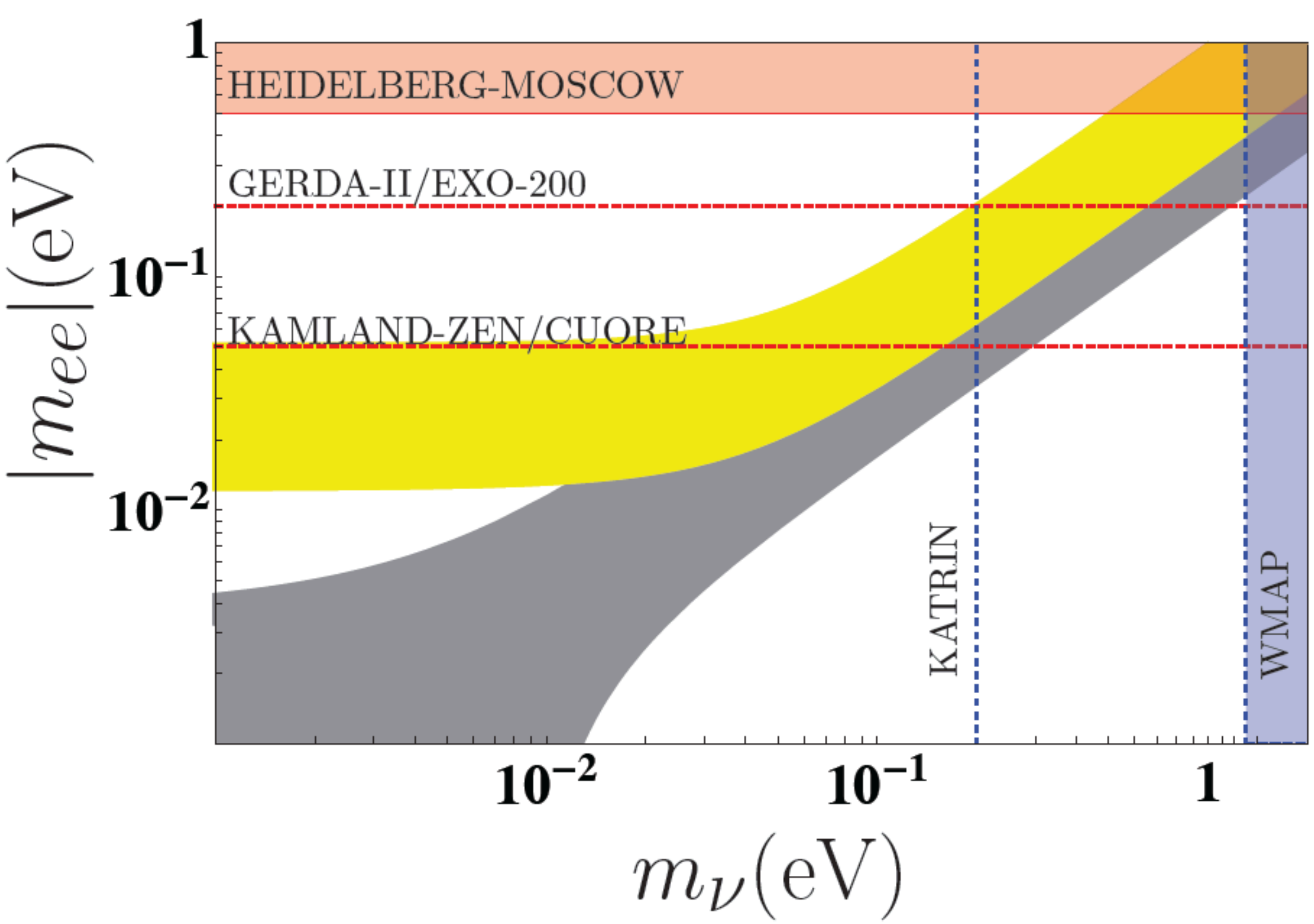
Rodejohann, Acta Phys.Polon. B43 (2012) 71-78

