

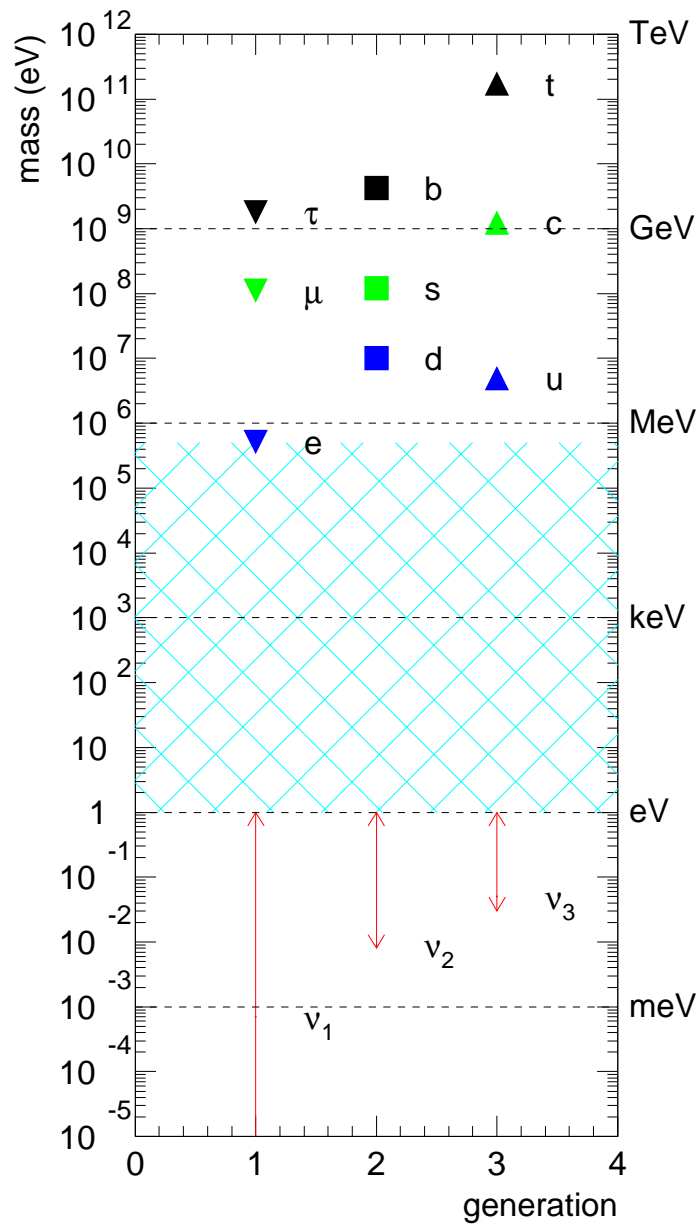
On Understanding Neutrino Masses and Mixing

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What We Are Trying To Understand:

⇐ **NEUTRINOS HAVE TINY MASSES**

⇓ **LEPTON MIXING IS “WEIRD”** ⇓

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

What Does It Mean?

Who Cares About Neutrino Masses: Only* “Palpable” Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

* There is only a handful of questions our model for fundamental physics cannot explain properly. These are, in order of “palpability” (my opinion):

- What is the physics behind electroweak symmetry breaking? (Higgs *or* not in SM).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM – is this “particle physics?”).

What is the New Standard Model? [ν SM]

The short answer is – WE DON'T KNOW. Not enough available info!



Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the ν SM candidates can do. [are they falsifiable?, are they “simple”?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

Options include:

[J. Valle's talk]

- modify SM Higgs sector (e.g. Higgs triplet) and/or
- modify SM particle content (e.g. $SU(2)_L$ Triplet or Singlet) and/or
- modify SM gauge structure and/or
- supersymmetrize the SM and add R-parity violation and/or
- augment the number of space-time dimensions and/or
- etc

Important: different options \rightarrow different phenomenological consequences

Candidate ν SM: The One I'll Concentrate On

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\text{SM}} \supset -y_{ij} \frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB: } \mathcal{L}_{\nu\text{SM}} \supset \frac{m_{ij}}{2} \nu^i \nu^j; \quad m_{ij} = y_{ij} \frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- ν SM effective theory – not valid for energies above *at most* Λ/y .
- Define $y_{\text{max}} \equiv 1 \Rightarrow$ data require $\Lambda \sim 10^{14}$ GeV.

What else is this “good for”? Depends on the ultraviolet completion!

The Seesaw Lagrangian

A simple^a, renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\text{old}} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^3 \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions.

\mathcal{L}_ν is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, \mathcal{L}_ν describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos**.

^aOnly requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M .

The data can be summarized as follows: there is evidence for three neutrinos, mostly “active” (linear combinations of ν_e , ν_μ , and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be “sterile.”

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large M : $M \gg v$. Popular examples include $M \sim M_{\text{GUT}}$ (GUT scale), or $M \sim 1 \text{ TeV}$ (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14} \text{ GeV}$, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV .

we can impose very, very few experimental constraints on M

What We Know About M :

- $M = 0$: the six neutrinos “fuse” into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} \nu$.

The symmetry of \mathcal{L}_ν is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ $[m \propto 1/\Lambda \Rightarrow \Lambda = M/\mu^2]$.

This the **seesaw mechanism**. Neutrinos are Majorana fermions.

Lepton number is not a good symmetry of \mathcal{L}_ν , even though L -violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale M ,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); **or**
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); **or**
- cancellations among different contributions render neutrino masses accidentally small (“fine-tuning”).

High-Energy Seesaw: Brief Comments

[J. Valle's talk]

- This is everyone's favorite scenario.
- Upper bound for M (e.g. Maltoni, Niczyporuk, Willenbrock, hep-ph/0006358):

$$M < 7.6 \times 10^{15} \text{ GeV} \times \left(\frac{0.1 \text{ eV}}{m_\nu} \right).$$

- Hierarchy problem hint (e.g., Casas, Espinosa, Hidalgo, hep-ph/0410298):

$$M < 10^7 \text{ GeV}.$$

- Physics “too” heavy! No observable consequence other than leptogenesis. From thermal leptogenesis $M > 10^9 \text{ GeV}$. Will we ever convince ourselves that this is correct? (e.g., Buckley, Murayama, hep-ph/0606088)

Low-Energy Seesaw [AdG PRD72, 033005 (2005)]

The other end of the M spectrum ($M < 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light? [For a possible alternative see Canetti, Shaposhnikov, arXiv: 1006.0133 and reference therein.]
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos \Rightarrow sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the ‘tHooft sense). In fact, theoretically, no value of M should be discriminated against!

More Details, assuming three right-handed neutrinos N :

$$m_\nu = \begin{pmatrix} 0 & \lambda v \\ (\lambda v)^t & M \end{pmatrix},$$

M is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where m_a is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of M .

6×6 mixing matrix U [$U^t m_\nu U = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$] is

$$U = \begin{pmatrix} V & \Theta \\ -\Theta^\dagger V & 1_{n \times n} \end{pmatrix},$$

where V is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1, m_2, m_3),$$

and the matrix that governs active–sterile mixing is

$$\Theta = (\lambda_\nu)^* M^{-1}.$$

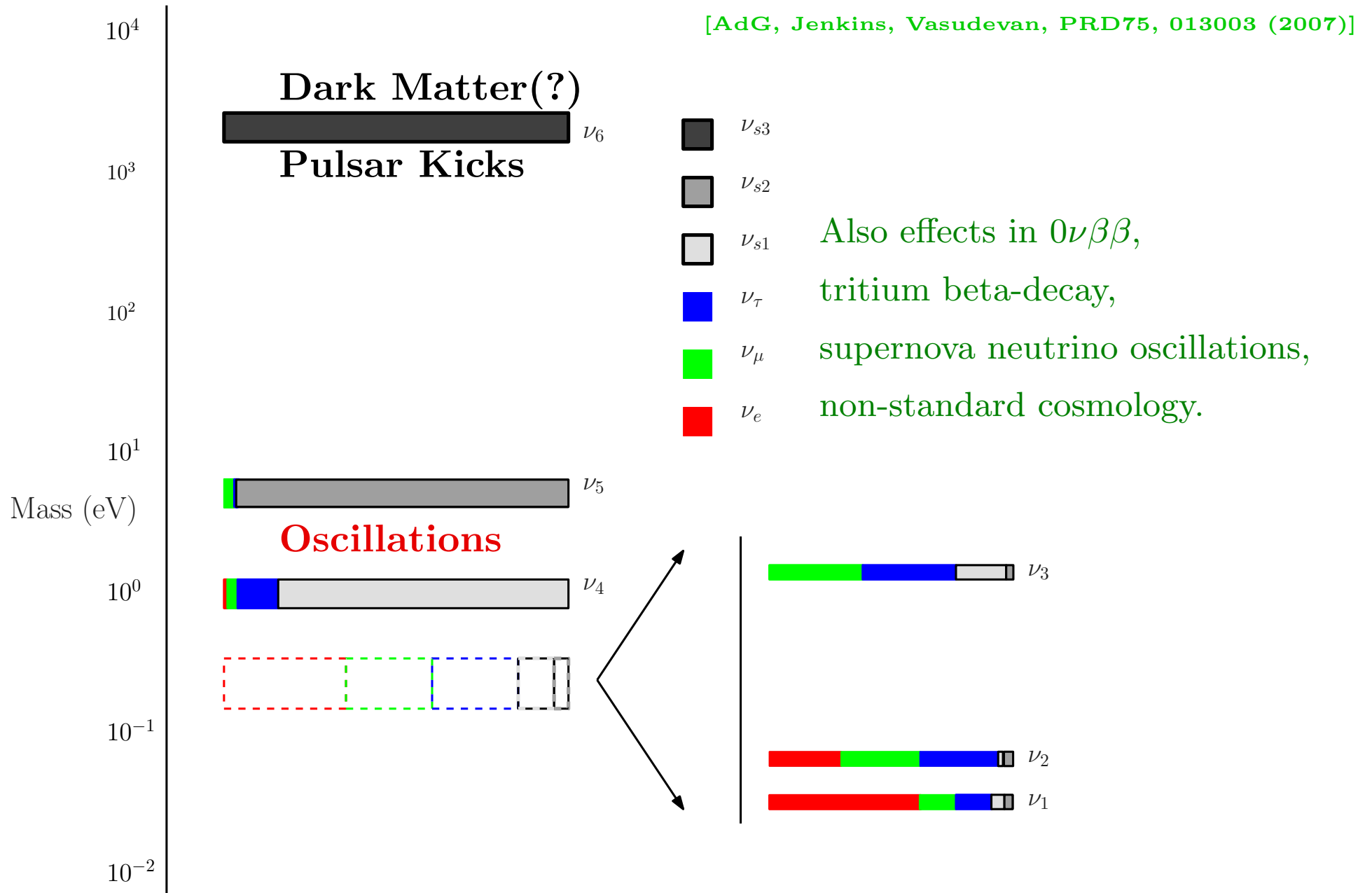
One can solve for the Yukawa couplings and re-express

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},$$

where R is a complex orthogonal matrix $RR^t = 1$.

