The Axiverse and Moduli Stabilisation for Chiral Global Models

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Based on:

1. MC,Mayrhofer,Valandro, arXiv:1110.3333 [hep-th]
2. MC,Goodsell,Ringwald, arXiv:1206.0819 [hep-th] $\longrightarrow$ see Ringwald's talk

Introduction

Two longstanding problems of CY compactifications:

1. Moduli stabilisation
2. Derivation of GUT- or MSSM-like constructions

- Type II theories promising because of D-branes and O-planes
- Moduli stabilisation is a global issue $\leftrightarrow$ model building is a local issue
  $\Rightarrow$ physics decouples, separate study
- Type IIB: viable mechanisms to fix the moduli and construct semi-realistic models
  $\Rightarrow$ It is time to combine the two solutions!
- BUT moduli stab and model building are not completely decoupled! Three problems:
  - Tension between moduli stab via NP effects and chirality [Blumenhagen,Moster,Plauschinn]
  - Tension between moduli stab via NP effects and the cancellation of Freed-Witten anomalies [Blumenhagen,Braun,Grimm,Weigand][Collinucci,Kreuzer,Mayrhofer,Walliser]
  - D-term induced shrinking of various divisors (the one supporting the visible sector) [Blumenhagen,Braun,Grimm,Weigand][Collinucci,Kreuzer,Mayrhofer,Walliser][MC,Kreuzer,Mayrhofer]

Solve them with control over the EFT, mod stab inside the Kähler cone and interesting phenomenological scales!

Main results

- Type IIB/F-theory compactifications with D3/D7-branes and O3/O7-planes
- Explicit description of the compact CY by means of toric geometry [MC,Kreuzer,Mayrhofer]
- Brane set-ups and world-volume fluxes that yield a chiral $SU(5)$- or MSSM-like model
- Global consistency: D7-tadpole, torsion charges and FW anomaly cancellation (leave space in D3-tadpole to turn on three-form fluxes)
- Kähler moduli fixed within the Kähler cone and the regime of validity of the EFT in a way compatible with chirality + No D-term induced shrinking of any divisor
- Realisation of LVS axiverse for globally consistent chiral models [MC,Goodsell,Ringwald]
- Get a visible sector gauge coupling of the correct size
- Three choices of underlying parameters which give:
  1. GUT-scale M_s and TeV-scale SUSY by fine-tuning the background fluxes
  2. Intermediate scale M_s and TeV-scale SUSY for natural fluxes
  3. TeV-scale M_s and micron-sized EDs for anisotropic CYs [MC,Burgess,Quevedo]
- dS vacua via D3/E(-1) instantons at a singularity [MC,Maharana,Quevedo,Burgess]

Kähler moduli stabilisation

- Type IIB closed string moduli: axio-dilaton S, cx str moduli U_α, $\alpha = 1, ..., h^{2,1}$,
  Kähler moduli $\quad T_i = \tau_i + \mathrm{i}\, c_i, \quad \tau_i = \mathrm{Vol}(D_i), \quad c_i = \int_{D_i} C_4, \quad i = 1, ..., h^{1,1}$
- Fluxes $G_3 = F_3 + \mathrm{i} S H_3$ generate $W_{\rm tree}(S,U)$ which fixes S and U at $D_{S,U} W = 0$
- No-scale structure $\Rightarrow$ T-moduli flat at tree-level
- S and U fixed at their flux-stabilised values $\Rightarrow W_0 = \langle W_{\rm tree} \rangle$, $K_{\rm tree} = -2 \ln \mathcal{V}$
- Sources for Kähler moduli stabilisation:

$$V = V_D + V_F^{\rm tree} + V_F^{\rm pert} + V_F^{\rm np}$$

  - $V_D \sim \mathcal{O}(1/\mathcal{V}^2)$: D-term potential (generated by fluxes on D7-branes)
  - $V_F^{\rm tree} \sim \mathcal{O}(1/\mathcal{V}^2) = 0$: no-scale structure
  - $V_F^{\rm pert} \lesssim \mathcal{O}(1/\mathcal{V}^3)$: perturbative ($\alpha'$ and g_s) corrections to K
  - $V_F^{\rm np} \sim \mathcal{O}(1/\mathcal{V}^3)$: non-perturbative corrections to W (E3-instantons or gaugino condensation on a D7-stack)
- At leading order in $1/\mathcal{V}$: $V_D = 0$
- At subleading order minimise $V_F \Rightarrow$ Break SUSY