Neutrinoless double beta decay

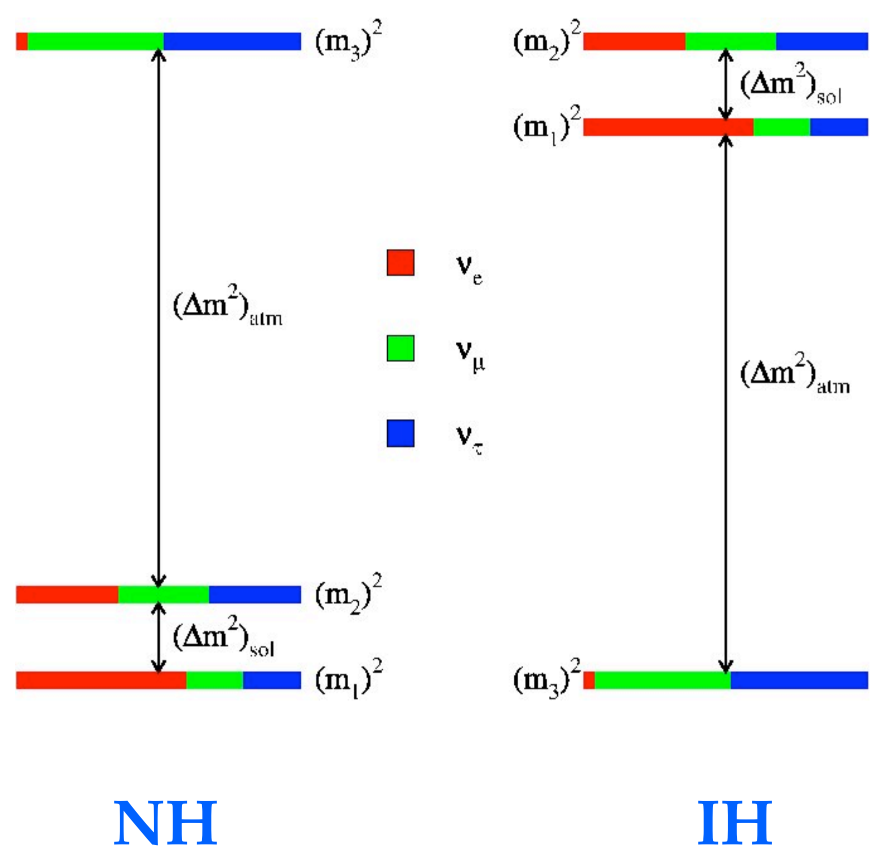
$0\nu\beta\beta$

Effective Majorana mass term

$$\langle m\rangle = \left|\sum_j U_{ej}^2\, m_j\right| = \begin{cases} \left|c_{12}^2 c_{13}^2\, m_1\, e^{2i\alpha} + s_{12}^2 c_{13}^2\, m_2\, e^{2i\beta} + s_{13}^2\, m_3\, e^{2i\delta}\right| & \text{(PDG)}, \\ \left|c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{2i\phi_{12}} + s_{13}^2\, m_3\, e^{2i\phi_{13}}\right| & \text{(symmetrical)} \end{cases}$$



Mass Hierarchy



An example
Case A) and NH

$$\chi\, m_2^{\nu} + \xi\, m_3^{\nu} = m_1^{\nu}$$

$m_1^{\nu} = m_1^0 \quad m_2^{\nu} = m_2^0 e^{i\alpha} \quad m_3^{\nu} = m_3^0 e^{i\beta}$

An example
Case A) and NH

$$\chi\, m_2^{\nu} + \xi\, m_3^{\nu} = m_1^{\nu}$$

$$m_1^{\nu} = m_1^0 \quad m_2^{\nu} = m_2^0 e^{i\alpha} \quad m_3^{\nu} = m_3^0 e^{i\beta}$$

Real equation

$$\xi \cos\alpha\, m_2^0 + \chi \cos\beta\, m_3^0 = m_1^0$$

Imaginary equation

$$\xi \sin\alpha\, m_2^0 + \chi \sin\beta\, m_3^0 = 0$$

An example
Case A) and NH

$$\chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$$

$m_1^\nu \;= m_1^0 \quad m_2^\nu \;= m_2^0 e^{i\alpha} \quad m_3^\nu \;= m_3^0 e^{i\beta}$

Real equation

$$\xi \cos\alpha\, m_2^0 + \chi \cos\beta\, m_3^0 = m_1^0$$

Imaginary equation

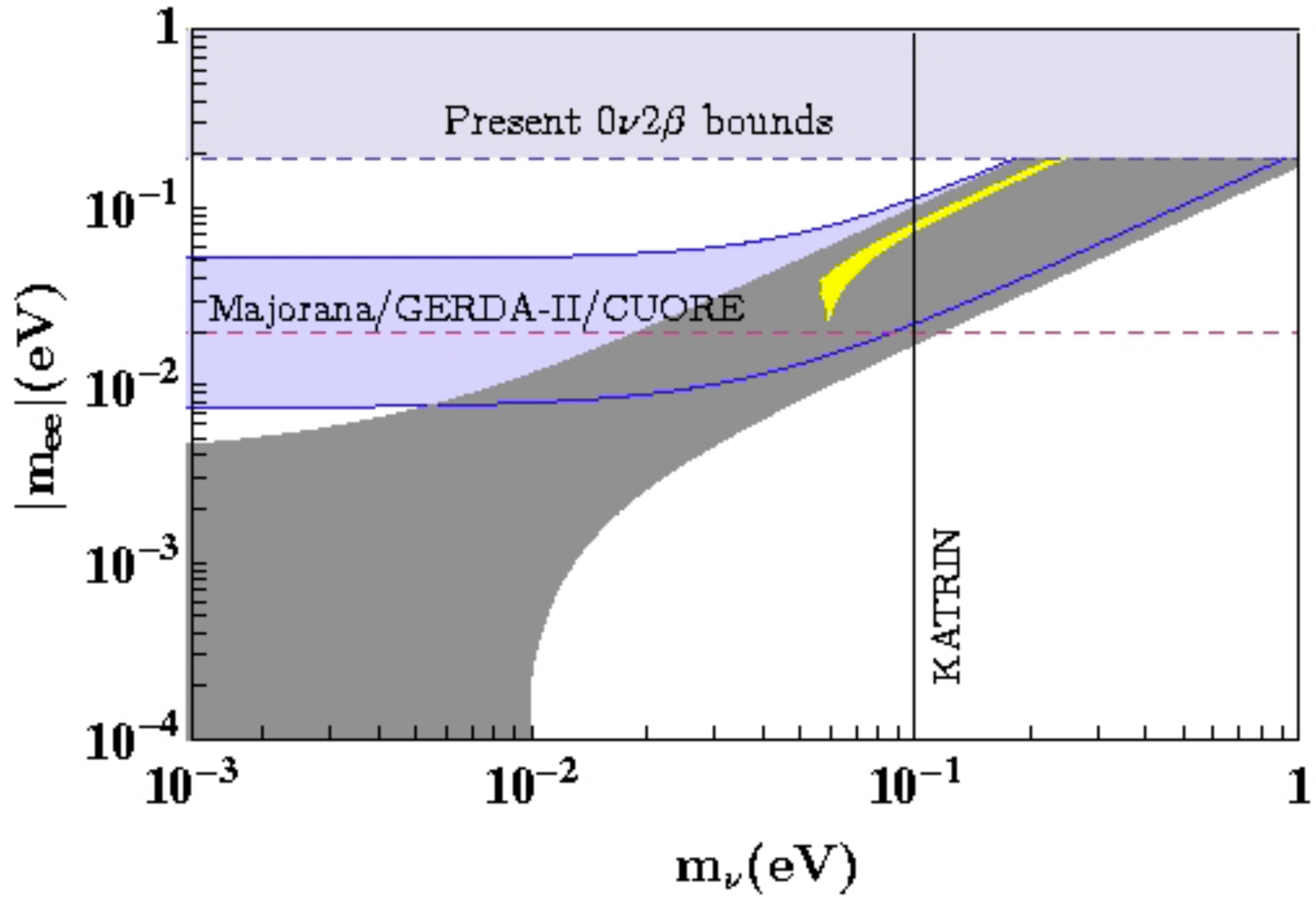
$$\xi \sin\alpha\, m_2^0 + \chi \sin\beta\, m_3^0 = 0$$