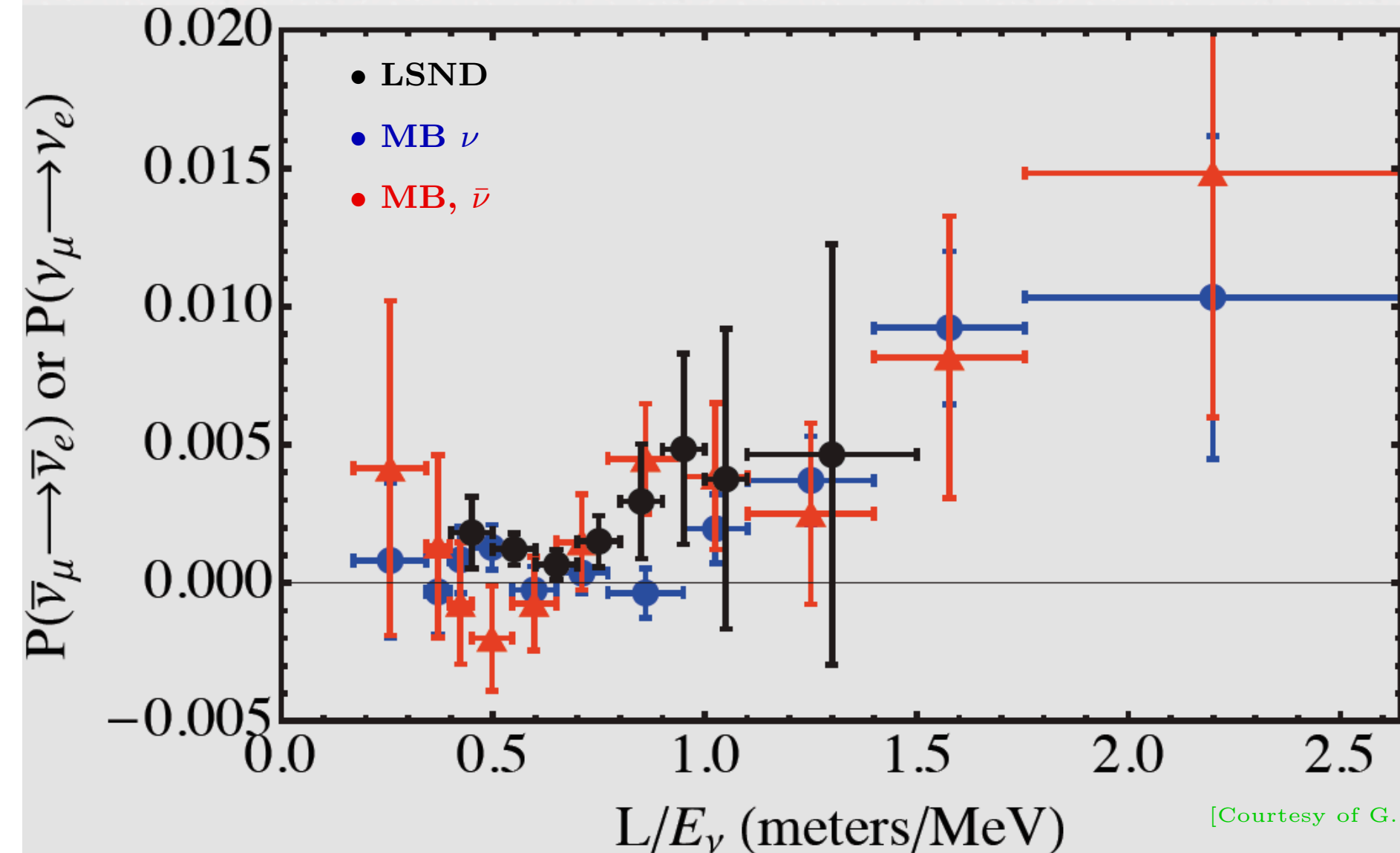
[Casas-Ibarra parameterization]

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



MiniBooNE & LSND

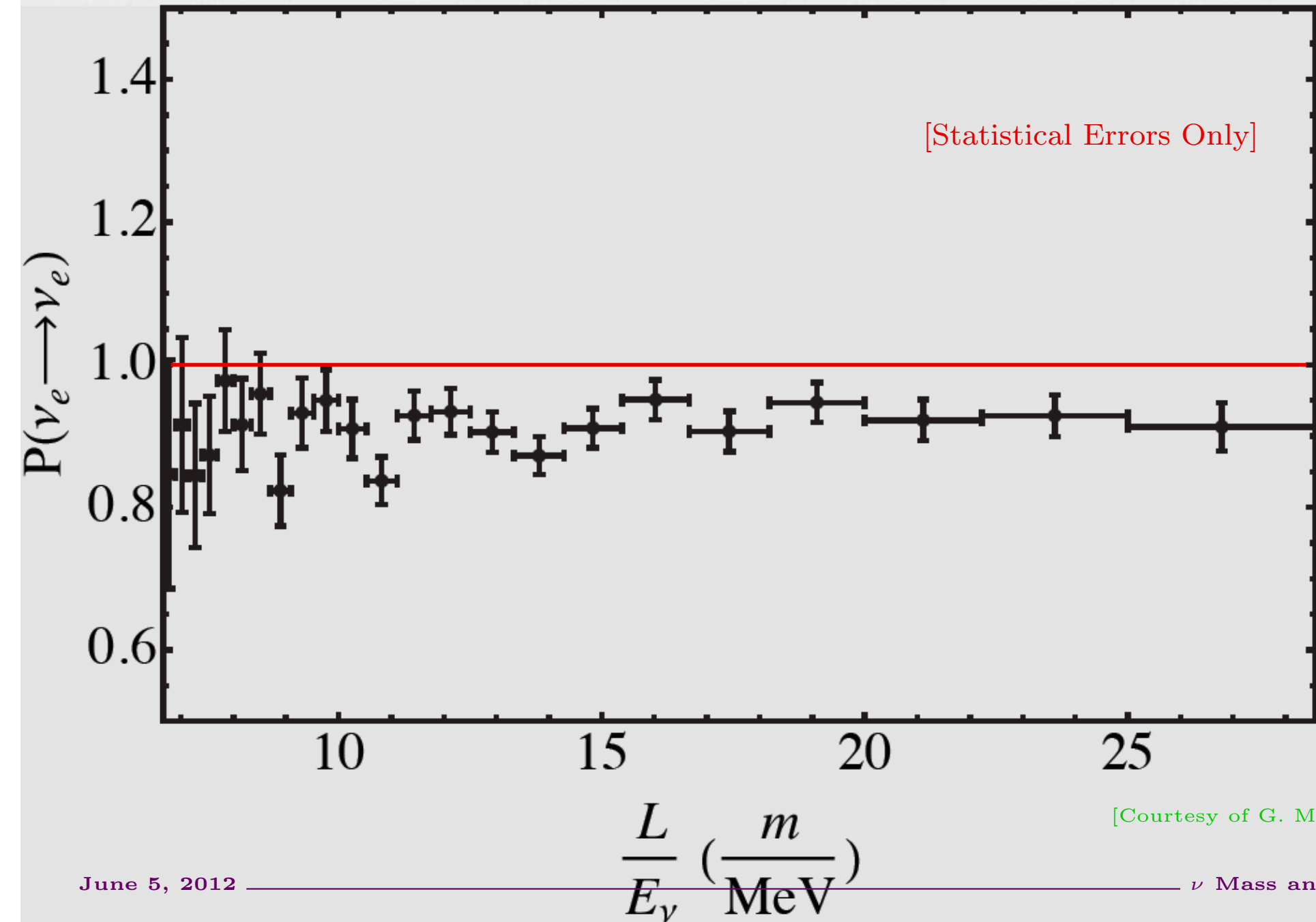
stern



[Courtesy of G. Mills]

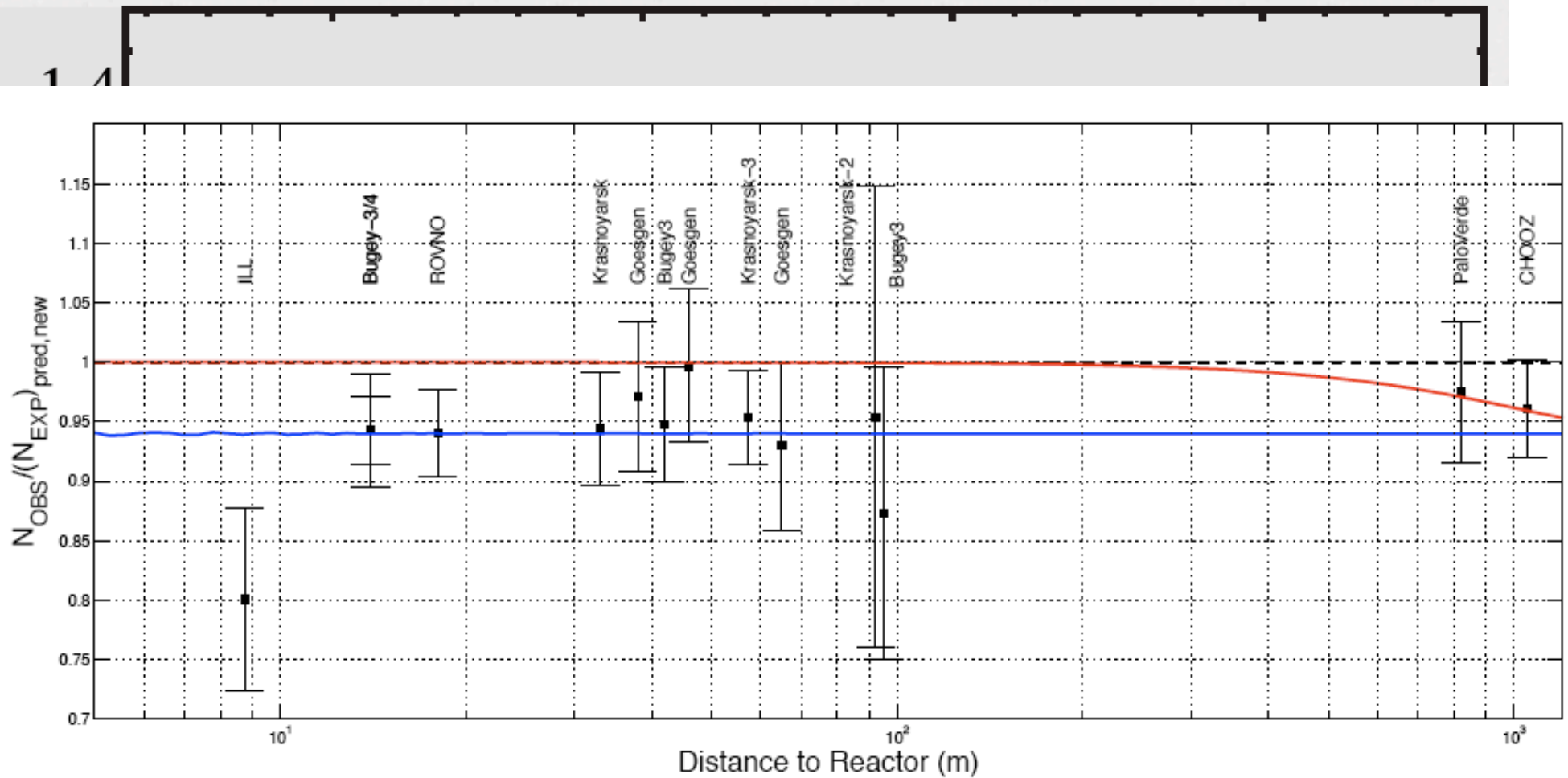
Bugey 40 m

stern



Bugey 40 m

tern



10

15

20

25

$$\frac{L}{E_\nu} \left(\frac{m}{\text{MeV}} \right)$$

More Room For New Neutrinos (?)