NP effects and chirality

Tension between Kähler md stab by NP effects and chirality

- Chirality induced by non-zero flux on intersections of branes $\Rightarrow$ visible sector with $\mathcal{F} \neq 0$
- $W_{\rm np} = \sum_i A_i\, e^{-a_i T_i}$. If chiral modes on intersection between NP-cycle and visible sector, A_i depend on visible sector modes ϕ
- To preserve visible sector gauge group, $\langle \phi \rangle = 0$ but then $A_i = 0$ and no contribution from i-cycle

Constraint on the flux choice: no chirality at possible intersection between NP effect cycle and visible sector

Best place to place NP effects: 'diagonal' del Pezzo divisors [MC,Kreuzer,Mayrhofer]

NP effects and Freed-Witten anomaly

Turn on half-integer flux on any *non-spin* 4-cycle D ($c_1(D)$ is odd) to cancel worldsheet anomalies [Minasian,Moore][Freed,Witten]:

$$F = f^i \eta_i + \frac{1}{2} c_1(D) \quad f^i \in \mathbb{Z} \quad \eta_i \in H^2(D, \mathbb{Z})$$

- $\mathcal{F} = F - B = 0$ on the E3-instanton or gaugino condensation stack, wrapping invariant cycle
- FW $\Rightarrow F \neq 0$

Need a proper choice of B to cancel F

BUT once B is fixed to cancel half-integral F on stack a, generically forces $\mathcal{F} \neq 0$ on a second non-spin stack b (unless they do not intersect)

$\Rightarrow$ FW anomaly generically prevents to have more than one NP effect to fix Kähler moduli

$\Rightarrow$ Kähler moduli stabilisation by only one NP effect!

This leads to the LARGE Volume Scenario ($\mathcal{V}$ fixed by interplay of α'-corr and NP effect supported by a single diagonal del Pezzo) [Balasubramanian,Berglund,Conlon,Quevedo] [MC,Conlon,Quevedo]

‘D-term problem’

Flux generates FI-term $\xi_a = \frac{1}{\mathcal{V}} \int_{D_a} J \wedge \mathcal{F}_a \quad \Rightarrow \quad V_D = \sum_a \frac{g_a^2}{2} \left(\sum_b q_{ab} |\phi_b|^2 - \xi_a \right)^2$

- If VEV of charged fields $\langle \phi \rangle = 0$, D-term conditions imply $\xi_a = 0$
- $\xi_a = 0 \to$ generically some 4-cycles shrink (away sugra approx)

$\xi_a \propto \int_{D_a} J \wedge \mathcal{F}_a = k_{ajk} \mathcal{F}_a^k t^j = 0$ homogeneous linear eqs in the $h^{1,1}$ Kähler md

- NP cycle does not enter in $\xi_a = 0$ eqs (diag dP, no chiral inters)
- In general we have $n = h^{1,1} - 1$ unknowns in eqs $\xi_a = 0$
- The matrix of the system $\xi_a = 0$ will have rank d
- If $d = n$, then $t^j = 0 \Rightarrow d < n$, $(n - d)$ flat directions
- $n - d = 1 \Rightarrow$ all of the same size: $t_j = t_* \; \forall j$
  $\Rightarrow$ no LVS due to visible gauge coupling: $g^{-2} \sim t_*^2$
- $n - d = 2 \Rightarrow$ can get LVS

If $d = 1$, the minimal n to allow for LVS is $n = 3 \Rightarrow h^{1,1} = 4$

Get LVS by choosing D7-brane config and fluxes such that $d = 1$

The axiverse and moduli stabilisation

Kähler moduli fixed by combination of different effects for $W_0 \sim \mathcal{O}(1)$:

- d combinations are fixed by leading D-term potential
  $\Rightarrow$ d axions get eaten up by anomalous $U(1)$s
- 'Diagonal' dP fixed by NP effects $W_{\rm np} = A\, e^{-aT_{\rm dP}}$
  $\Rightarrow$ Corresponding axion gets the same mass of the order $m_{3/2}$
- Remaining $n_{\rm ax} = h^{1,1} - 1 - d \geq 2$ moduli fixed perturbatively:
  - Volume mode fixed by α' corrections to K
  - Remaining moduli fixed by subleading g_s corrections to K