Majorana CP phases:

$$\cos\alpha \;=\; \frac{m_1^2 - \chi^2\left(\Delta m^2_{atm} + m_1^2\right) + \xi^2\left(\Delta m^2_{sol} + m_1^2\right)}{2 m_1 \xi \sqrt{\Delta m^2_{sol} + m_1^2}}$$

$$\cos\beta \;=\; \frac{m_1^2 + \chi^2\left(m_1^2 + \Delta m^2_{atm}\right) - \xi^2\left(m_1^2 + \Delta m^2_{sol}\right)}{2 m_1 \chi \sqrt{\Delta m^2_{atm} + m_1^2}}$$

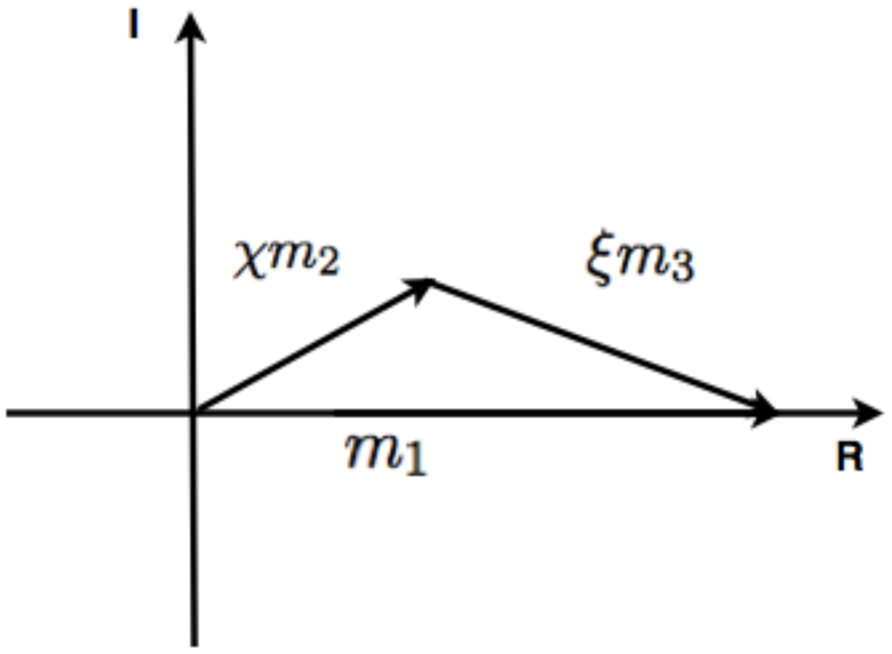
Lower limit on m_1



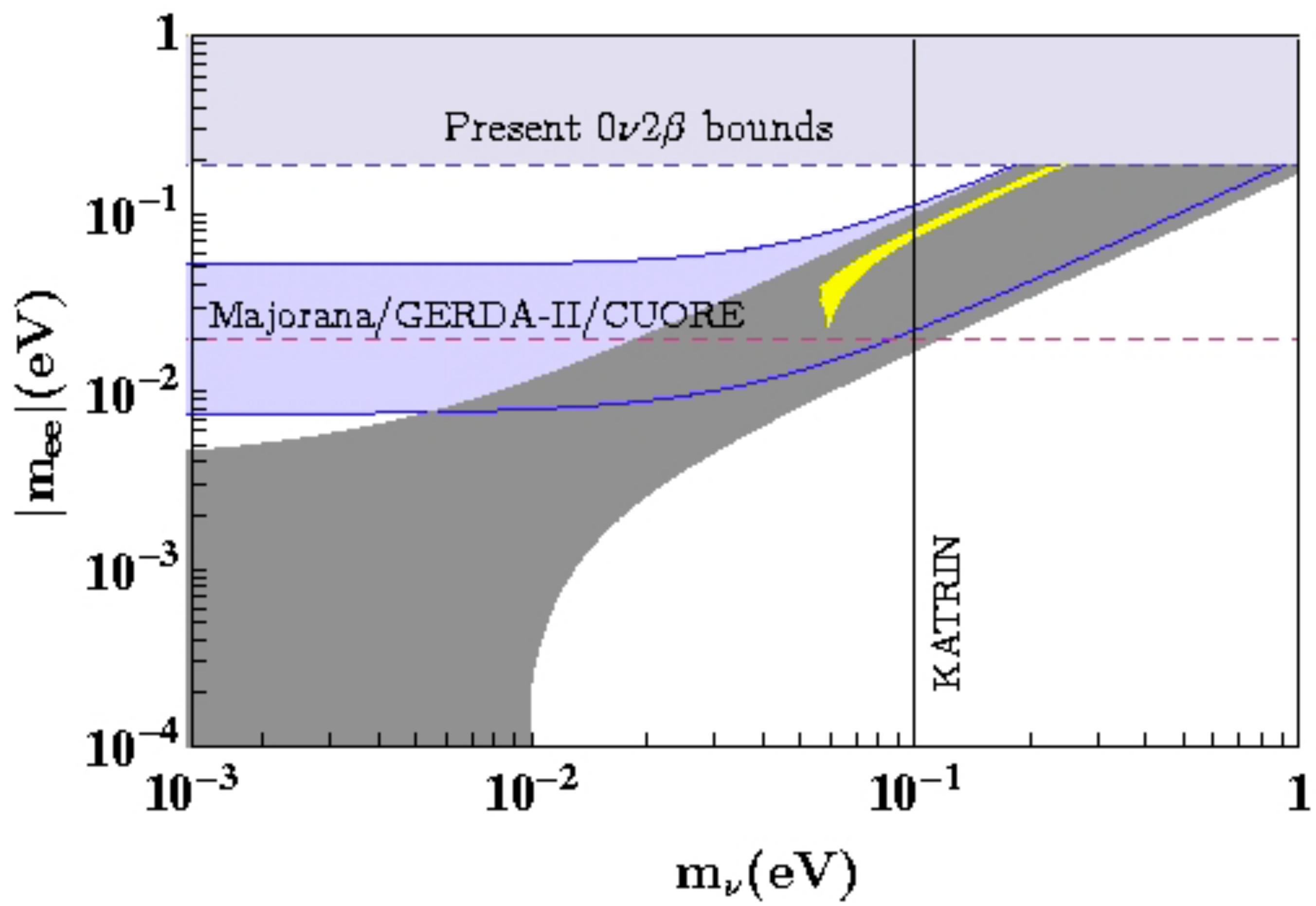
Lower limit on m_1

$m_2 = m_2^0 e^{i\alpha}$ $\quad$ $m_3 = m_3^0 e^{i\beta}$

Triangle Inequality



$$|\xi m_3^\nu| \leq |\chi m_2^\nu| + |m_1^\nu|$$

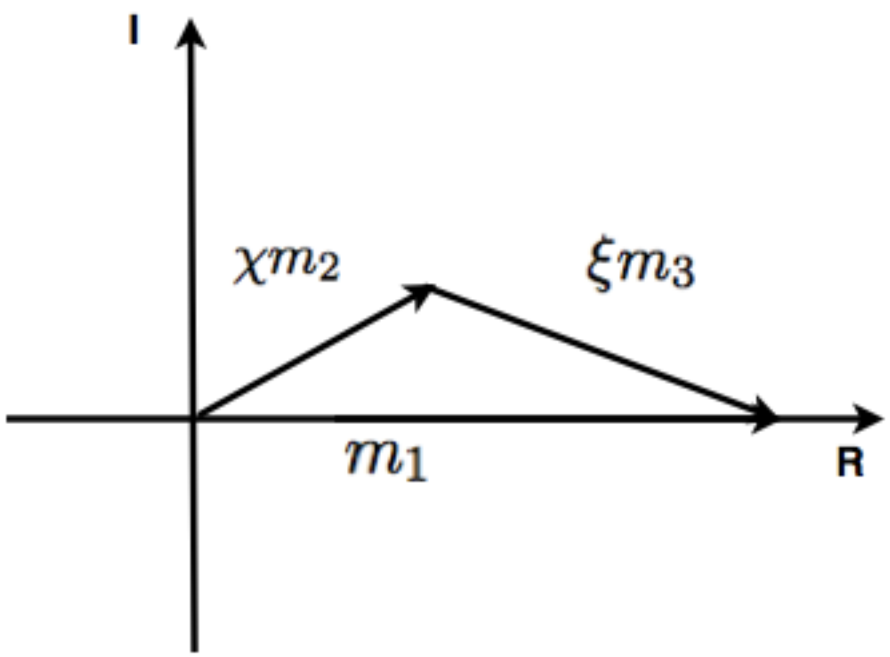


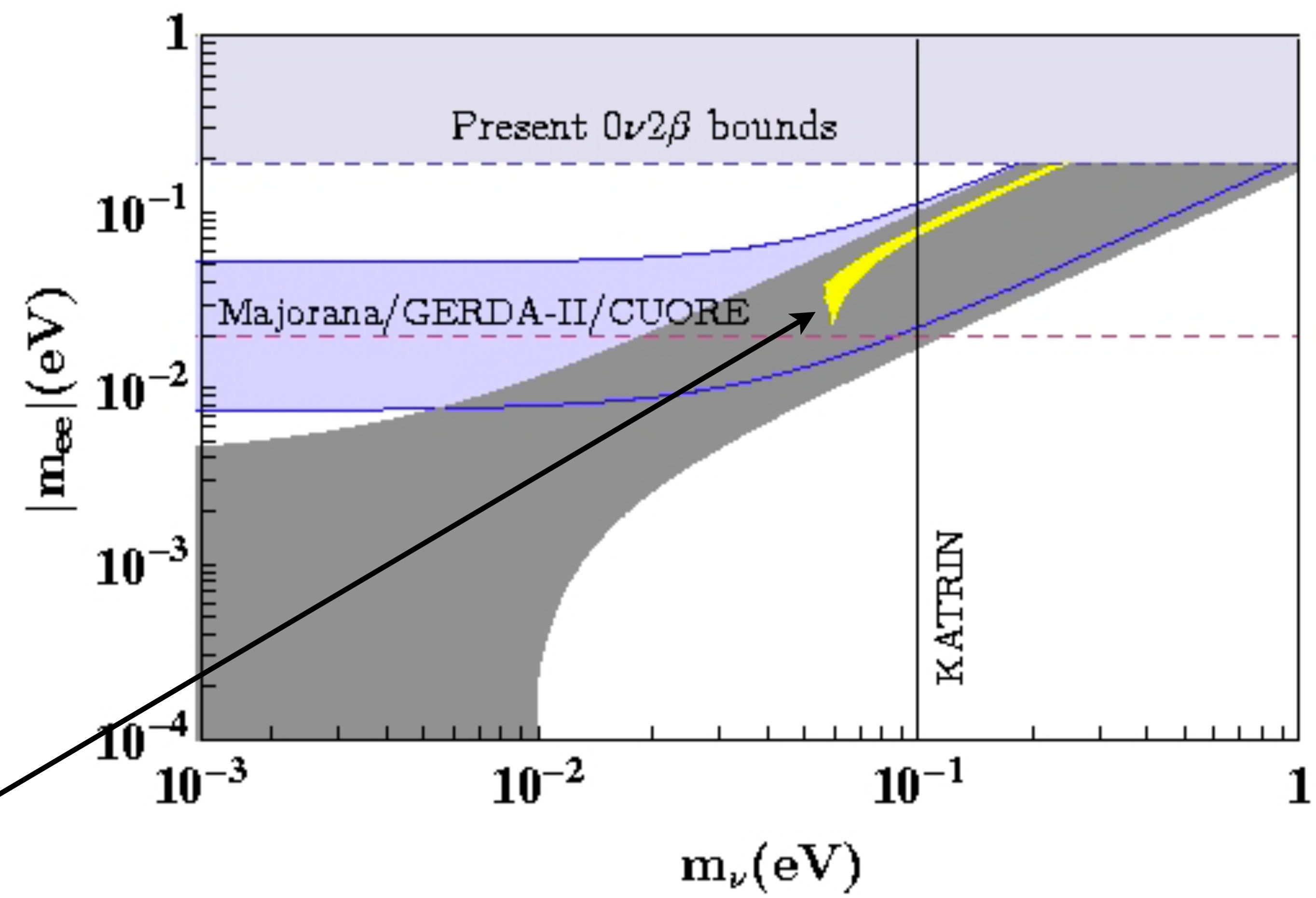
J. Barry, W. Rodejohann, 2010

Lower limit on m_1

$$m_2 = m_2^0 e^{i\alpha} \qquad m_3 = m_3^0 e^{i\beta}$$

Triangle Inequality





$$|\xi m_3^\nu| \le |\chi m_2^\nu| + |m_1^\nu|$$

J. Barry, W. Rodejohann, 2010

Minimal values for the effective $0\nu\beta\beta$ decay parameter

We analyse the four MSR taking the present values of the neutrino oscillations parameters at 3σ

A)	$\chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$