[Kopp, Maltoni, Schwetz, 1103.4570]

	LSND+MB($\bar{\nu}$) vs rest appearance vs disapp.			
	old	new	old	new
$\chi_{\text{PG},3+2}^2/\text{dof}$	25.1/5	19.9/5	19.9/4	14.7/4
PG ₃₊₂	10^{-4}	0.13%	5×10^{-4}	0.53%
$\chi_{\text{PG},1+3+1}^2/\text{dof}$	19.6/5	16.0/5	14.4/4	10.6/4
PG ₁₊₃₊₁	0.14%	0.7%	0.6%	3%

Table III: Compatibility of data sets [23] for 3+2 and 1+3+1 oscillations using old and new reactor fluxes.

data, although in this case the fit is slightly worse than a fit to appearance data only (dashed histograms). Note that MiniBooNE observes an event excess in the lower part of the spectrum. This excess can be explained if only appearance data are considered, but not in the global analysis including disappearance searches [8]. Therefore, we follow [19] and assume an alternative explanation for this excess, e.g. [25]. In Tab. III we show the compatibility of the LSND/MiniBooNE($\bar{\nu}$) signal with the rest of the data, as well as the compatibility of appearance and disappearance searches using the PG test from [23].

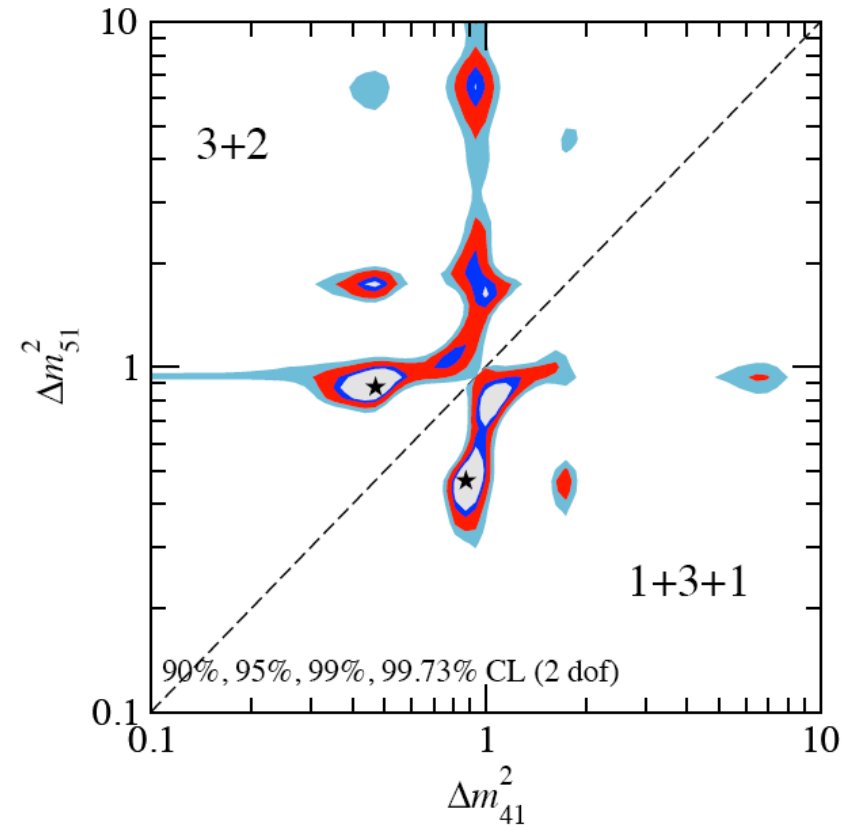


Figure 5: The globally preferred regions for the neutrino mass squared differences Δm_{41}^2 and Δm_{51}^2 in the 3+2 (upper left) and 1+3+1 (lower right) scenarios.

Predictions: Neutrinoless Double-Beta Decay

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \rightarrow (Z + 2)e^-e^-$.

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

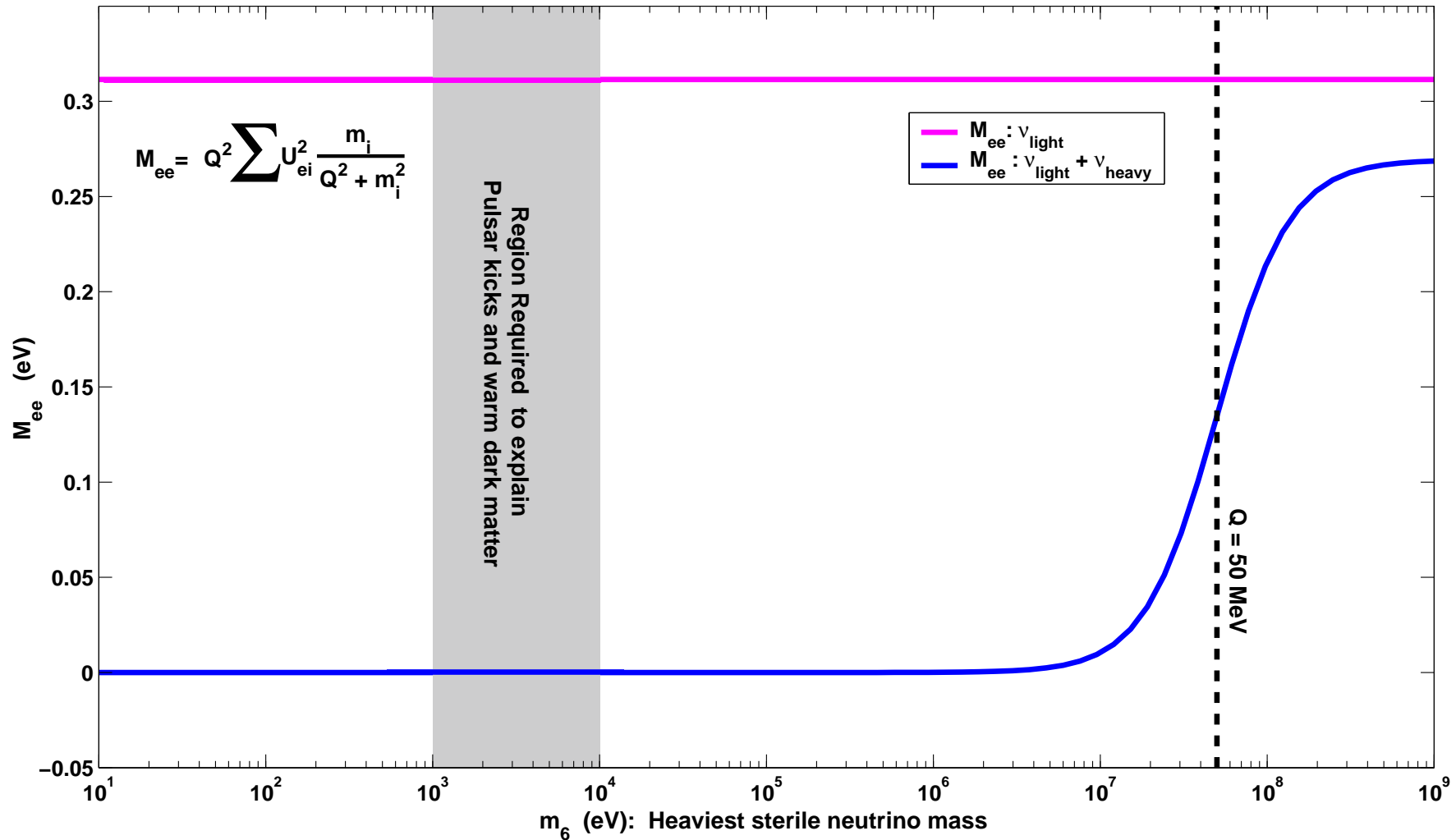
$$m_{ee} = \left| \sum_{i=1}^6 U_{ei}^2 m_i \right| \sim \left| \sum_{i=1}^3 U_{ei}^2 m_i + \sum_{i=1}^3 \vartheta_{ei}^2 M_i \right|.$$

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. **The contribution of light and heavy neutrinos exactly cancels!** This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\left[\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \rightarrow m_{ee} \text{ is identically zero!} \right]$$

(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



On Early Universe Cosmology / Astrophysics

[S. Hannestad's talk]

A combination of the SM of particle physics plus the “concordance cosmological model” severely constrain light, sterile neutrinos with significant active-sterile mixing. Taken at face value, not only is the eV-seesaw ruled out, but so are all oscillation solutions to the LSND anomaly.

Hence, eV-seesaw \rightarrow nonstandard particle physics and cosmology.

On the other hand...

- Right-handed neutrinos may make good warm dark matter particles.

Asaka, Blanchet, Shaposhnikov, hep-ph/0503065; M. Lindner's talk

- Sterile neutrinos are known to help out with r-process nucleosynthesis in supernovae, ...
- ...and may help explain the peculiar peculiar velocities of pulsars ...

What if $1 \text{ GeV} < M < 1 \text{ TeV}$?

Naively, one expects

$$\Theta \sim \sqrt{\frac{m_a}{M}} < 10^{-5} \sqrt{\frac{1 \text{ GeV}}{M}},$$

such that, for $M = 1 \text{ GeV}$ and above, sterile neutrino effects are mostly negligible.

However,

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},$$

and the magnitude of the entries of R can be arbitrarily large [$\cos(ix) = \cosh x \gg 1$ if $x > 1$].

This is true as long as

- $\lambda v \ll M$ (seesaw approximation holds)
- $\lambda < 4\pi$ (theory is “well-defined”)

This implies that, in principle, Θ is a quasi-free parameter – independent from light neutrino masses and mixing – as long as $\Theta \ll 1$ and $M < 1 \text{ TeV}$.

What Does $R \gg 1$ Mean?