  $\Rightarrow$ $n_{\rm ax} \geq 2$ light axions
- For $h^{1,1} \sim \mathcal{O}(100)$ expect $n_{\rm ax}$ very large
- One axion is the QCD axion and the others get a tiny mass via higher order NP effects
  $W_{\rm np} = A\, e^{-aT_{\rm dP}} + \sum_i^{n_{\rm ax}} A_i\, e^{-n_i a_i T_i}$
  $\Rightarrow$ Axiverse with many light axions whose masses are logarithmically hierarchical

Explicit example: the Calabi-Yau

The CY$_3$ X is a hypersurface in a 4D toric ambient variety. Weight matrix:

z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	D_X
1	1	1	0	0	0	1	4	8
1	1	0	0	0	1	0	3	6
0	1	1	1	0	0	0	3	6
0	1	0	0	1	0	0	2	4

,

Toric variety is the moduli space of a GLSM:

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_7|^2 + 4|z_8|^2 = \xi_1, \quad |z_1|^2 + |z_2|^2 + |z_6|^2 + 3|z_8|^2 = \xi_2, \ldots$$

CY data obtained from PALP [Kreuzer,Skarke][Braun,Walliser]

- Hodge numbers: $h^{1,1}(X) = 4$, $h^{1,2}(X) = 106$
- Basis of $H_4(X, \mathbb{Z})$:

$$\Gamma_1 = D_7\,, \qquad \Gamma_2 = D_2 + D_7\,, \qquad \Gamma_3 = D_1\,, \qquad \Gamma_4 = D_5$$

- Intersection form: $I_3 = 2\Gamma_1^3 + 4\Gamma_2^3 + 4\Gamma_4^3 + 2\Gamma_2^2\Gamma_3 - 2\Gamma_4^2\Gamma_3$

A K3-fibred Calabi-Yau

X is a K3 fibration with a diagonal del Pezzo [MC,Kreuzer,Mayrhofer]

- K3 fibre divisor is D_1
- One 'diagonal' dP$_7$ corresponding to $\Gamma_1 = D_7$
- Three other rigid (non-dP) divisors: D_4, D_5, D_6 with $h^{2,0} = h^{1,0} = 0$

Kähler form expanded as $J = \sum_{i=1}^{4} t_i \Gamma_i$

- Volume of X: $\mathcal{V} = \frac{1}{3}\left[2t_2^3 + 3t_2^2 t_3 + t_4^2(2t_4 - 3t_3) + t_1^3\right]$
- The volume of the 'diagonal' dP is $\tau_7 = t_1^2$
- Kähler cone:

  $$r_1 \equiv -t_1 > 0\,, \qquad r_2 \equiv t_1 + t_2 + t_4 > 0\,, \qquad r_3 \equiv t_3 - t_4 > 0\,, \qquad r_4 \equiv -t_4 > 0$$

- K3-fibred CYs promising for particle phenomenology and cosmology:
  1. Anisotropy: ADD [MC,Burgess,Quevedo 2011], hidden photons [MC,Goodsell,Jäckel,Ringwald]
  2. Inflation: single-field [MC,Burgess,Quevedo 2008] [MC,Pedro,Tasinato 2011]; multi-field [Burgess,MC,Gomez-Reino,Quevedo,Tasinato,Zavala] [MC,Tasinato,Zavala,Burgess,Quevedo]
  3. Quintessence [MC,Pedro,Tasinato 2012]

Model building with D7-branes

GUT or MSSM on D7-branes wrapped around 4-cycles

Orientifold $O = (-1)^{F_L}\Omega_p\sigma$ where σ is holomorphic involution of X

- N_a D7-branes (plus images) wrapping invariant divisor $D \Rightarrow Sp(2N_a)$ gauge group

Switch on flux $\mathcal{F}$ on the brane wrapping D

- If diagonal $\mathcal{F} \Rightarrow Sp(2N_a) \to SU(N_a) \times U(1)$
- D7-brane flux generates chiral modes
- Number of chiral zero-modes in symmetric and antisymmetric $U(N_a)$ reps:

$$I_a^{(S,A)} = \mp\frac{1}{2}\int_X [D7_a] \wedge [O7] \wedge \mathcal{F}_a - \int_X [D7_a] \wedge [D7_a] \wedge \mathcal{F}_a$$