B)	$\dfrac{\chi}{m_2^\nu} + \dfrac{\xi}{m_3^\nu} = \dfrac{1}{m_1^\nu}$
C)	$\chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} = \sqrt{m_1^\nu}$
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Minimal values for the effective $0\nu\beta\beta$ decay parameter

χ,ξ	A–NH	A–IH	Ref.	B–NH	B – IH	Ref.	C– NH	C –IH	Ref.	D –NH	D– IH	
1,1	0.010	0.044	[†]	0.008	0.036	[††]	0.006	0.029	-	.005	.008	
1,2	⋆	0.046	-	0.008	0.027	-	⋆	0.014	-	.004	.026	#
1,3	⋆	0.011	-	0.030	0.005	-	⋆	0.014	-	.018	.025	
2,1	0.006	⋆	[‡]	0.006	0.007	[‡‡]	0.000	⋆	[‡‡‡]	⋆	.007	
2,2	0.019	0.026	-	0.023	0.008	-	0.017	⋆	-	.003	.015	
2,3	⋆	0.046	-	0.007	0.008	-	⋆	0.031	-	.005	.026	
3,1	0.004	⋆	-	0.004	0.008	-	⋆	⋆	-	⋆	⋆	
3,2	0.011	⋆	-	0.004	0.021	-	0.000	⋆	-	⋆	.007	
3,3	0.023	0.061	-	0.029	0.031	-	0.011	0.019	-	.018	.016	

[†] E. Ma, G. Altarelli, F. Feruglio, Y. Lin, F Bazzochi, S Kaneko, S Morisi, M. Honda, ...

[††] J. Barry, W. Rodejohann, G. J. Ding, F. Bazocchi, L. Merlo, S. Morisi

[‡] G. Altarelli, F. Feruglio, Y. Lin, F Bazzochi, S Kaneko, S Morisi, M. Honda, M. Tanimoto

[‡‡] G. Altarelli, F. Feruglio, Y. Lin, F Bazzochi, S Kaneko, S Morisi, M. Honda, ...