It is illustrative to consider the case of one active neutrino of mass m_3 and two sterile ones, and further assume that $M_1 = M_2 = M$. In this case,

$$\Theta = \sqrt{\frac{m_3}{M}} \begin{pmatrix} \cos \zeta & \sin \zeta \end{pmatrix},$$

$$\lambda v = \sqrt{m_3 M} \begin{pmatrix} \cos \zeta^* & \sin \zeta^* \end{pmatrix} \equiv \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix}.$$

If ζ has a large imaginary part $\Rightarrow \Theta$ is (exponentially) larger than $(m_3/M)^{1/2}$, λ_i neutrino Yukawa couplings are much larger than $\sqrt{m_3 M}/v$

The reason for this is a strong cancellation between the contribution of the two different Yukawa couplings to the active neutrino mass

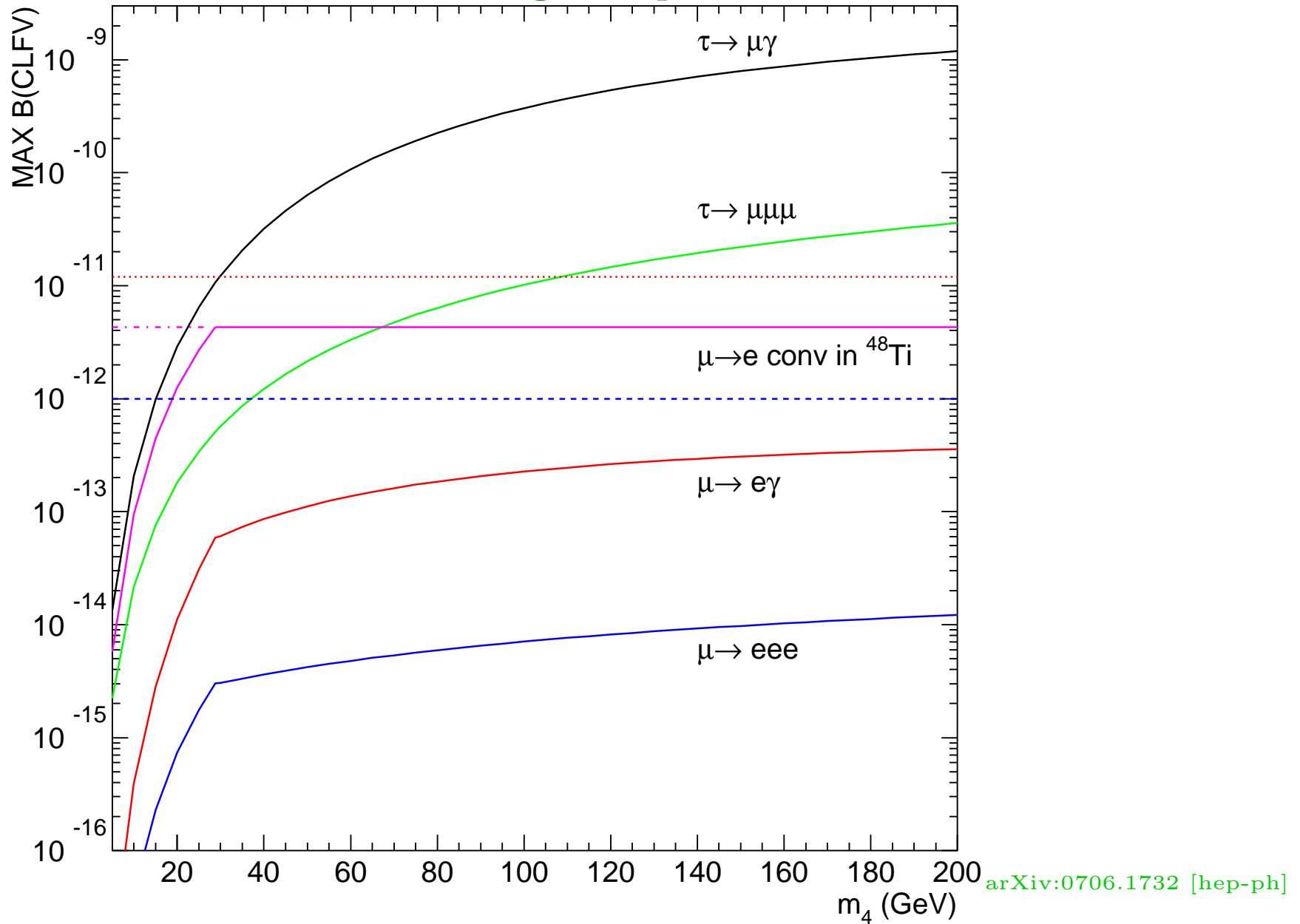
$$\Rightarrow m_3 = \lambda_1^2 v^2 / M + \lambda_2^2 v^2 / M.$$

For example: $m_3 = 0.1$ eV, $M = 100$ GeV, $\zeta = 14i \Rightarrow \lambda_1 \sim 0.244$, $\lambda_2 \sim -0.244i$, while $|y_1| - |y_2| \sim 3.38 \times 10^{-13}$.

NOTE: cancellation may be consequence of a symmetry (say, lepton number).

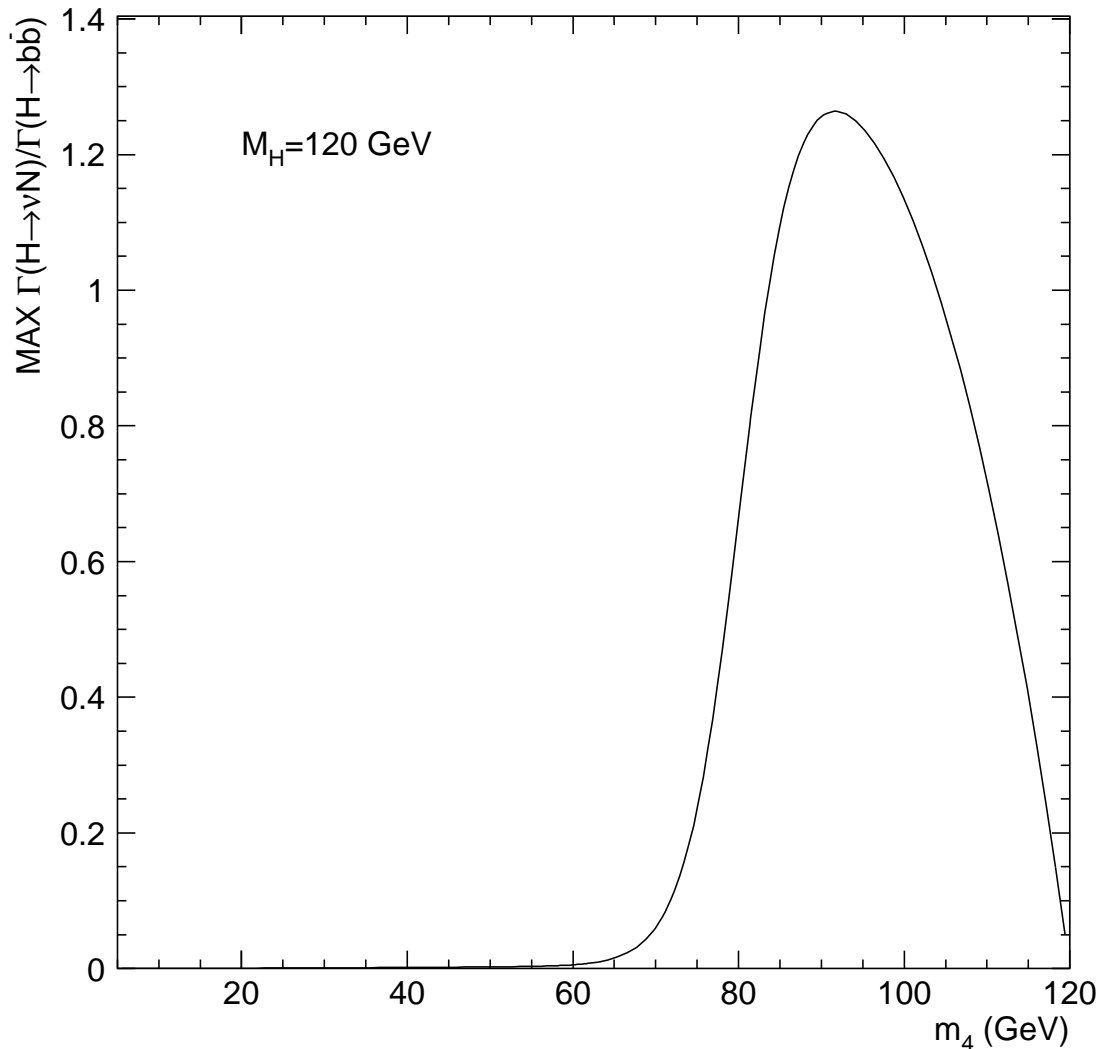
See, for example, the “inverse seesaw” [Mohapatra and Valle, PRD34, 1642 \(1986\)](#).

Constraints From Charged Lepton Flavor Violation



Weak Scale Seesaw, and Accidentally Light Neutrino Masses

[AdG arXiv:0706.1732 [hep-ph]]



What does the seesaw Lagrangian predict for the LHC?

Nothing much, unless...

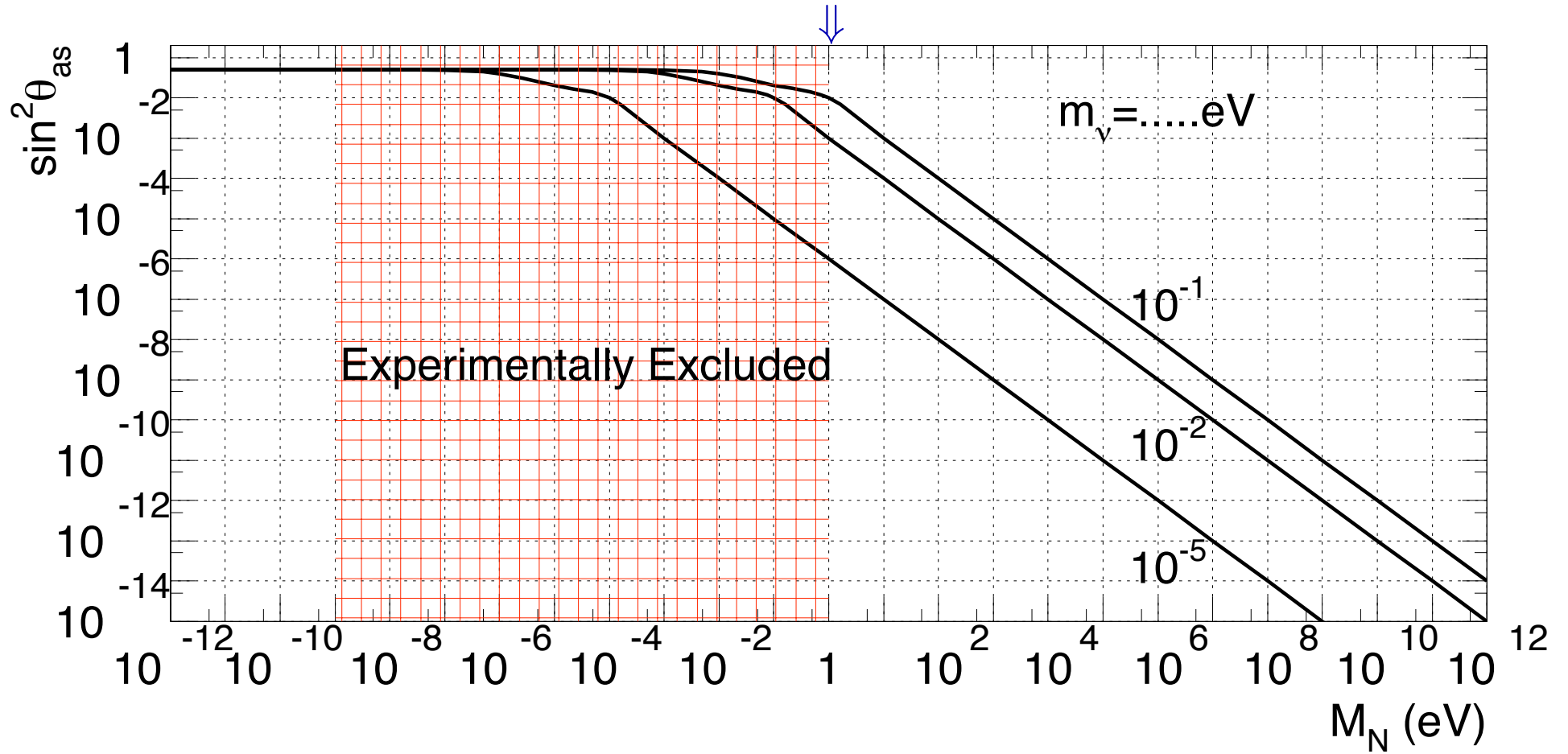
- $M_N \sim 1 - 100$ GeV,
- Yukawa couplings larger than naive expectations.