- At intersection, chiral zero-modes in bi-fundamental reps $(N_a, \overline{N}_b)$ and (N_a, N_b):

$$\begin{aligned} I_{a\bar{b}} &= \int_X [D7_a] \wedge [D7_b] \wedge (\mathcal{F}_a - \mathcal{F}_b) \\ I_{ab} &= \int_X [D7_a] \wedge [D7_b] \wedge (\mathcal{F}_a + \mathcal{F}_b) \end{aligned}$$

Charge cancellation

Homological charges must be cancelled:

$$\Gamma_{D7} = [D7] + [D7] \wedge \mathcal{F} + [D7] \wedge \left(\frac{1}{2} \mathcal{F} \wedge \mathcal{F} + \frac{\chi(D7)}{24} \right)$$
$$\Gamma_{O7} = -8[O7] + [O7] \wedge \frac{\chi(O7)}{6}$$

- D7-charge: $\Sigma_{D7}[D7] = 8[O7]$
- D5-charge: $\mathcal{F}' = -\mathcal{F} \Rightarrow$ zero if all branes and image-branes wrap same divisor
- D3-charge: contributions from fluxes and geometry: leave space to turn on H_3 and F_3

Also K-theoretic torsion charges must sum to zero:

- Probe argument [Uranga]: equivalent to require absence of $SU(2)$ gauge anomaly on any probe Sp-brane $\rightarrow$ even number of chiral fundamental rep

Orientifold projection

Choice for holomorphic orientifold involution σ:

$$\sigma: \quad z_8 \mapsto -z_8$$

- O7-plane at $z_8 = 0 \Rightarrow [O7] = D_8$
- No O3-planes
- $h^{1,1}_-(X) = 0$ and so $h^{1,1}_+(X) = h^{1,1}(X)$
- Equation for CY$_3$:
$$z_8^2 = P_{8,6,6,4}(z_1, ..., z_7)$$
- To cancel D7-charge of O7, D7-config on divisor class $8[D_8]$

D7-brane stacks

D7-config described by polynomial (Whitney brane with double intersection with O7):

$$\eta^2 - z_8^2 \chi = 0$$

To have different stacks, this polynomial has to factorise

- Special forms for polynomials η and χ:

  $$\eta = z_i^m \tilde{\eta}, \quad \chi = z_i^{2m} \tilde{\chi} \qquad \Rightarrow \qquad \eta^2 - z_8^2 \chi = z_i^{2m} \left(\tilde{\eta}^2 - z_8^2 \tilde{\chi} \right)$$

  $\rightarrow$ one $Sp(2m)$ stack along $z_i = 0$ plus a *Whitney brane*

  N_a branes on D_4, N_b on D_5, N_{k3} on D_1 and N_{gc} on D_7 (plus images):

  $$\eta^2 - z_8^2 \chi \qquad \rightarrow \qquad z_1^{2N_{k3}} z_4^{2N_a} z_5^{2N_b} z_7^{2N_{gc}} \left(\tilde{\eta}^2 - z_8^2 \tilde{\chi} \right)$$

  No further factorisation if:

  $$N_{gc} \leq 4 \qquad N_{gc} + N_{k3} \leq 4 + N_a \qquad N_a - N_b \leq N_{gc}$$

- The Whitney brane has zero flux
- Set $\mathcal{F}_{gc} = 0 \Rightarrow B = F_{gc}$

K-theory constraints solved if N_b is an even number

Example with one D-term

Choice of set-up:

$$N_a = 3\,, \qquad N_{k3} = 1\,, \qquad N_{gc} = 3 \qquad \text{and} \qquad N_b = 0$$

Fluxes:

- $\mathcal{F}_a^\sigma = \mathcal{F}_a$ $\sigma = 1, \cdots, 3$, (diag) on D_4
- $\mathcal{F}_{k3} = 0$ on D_1

Gauge group broken to:

$$Sp(6) \times SU(2) \times Sp(6) \to SU(3) \times U(1) \times SU(2) \times Sp(6) \to SU(3) \times SU(2) \times Sp(6)\,,$$

Also GUT-like example with two D-terms: $SU(5) \times U(1) \times Sp(8)$

Chiral matter and D3-charge

Chiral modes:

$$\begin{aligned} I_a^{(A)} &= 2\beta_a - \nu & I_a^{(S)} &= -2\beta_a + 3\nu \\ I_{ak3} &= 2\alpha_a & I_{aW} &= 4(4\beta_a - \alpha_a) + 8\,\nu \\ I_{k3W} &= 0 & I_{agc} &= \nu \end{aligned}$$

α_a, β_a and ν are integral combinations of flux numbers

Gaugino condensation without chiral intersections $\Rightarrow \nu = 0$.