[‡‡‡] M. Hirsch, S. Morisi and J. W. F. Valle, 2008

S. Morisi, E. Peinado, A. Rojas, J. W. F. Valle, LD 2012

Minimal values for the effective $0\nu\beta\beta$ decay parameter

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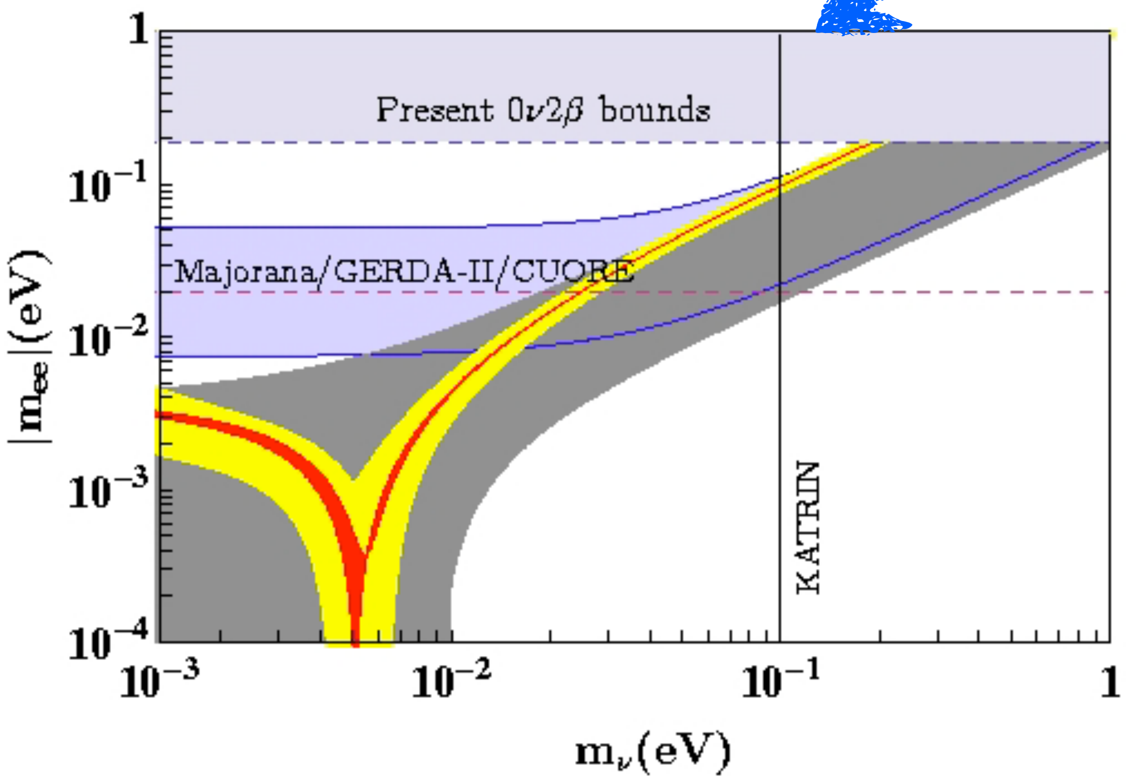
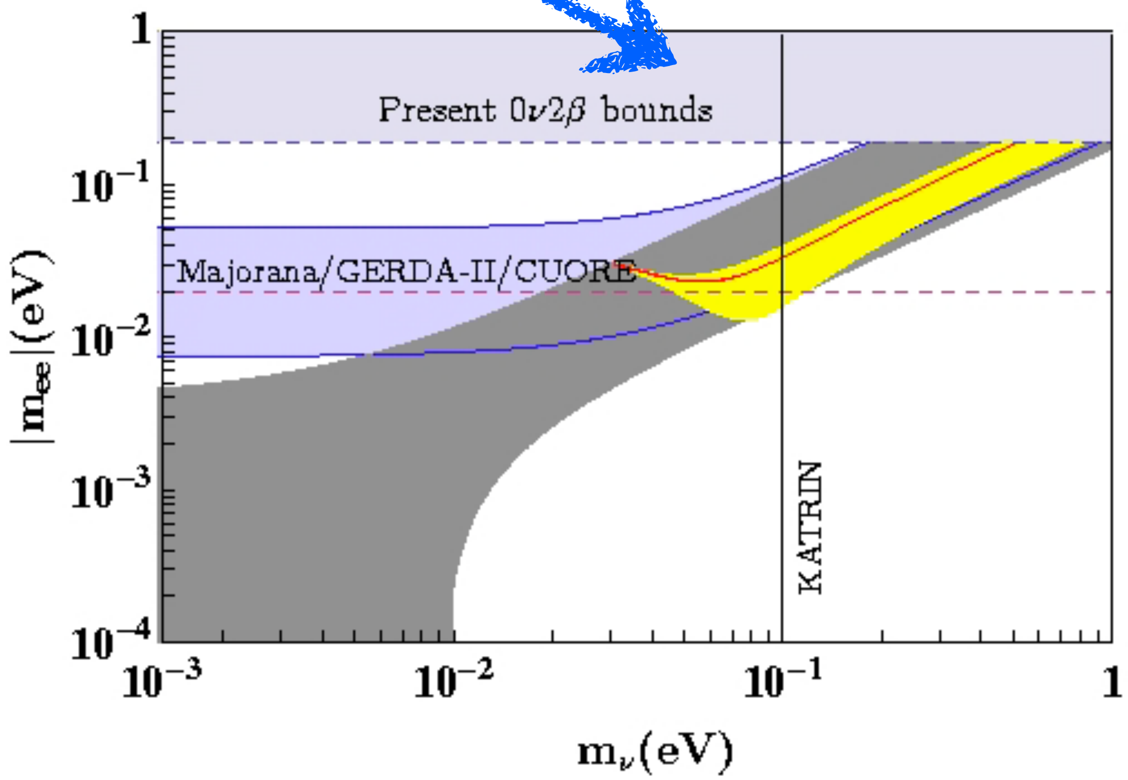
2,1	0.006	⋆	[‡]	0.006	0.007	[‡‡]	0.000	⋆	[‡‡‡]	⋆	.007
2,2	0.019	0.026	-	0.023	0.008	-	0.017	⋆	-	.003	.015
2,3	⋆	0.046	-	0.007	0.008	-	⋆	0.031	-	.005	.026
3,1	0.004	⋆	-	0.004	0.008	-	⋆	⋆	-	⋆	⋆
3,2	0.011	⋆	-	0.004	0.021	-	0.000	⋆	-	⋆	.007