$\Leftarrow H \rightarrow \nu N$ as likely as $H \rightarrow b\bar{b}$!

(NOTE: $N \rightarrow \ell q' \bar{q}$ or $\ell \ell' \nu$ (prompt)
 “Weird” Higgs decay signature!)

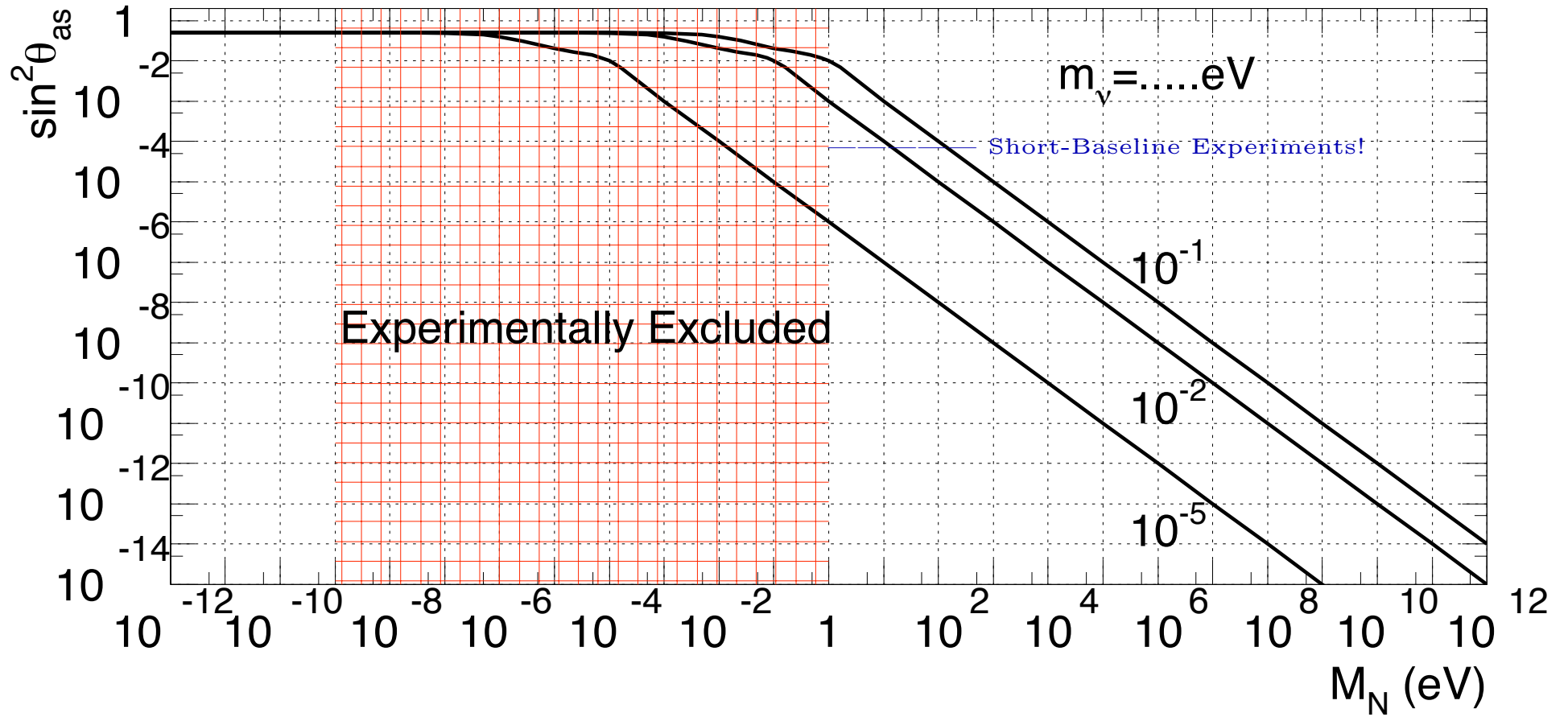
Constraining the Seesaw Lagrangian

[rough upper bound, see Donini et al, arXiv:1106.0064]



[AdG, Huang, Jenkins, arXiv:0906.1611]

Can we improve our sensitivity?



[AdG, Huang, Jenkins, arXiv:0906.1611]

Model independent constraints

Constraints depend, unfortunately, on m_i and M_i and R . E.g.,

$$U_{e4} = U_{e1}A\sqrt{\frac{m_1}{m_4}} + U_{e2}B\sqrt{\frac{m_2}{m_4}} + U_{e3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\mu4} = U_{\mu1}A\sqrt{\frac{m_1}{m_4}} + U_{\mu2}B\sqrt{\frac{m_2}{m_4}} + U_{\mu3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\tau4} = U_{\tau1}A\sqrt{\frac{m_1}{m_4}} + U_{\tau2}B\sqrt{\frac{m_2}{m_4}} + U_{\tau3}C\sqrt{\frac{m_3}{m_4}},$$

where

$$A^2 + B^2 + C^2 = 1.$$

One can pick A, B, C such that two of these vanish. But the other one is maximized, along with $U_{\alpha5}$ and $U_{\alpha6}$.

Can we (a) constrain the seesaw scale with combined bounds on $U_{\alpha4}$ or (b) test whether the low energy seesaw is “correct” if nonzero $U_{\alpha4}$ are discovered?

Concrete Example: 2 right-handed neutrinos

$$X_{\text{normal}} = \begin{pmatrix} 0.23e^{i\phi} & 0.1e^{i\delta} \\ (0.25 - 0.02e^{-i\delta})e^{i\phi} & 0.70 \\ -(0.25 + 0.02e^{-i\delta})e^{i\phi} & 0.70 \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix}$$

$$X_{\text{inverted}} = \begin{pmatrix} 0.83e^{i\psi} & 0.55 \\ -(0.39 + 0.06e^{-i\delta})e^{i\psi} & 0.59 - 0.04e^{-i\delta} \\ (0.39 - 0.06e^{-i\delta})e^{i\psi} & -0.59 - 0.04e^{-i\delta} \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix}$$

$$\zeta \in \mathcal{C}$$

where

$$X_{\text{normal (inverted)}} = \Theta \sqrt{\frac{m_{\text{heavy}}}{m_3 (m_2)}}$$

Some Relevant Examples: [AdG, W-C Huang, arXiv:1110.6122]

$\zeta = 3/4\pi + i$, $\delta = 6/5\pi$, $\phi = \pi/2$ and a normal mass hierarchy,

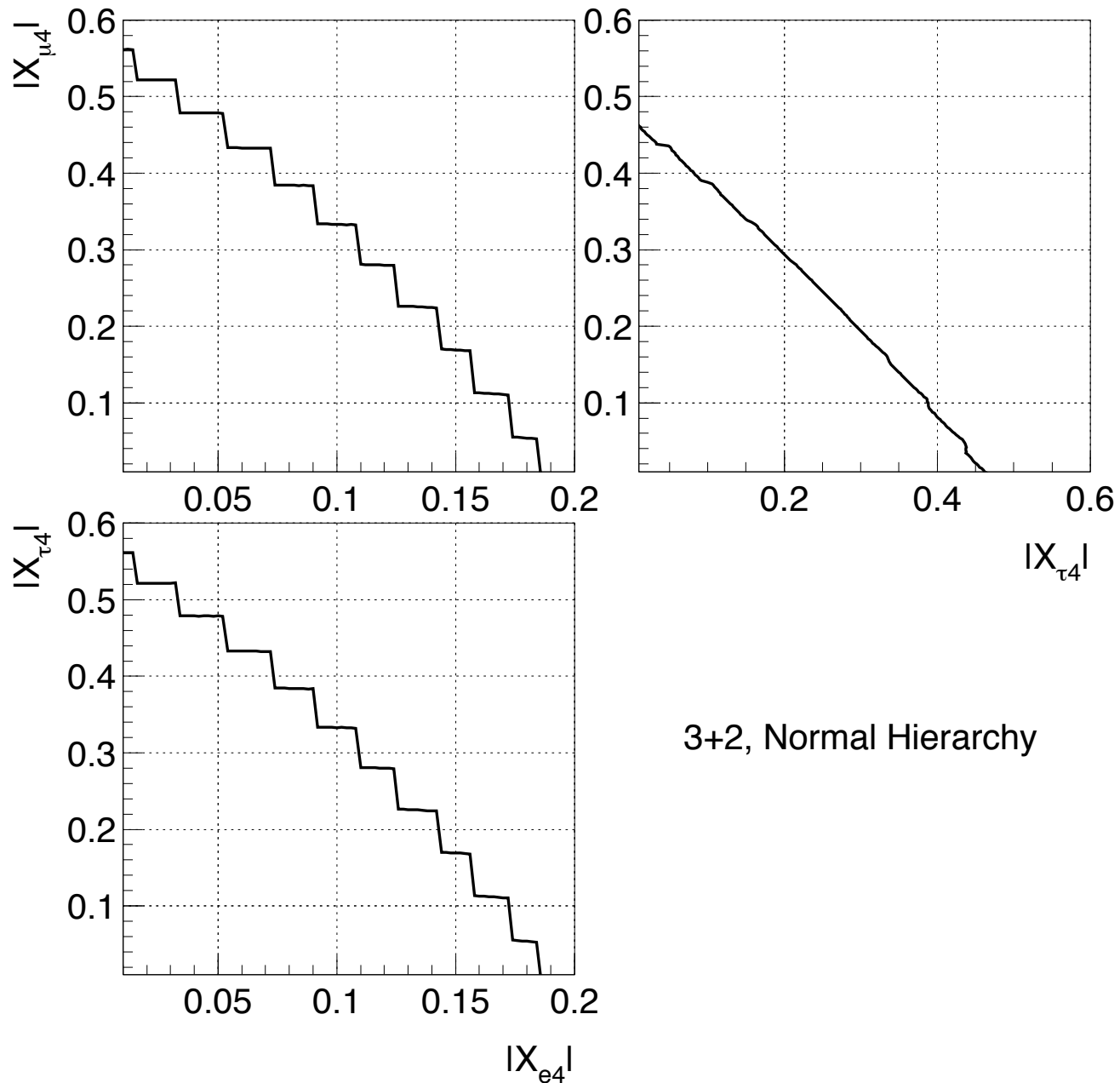
$$X_{\text{normal}} = \begin{pmatrix} 0.41e^{-0.66i} & 0.45e^{1.03i} \\ 0.62e^{2.67i} & 0.61e^{-2.62i} \\ 1.27e^{2.44i} & 1.26e^{-2.41i} \end{pmatrix}.$$

$\zeta = 2/3\pi + 0.3i$, $\delta = 0$, $\psi = \pi/2$, and an inverted mass hierarchy,

$$X_{\text{inverted}} = \begin{pmatrix} 0.44e^{-2.24i} & 0.62e^{1.83i} \\ 0.69e^{2.66i} & 0.66e^{-2.14i} \\ 0.71e^{-0.39i} & 0.60e^{0.89i} \end{pmatrix}.$$

both accommodate 3+2 fit for $m_4^2 = 0.5 \text{ eV}^2$ and $m_5^2 = 0.9 \text{ eV}^2$. Furthermore, $|U_{\tau 4}|$ and $|U_{\tau 5}|$ are completely fixed. No more free parameters. They are also both larger than (or at least as large as $|U_{\mu 4}|$ and $|U_{\mu 5}|$).