Choice of flux numbers consistent with requirements:

$$\alpha_a = 1\,, \qquad \beta_a = -1\,, \qquad \nu = 0$$

- Total D3-charge (including the geometric contribution):

$$Q^{\mathrm{tot}}_{(D3)} = -606$$

- Non-zero chiral intersections:

$$I_a^{(A)} = -2\,, \qquad I_a^{(S)} = 2\,, \qquad I_{ak3} = 2\,, \qquad I_{aW} = -20$$

D-term potential

Just one non-trivial FI-term:

$$\xi_a = \frac{1}{4\pi\mathcal{V}} \int_X [D7_a] \wedge J \wedge \mathcal{F}_a = \frac{1}{4\pi\mathcal{V}} \left[(\beta_a - \alpha_a)(r_1 + r_2) + 2\alpha_a r_3\right] .$$

Solution of $\xi_a = 0$:

$$r_3 = \left(1 - \frac{\beta_a}{\alpha_a}\right) \frac{r_1 + r_2}{2}$$

Substituting flux choice, following relations between divisor volumes:

$$\tau_4 = 3\,(\tau_1 - \tau_5) - \tau_7$$

$\Rightarrow$ Plug them in subleading F-term potential:

$$\mathcal{V} = \frac{1}{6}\sqrt{\tau_1 - \tau_5}\,(10\tau_1 - \tau_5) - \frac{1}{3}\,\tau_7^{3/2}$$

Since $\alpha_{\rm vis}^{-1} = \tau_4$, the combination $(\tau_1 - \tau_5)$ has to be fixed small:

$$\tau_s \equiv \tau_1 - \tau_5 \qquad\qquad \tau_b \equiv \frac{10\tau_1 - \tau_5}{2}$$

$$\mathcal{V} = \frac{1}{3}\left(\sqrt{\tau_s}\,\tau_b - \tau_7^{3/2}\right)$$

F-term potential

- F-term potential depends on τ_s, τ_b and τ_7
- F-term potential given by NP and α' pert corrections

$$K = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right), \qquad W = W_0 + A\, e^{-\frac{2\pi T_7}{N g c + 1}} = W_0 + A\, e^{-\frac{\pi T_7}{2}}$$

- Potential minimised both numerically for given value of parameters A, g_s, W_0 and analytically, using leading approximation
- Can find LVS!

$$V \simeq 2\pi^2 A^2 \frac{\sqrt{\tau_7}}{\mathcal{V}}\, e^{-\pi\tau_7} - 2\,\pi A W_0\, \frac{\tau_7}{\mathcal{V}^2}\, e^{-\frac{\pi\tau_7}{2}} + \frac{3W_0^2\hat{\xi}}{4\mathcal{V}^3}$$

V depends only on $\mathcal{V}$ and τ_7. One flat direction

- Solution:

$$\mathcal{V} \simeq \frac{W_0\sqrt{\tau_7}}{2\pi A}\, e^{\frac{\pi\tau_7}{2}} \qquad \text{and} \qquad \tau_7 \simeq \left(\frac{3\xi}{2}\right)^{2/3}\frac{1}{g_s}$$

F-term potential

- For choice:

$$W_0 = 1 \qquad A = 0.1 \qquad g_s = 0.05$$

α' + NP corrections stabilise τ_7 and the product $\sqrt{\tau_s}\tau_b$. Find (both numerically and analytically):

$$\langle \tau_7 \rangle \simeq 16 \qquad \langle \mathcal{V} \rangle \simeq 1 \cdot 10^{12}$$ Justify validity of approximations

- Get TeV-scale SUSY:

$$m_{3/2} = e^{K/2} W_0 M_P = \sqrt{\frac{g_s}{8\pi}} \frac{W_0 M_P}{\mathcal{V}} \simeq 100\,\mathrm{TeV}$$

by gravity mediation $M_{\rm soft} \simeq \frac{m_{3/2}}{\ln\left(M_P/m_{3/2}\right)} \simeq 3$ TeV

- Get intermediate string scale:

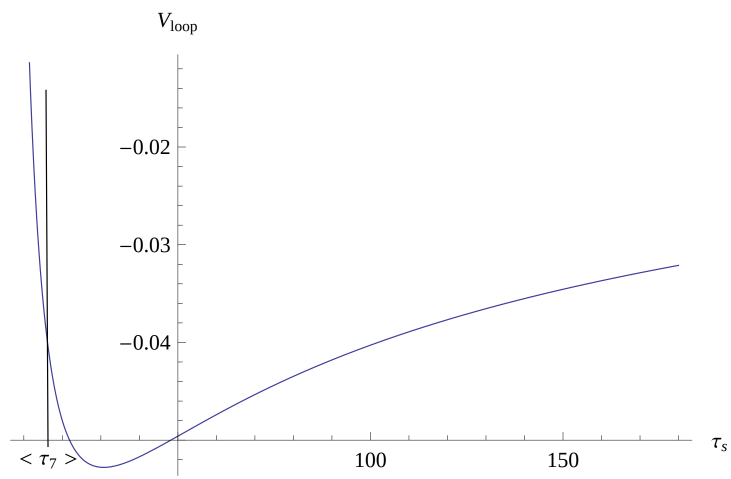
$$M_s \simeq \frac{M_P}{\sqrt{4\pi\mathcal{V}}} \simeq 10^{11}\,\mathrm{GeV}$$

- Perfect intermediate scale decay constant for local axions: $f_a \sim M_s$

String loop corrections

- g_s corrections can stabilise τ_s small ($\Rightarrow \tau_b$ large) [MC,Conlon,Quevedo]:

$$\delta V_{(g_s)}^{1-loop} = \left(\frac{c_1}{\sqrt{\tau_s}} + \frac{c_2}{5\sqrt{\tau_s} - 2\sqrt{\tau_7}} + \frac{c_3}{19\sqrt{\tau_s} - 8\sqrt{\tau_7}} \right) \frac{W_0^2}{\mathcal{V}^3} + \mathcal{O}\left(\frac{1}{\mathcal{V}^4} \right)$$



$\Rightarrow \tau_s \simeq 30$ and $\tau_b \simeq 10^{11}$ well inside the Kähler cone! Anisotropic CY!

- This keeps $\tau_4 = 3\tau_s - \tau_7$ small and $\alpha_{\rm vis}^{-1} = \langle \tau_4 \rangle - \frac{1}{2g_s} \int_{D_4} \mathcal{F}_4 \wedge \mathcal{F}_4 \simeq 100$
- Different choice of parameters: $g_s = 0.02$ instead of $g_s = 0.05 \Rightarrow \mathcal{V} \simeq 10^{29}$

  $\Rightarrow$ ADD scenarios from strings: $M_s \simeq \frac{M_P}{\sqrt{4\pi\mathcal{V}}} \simeq 2$ TeV + 2 micron-sized extra dimensions [MC,Burgess,Quevedo 2011]

Conclusions

We have presented a concrete chiral model with Kähler moduli stabilised

- General strategy to combine Kähler moduli stabilisation with chiral D7-brane models in Type IIB flux compactifications
- Geometric data described by toric geometry. This allowed to make specific choice of brane setup and fluxes that give rise to GUT- or MSSM-like models
- We have checked several consistency constraints
- We have computed the scalar potential and minimised it, obtaining different interesting scenarios
- First realisation of LARGE Volume Scenario in a concrete model
- This mod stab mechanism leads to an axiverse

Outlook

- Explicit analysis of three-form background fluxes to fix the cx str, the dilaton and D7-brane deformation moduli
- Realisation of a fluxed brane set-up that produces the right chiral spectrum and Yukawas
- There is a long list of CY_3 in PALP output: try to automatise the search for a consistent and phenomenological viable model
- Explicit realisation of dS vacua [MC,Maharana,Quevedo,Burgess]
- K3-fibration promising for cosmology: derive the details of the inflationary scenarios [MC,Burgess,Quevedo 2008] [Burgess,MC,Gomez-Reino,Quevedo,Tasinato,Zavala] [MC,Pedro,Tasinato 2011] [MC,Tasinato,Zavala,Burgess,Quevedo] [MC,Pedro,Tasinato 2012]
- Consider visible sector on shrinking (by D-term) divisor $\rightarrow$ quiver theories [Conlon,Maharana,Quevedo][Blumenhagen,Conlon,Krippendorf,Moster,Quevedo] [MC,Krippendorf,Mayrhofer,Quevedo,Valandro in preparation]
- Study of the phenomenology of light hidden sector particles [MC,Goodsell,Jäckel,Ringwald]
- Applications to F-theory