3,3	0.023	0.061	-	0.029	0.031	-	0.011	0.019	-	.018	.016



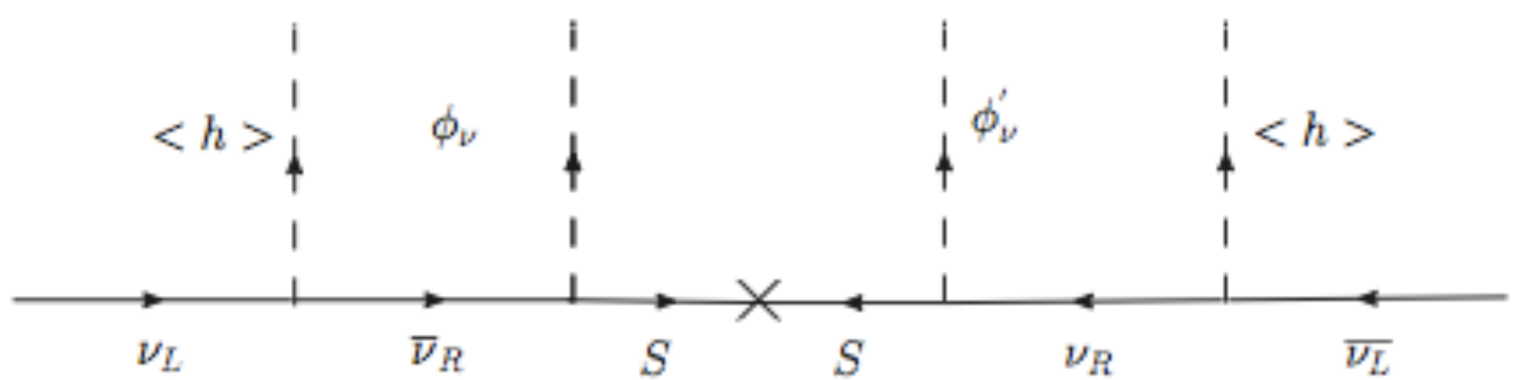


Renormalizable Lagrangian (Neutrinos)

	$\overline{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	2	1	1	2	1	1	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	-1	1	1	0	-1	0	0	0	0	0	2	1	-1

$\langle \phi_\nu \rangle \sim (1,0,0)$ $\quad \langle \phi_{\nu'} \rangle$

$$\mathcal{L}_\nu = Y_{D_{ij}} \overline{L}_i \nu_{R_j} h + Y^k_{\nu ij} \nu_{R_i} S_j \phi_{\nu_k} + Y'_{\nu ij} \nu_{R_i} S_j \phi'_\nu + \mu_{ij} S_i S_j \sigma$$