$\nu_\mu \rightarrow \nu_\tau$ MUST be observed if this is the origin of the two mostly sterile neutrinos.



Making Predictions, for an inverted mass hierarchy, $m_4 = 1 \text{ eV} (\ll m_5)$

- ν_e disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{ee} > 0.02$. An interesting new proposal to closely expose the Daya Bay detectors to a strong β -emitting source would be sensitive to $\sin^2 2\vartheta_{ee} > 0.04$;
- ν_μ disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{\mu\mu} > 0.07$, very close to the most recent MINOS lower bound;
- $\nu_\mu \leftrightarrow \nu_e$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{e\mu} > 0.0004$;
- $\nu_\mu \leftrightarrow \nu_\tau$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{\mu\tau} > 0.001$. A $\nu_\mu \rightarrow \nu_\tau$ appearance search sensitive to probabilities larger than 0.1% for a mass-squared difference of 1 eV^2 would definitively rule out $m_4 = 1 \text{ eV}$ if the neutrino mass hierarchy is inverted.

Understanding Fermion Mixing

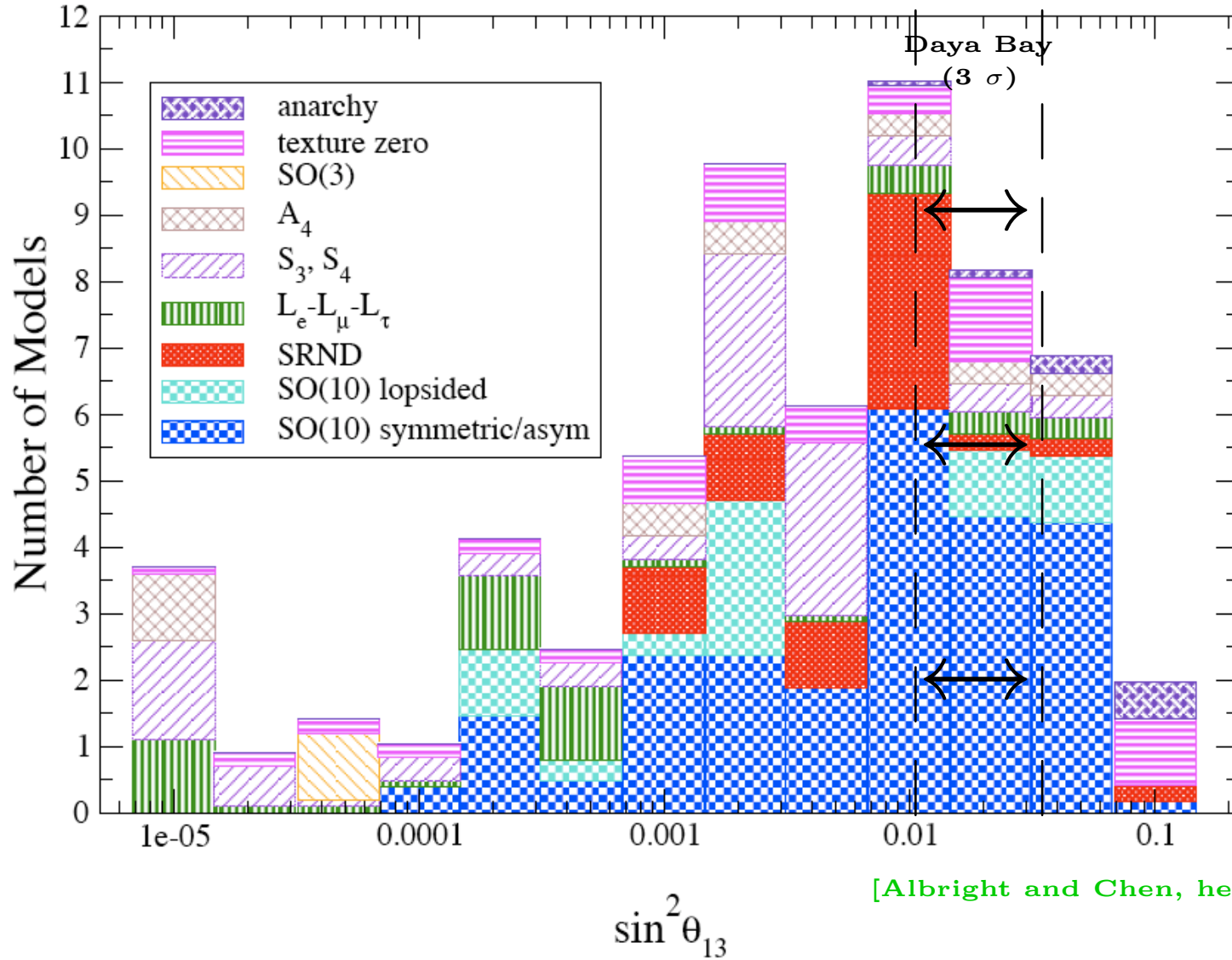
The other puzzling phenomenon uncovered by the neutrino data is the fact that **Neutrino Mixing is Strange**. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \quad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix} \quad \boxed{\text{WHY?}}$$

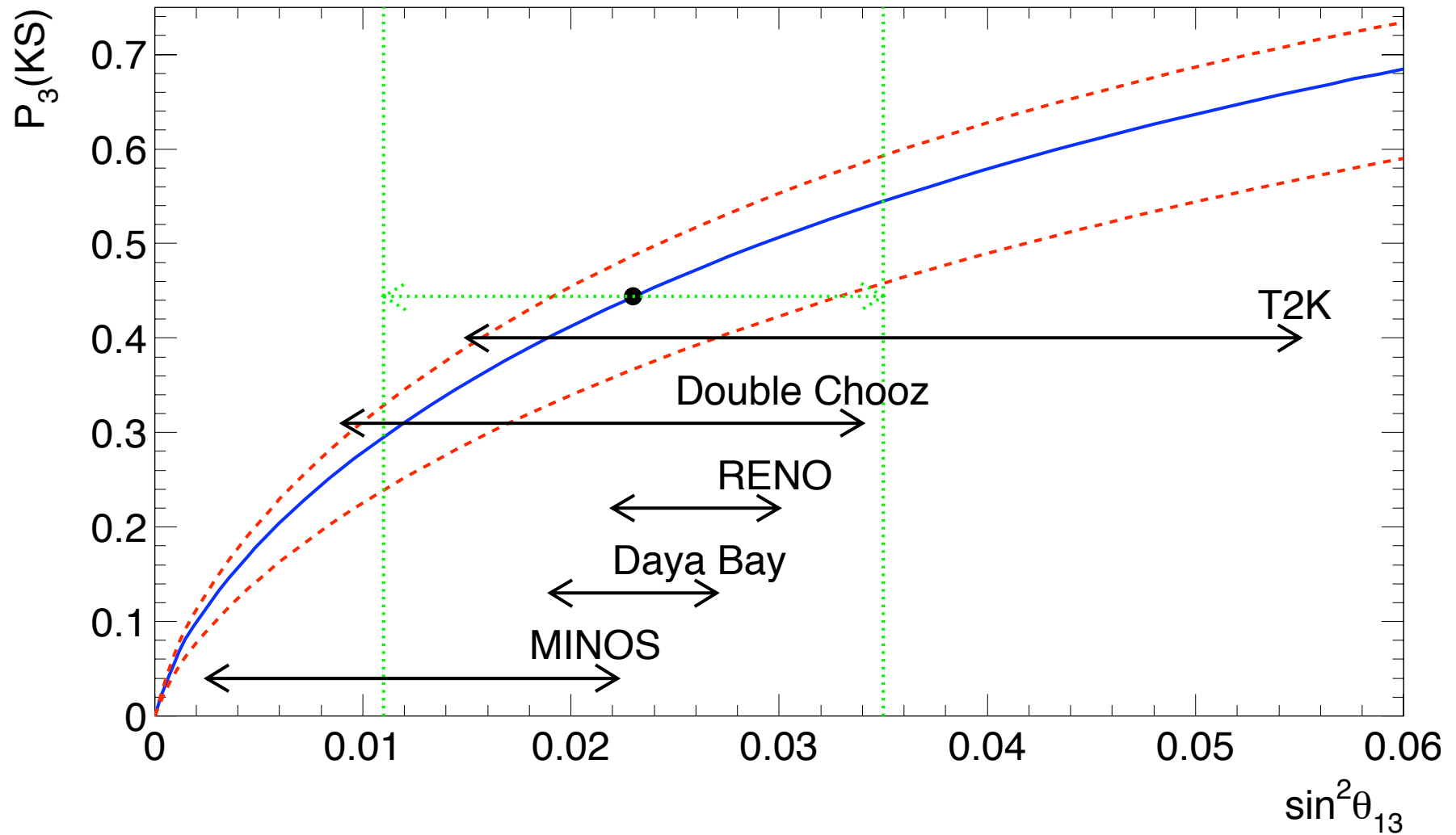
$[|(V_{MNS})_{e3}| < 0.2]$

They certainly look **VERY** different, but which one would you label as “strange”?

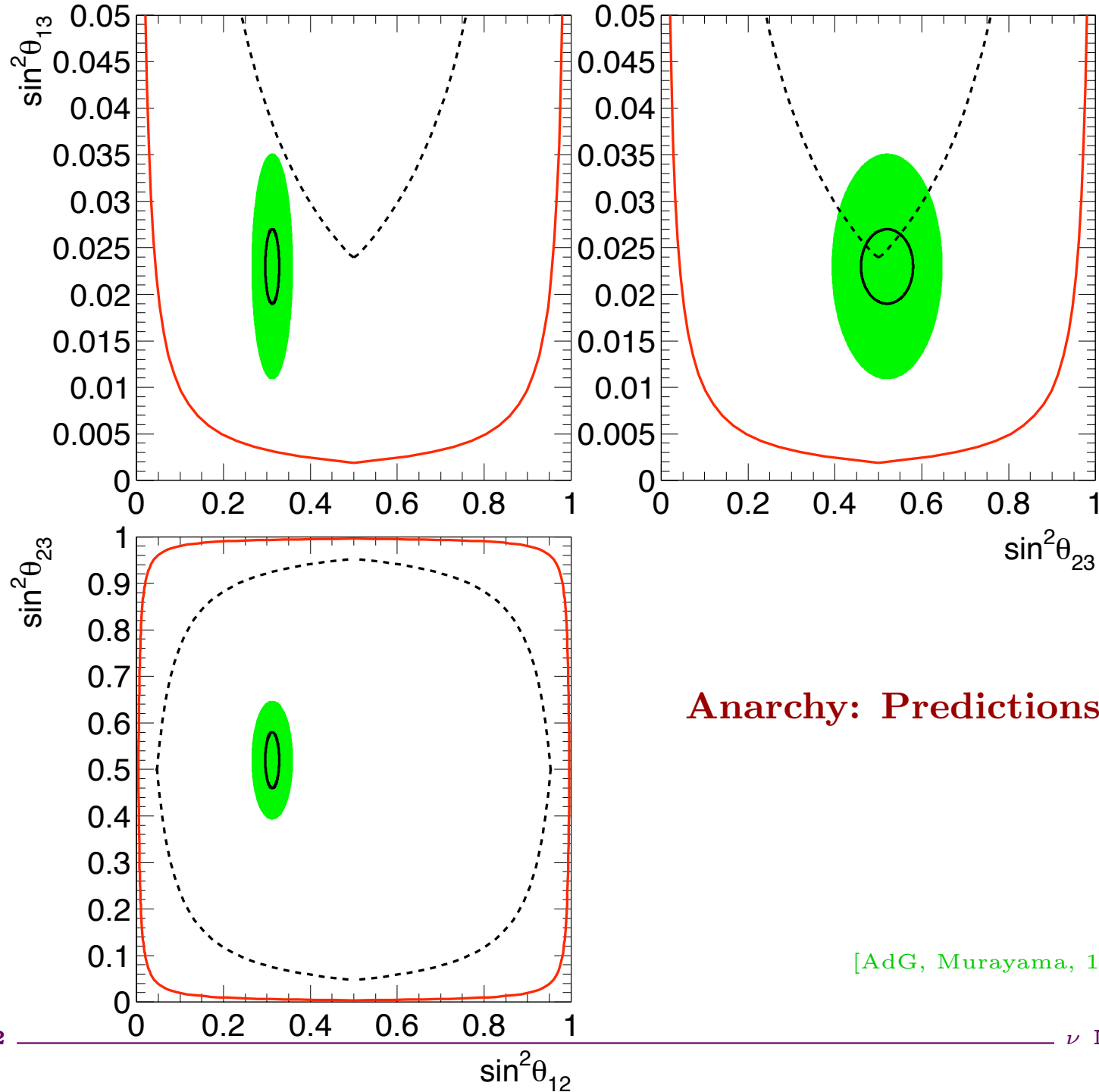


“Left-Over” Predictions: δ , mass-hierarchy, $\cos 2\theta_{23}$

Neutrino Mixing Anarchy: Alive and Kicking!



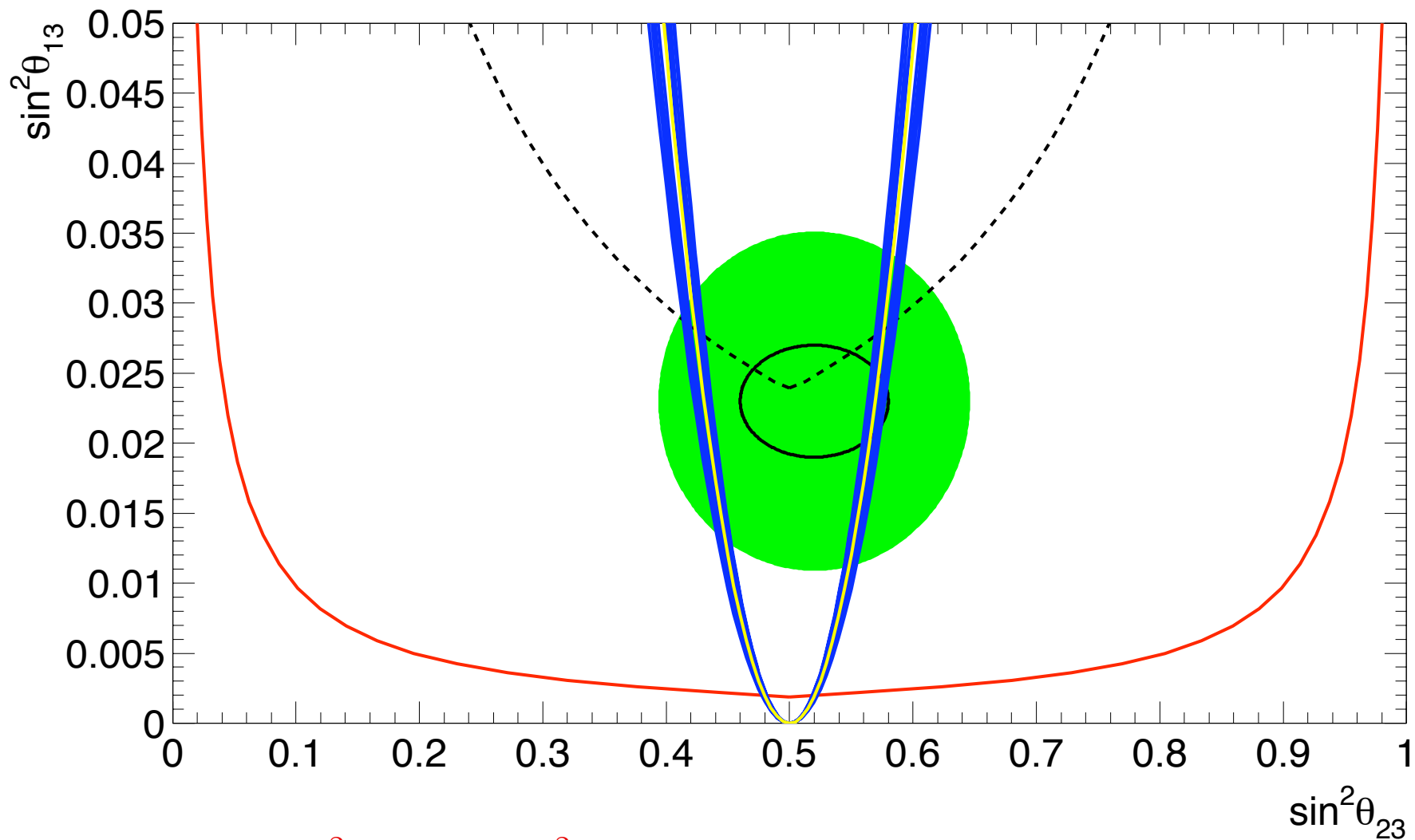
[AdG, Murayama, 1204.1249]



Anarchy: Predictions

[AdG, Murayama, 1204.1249]

Anarchy vs. Order — more precision required!



Order: $\sin^2 \theta_{13} = C \cos^2 2\theta_{23}$, $C \in [0.8, 1.2]$

[AdG, Murayama, 1204.1249]

How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation;
($\mu \rightarrow e\gamma$, $\mu \rightarrow e$ -conversion in nuclei, etc)
- searches for lepton number violation;
(neutrinoless double beta decay, etc)
- neutrino oscillation experiments;
(Daya Bay, NO ν A, etc)
- searches for fermion electric/magnetic dipole moments
(electron edm, muon $g - 2$, etc);

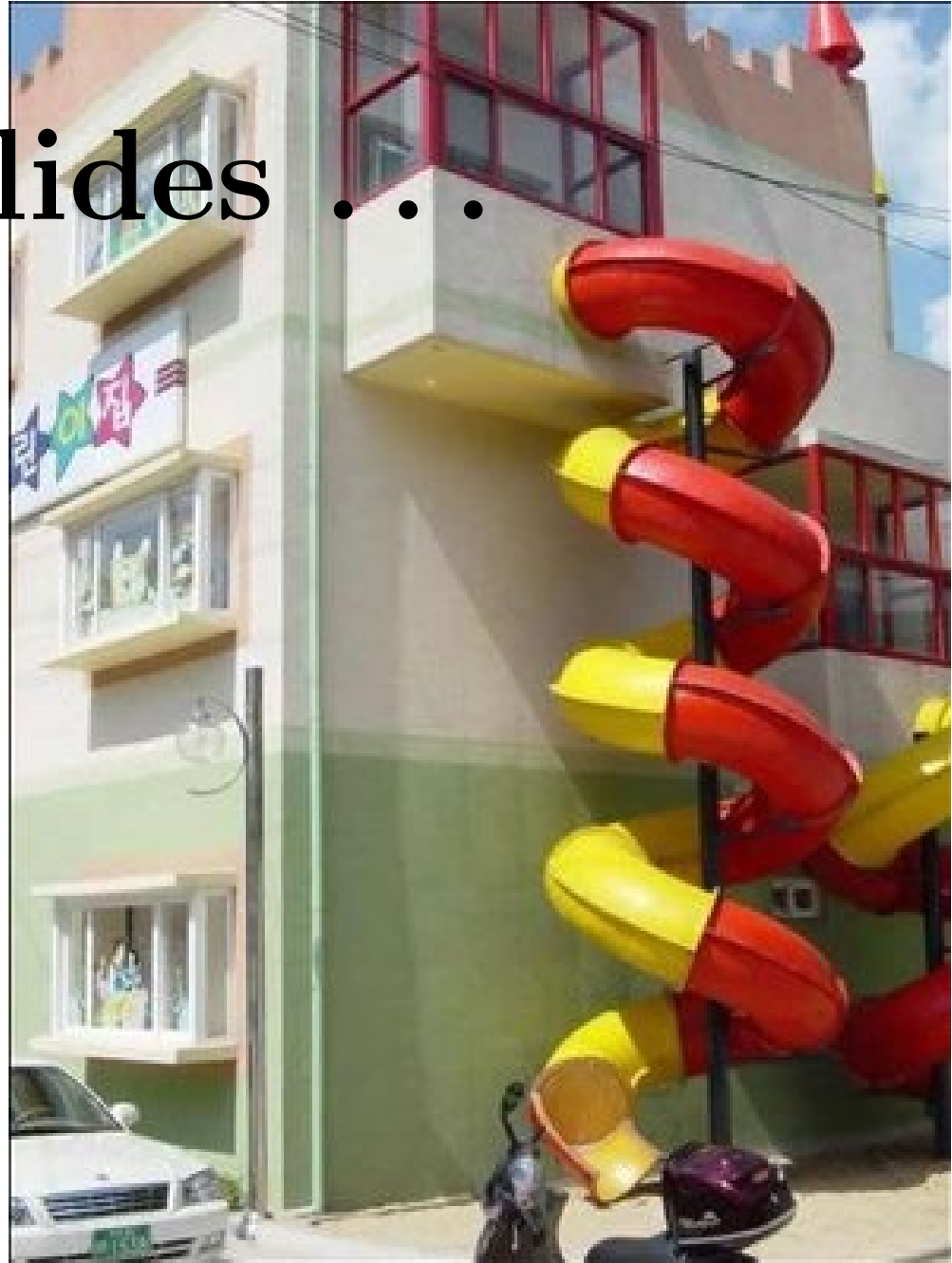
- precision studies of neutrino – matter interactions;
(Miner ν a, MicroBooNE, etc)

- collider experiments:
(LHC, etc)
 - *Can* we “see” the physics responsible for neutrino masses at the LHC?
– YES!
Must we see it? – NO, but we won’t find out until we try!
 - we need to understand the physics at the TeV scale before we can really understand the physics behind neutrino masses (is there low-energy SUSY?, etc).