A model for the case Case D)

From inverse seesaw

$$m_\nu \; = \; m_D \frac{1}{M} \mu \frac{1}{M} m_D^T$$

Light neutrinos eigenvalues

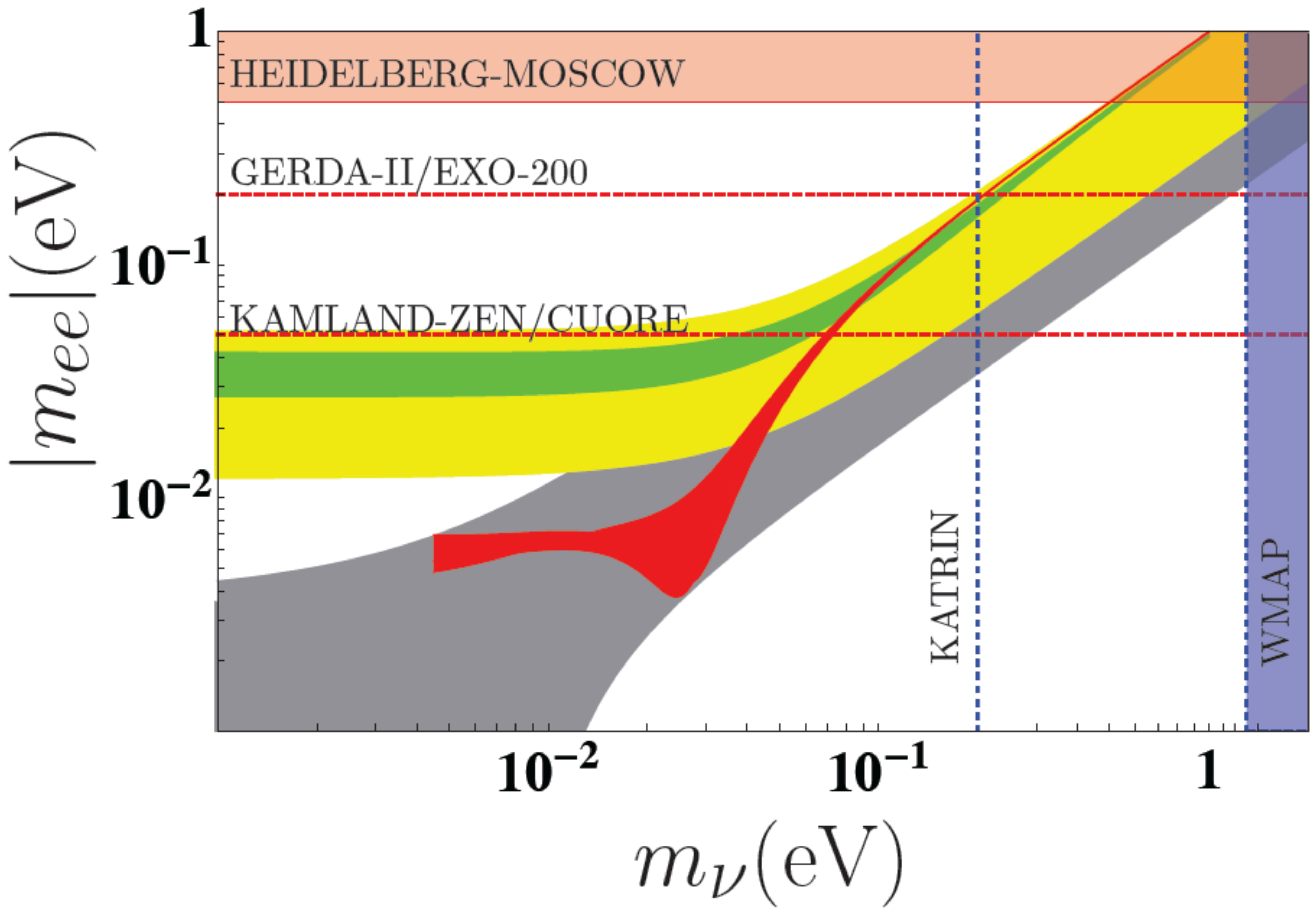
$$m_\nu = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix}$$

$$m_1 = \frac{1}{(a+b)^2}, \quad m_2 = \frac{1}{(a-b)^2}, \quad m_3 = \frac{1}{a^2}.$$

$$\frac{1}{\sqrt{m_1}} + \frac{1}{\sqrt{m_2}} = \frac{2}{\sqrt{m_3}}$$

Neutrinoless double beta decay

$0\nu\beta\beta$



Summary

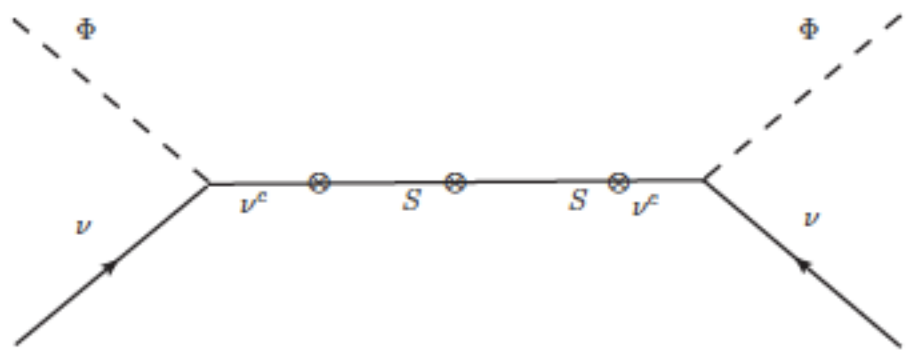
We have analysed the implications for the lower bound on the effective $0\nu\beta\beta$ mass parameter, arising from possible mass sum-rules obtained in the context of flavour models.

MSR were classified in four different categories, some have already been considered in the literature.

We construct a novel model using inverse seesaw to give mass to the neutrinos, such that the MSR will give a lower bound on the m_{ee} at reach of the future $0\nu\beta\beta$.

Thanks.

Backup slides



Case A)

Dimension five operators

$$\mathcal{L} = \frac{y_a}{M}\epsilon^a_{ij}(L_iL_j)_a HH + \frac{y_b}{M}\epsilon^b_{ij}(L_iL_j)_b HH$$

Effective neutrino mass matrix

$$M^\nu_{ij} = a\,\epsilon^a_{ij} + b\,\epsilon^b_{ij}$$

Clebsch-Gordan

ϵ^a_{ij} ϵ^b_{ij}

$a = y_a \langle H\rangle^2 / M$

$b = y_b \langle H\rangle^2 / M$

$$\chi\, m^\nu_2 + \xi\, m^\nu_3 = m^\nu_1$$

E. Ma, G. Altarelli, F. Feruglio, Y. Lin, F Bazzocchi, S Kaneko, S Morisi, M. Honda, ...

Case C)

From seesaw type I

$$M^{\nu} = -m_D M_R^{-1} m_D^T$$

$$M_R \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_D \sim TBM$$

$$m_i^{\nu} \propto (\alpha_i a + \beta_i b)^2$$

$$\chi \sqrt{m_2^{\nu}} + \xi \sqrt{m_3^{\nu}} = \sqrt{m_1^{\nu}}$$

M. Hirsch, S. Morisi and J. W. F. Valle, 2008

Case B)

(Assuming TBM)

From seesaw type I

$$M^\nu = -m_D M_R^{-1} m_D^T$$

$$M_R \sim TBM \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_i^\nu \propto 1/(\alpha_i a + \beta_i b)$$

$$\frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}$$

J. Barry, W. Rodejohann, G. J. Ding, F. Bazzocchi, L. Merlo, S. Morisi, ...

MSR of the model

Light neutrinos eigenvalues

$$M_\nu = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix}$$

$$m_1 = \frac{1}{(a+b)^2}, \quad m_2 = \frac{1}{(a-b)^2}, \quad m_3 = \frac{1}{a^2}.$$

$$\frac{1}{\sqrt{m_1}} + \frac{1}{\sqrt{m_2}} = \frac{2}{\sqrt{m_3}}$$

Particle content of the model

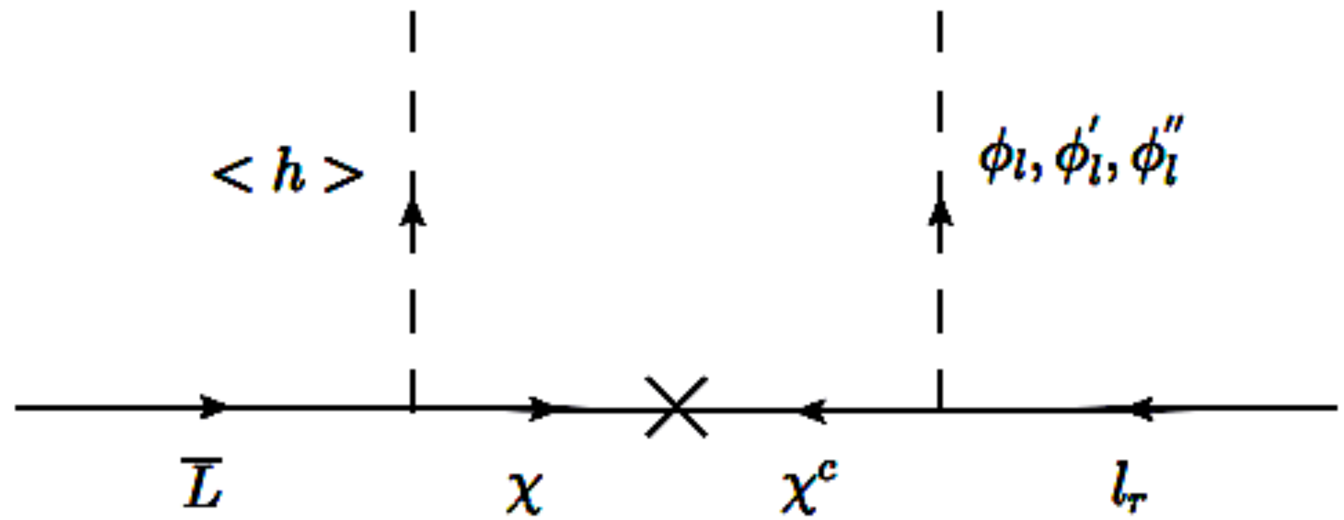
	$\bar{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	2	1	1	2	1	1	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	-1	1	1	0	-1	0	0	0	0	0	2	1	-1

Irreducible representations of S_4

1_1 , 1_2 , 2 , 3_1 , 3_2

Renormalizable Lagrangian (Charged leptons)

	$\overline{L}$	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	2	1	1	2	1	1	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	-1	1	1	0	-1	0	0	0	0	0	2	1	-1



Charged leptons

$$\mathcal{L}_l = \frac{y_l}{\Lambda}(\bar{L}l_R)h\phi_l + \frac{y'_l}{\Lambda}(\bar{L}l_R)h\phi'_l + \frac{y''_l}{\Lambda}(\bar{L}l_R)h\phi''_l$$

$\langle\phi_l\rangle \sim (1,1,1)$ $\quad \langle\phi_{l'}\rangle \sim (1,1,1)$ $\quad \langle\phi_{l''}\rangle$

Charged leptons mass matrix

$$M^l \sim \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix}$$

Diagonalized by the magic matrix

$$U_\omega = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

Case B)
(Assuming TBM)

From seesaw type I

$$M^\nu = -m_D M_R^{-1} m_D^T$$

$$M_R \sim \begin{pmatrix} a+\frac{2b}{3} & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2b}{3} & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2b}{3} \end{pmatrix} \qquad m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_i^\nu \sim \{\frac{1}{a-b}, \frac{1}{a+b}, \frac{1}{a}\}$$

$$-\frac{1}{m_2^\nu} + \frac{2}{m_3^\nu} = \frac{1}{m_1^\nu}$$

J. Barry, W. Rodejohann, G. J. Ding, F. Bazzocchi, L. Merlo, S. Morisi