CONCLUSIONS

1. We know very little about the new physics uncovered by neutrino oscillations, e.g.,
 - It could be renormalizable \rightarrow “boring” Dirac neutrinos.
 - It could be due to Physics at absurdly high energy scales $M \gg 1 \text{ TeV} \rightarrow$ high energy seesaw. How can we ever convince ourselves that this is correct?
 - It could be due to very light new physics \rightarrow low energy seesaw. Prediction: new light propagating degrees of freedom – sterile neutrinos.
2. We still don't understand the pattern of lepton mixing, but anarchical hypothesis works great. Can one do better? (θ_{23} , quarks, ...)
3. We need more experimental input!

Backup Slides . . .



Fourth Avenue: Higher Order Neutrino Masses from $\Delta L = 2$ Physics.

Imagine that there is **new physics that breaks lepton number by 2 units** at some energy scale Λ , but that it does not, in general, lead to neutrino masses **at the tree level**.

We know that neutrinos will get a mass at some order in perturbation theory – which order is model dependent!

For example:

- SUSY with trilinear R-parity violation – neutrino masses at one-loop;
- Zee model – neutrino masses at one-loop;
- Babu and Ma – neutrino masses at two loops;
- Chen, *et al.* 0706.1964 – neutrino masses at two loops;
- etc

André de Gouvêa
AdG, Jenkins,
0708.1344 [hep-ph]

**Effective
Operator
Approach**

(there are 129
of them if you
discount different
Lorentz structures!)

classified by Babu
and Leung in
NPB619,667(2001)

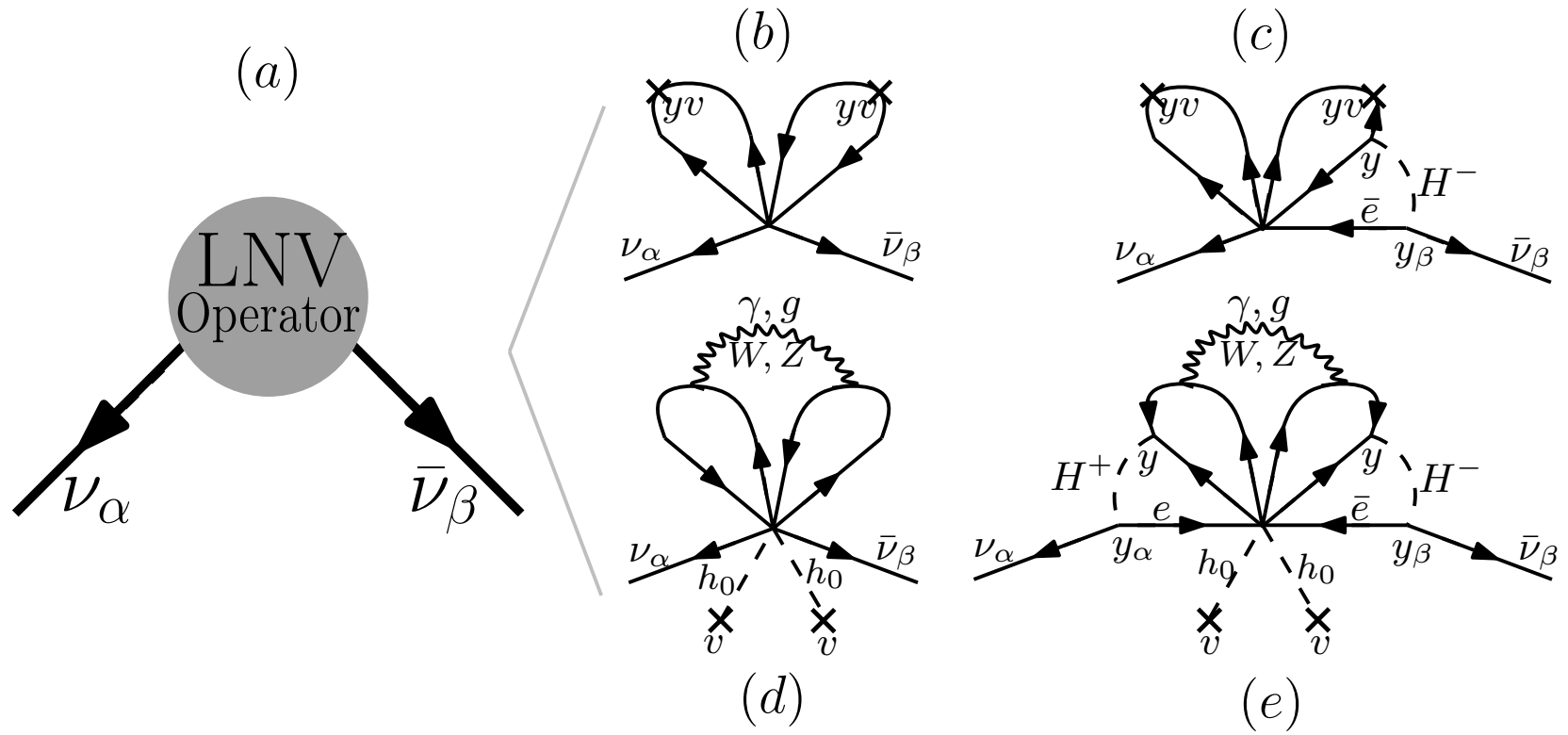
June 5, 2012

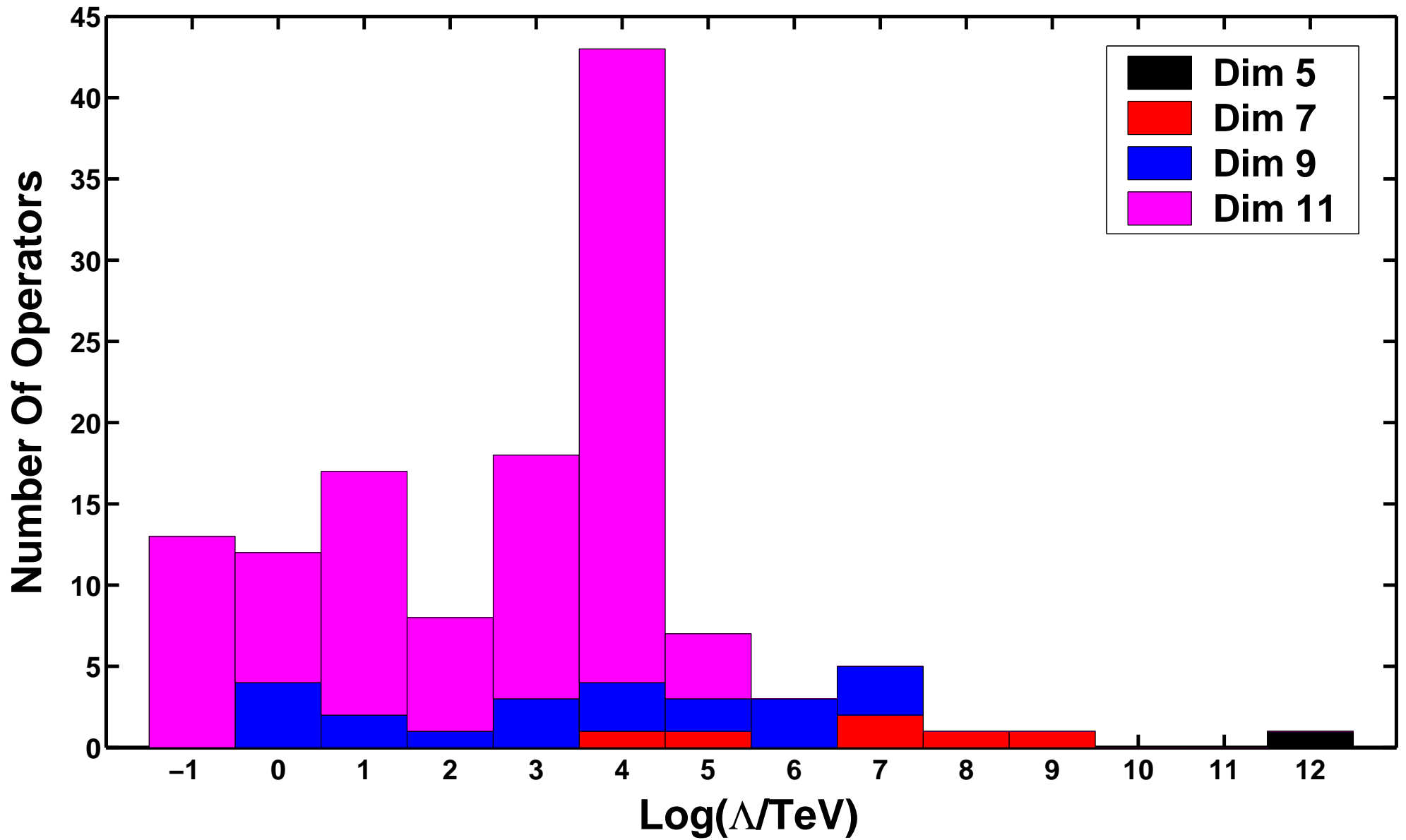
13	$L^i L^j \bar{Q}_i \bar{u}^c L^l e^c \epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda}$	2×10^5	$\beta\beta\nu$
14 _a	$L^i L^j \bar{Q}_k \bar{u}^c Q^k d^c \epsilon_{ij}$	$\frac{y_d y_u g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^3	Northwest $\beta\beta\nu$
14 _b	$L^i L^j \bar{Q}_i \bar{u}^c Q^l d^c \epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda}$	6×10^5	$\beta\beta\nu$
15	$L^i L^j L^k d^c \bar{L}_i \bar{u}^c \epsilon_{jk}$	$\frac{y_d y_u g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^3	$\beta\beta\nu$
16	$L^i L^j e^c d^c \bar{e}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
17	$L^i L^j d^c d^c \bar{d}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
18	$L^i L^j d^c u^c \bar{u}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
19	$L^i Q^j d^c d^c \bar{e}^c \bar{u}^c \epsilon_{ij}$	$y_\ell y_\beta \frac{y_d^2 y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1	$\beta\beta\nu$, HEInν, LHC, m
20	$L^i d^c \bar{Q}_i \bar{u}^c \bar{e}^c \bar{u}^c$	$y_\ell y_\beta \frac{y_d y_u^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	40	$\beta\beta\nu$, mix
21 _a	$L^i L^j L^k e^c Q^l u^c H^m H^n \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	$\beta\beta\nu$
21 _b	$L^i L^j L^k e^c Q^l u^c H^m H^n \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	$\beta\beta\nu$
22	$L^i L^j L^k e^c \bar{L}_k \bar{e}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
23	$L^i L^j L^k e^c \bar{Q}_k \bar{d}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	40	$\beta\beta\nu$
24 _a	$L^i L^j Q^k d^c Q^l d^c H^m \bar{H}_i \epsilon_{jk} \epsilon_{lm}$	$\frac{y_d^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^2	$\beta\beta\nu$
24 _b	$L^i L^j Q^k d^c Q^l d^c H^m \bar{H}_i \epsilon_{jm} \epsilon_{kl}$	$\frac{y_d^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^2	$\beta\beta\nu$
25	$L^i L^j Q^k d^c Q^l u^c H^m H^n \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	4×10^3	$\beta\beta\nu$
26 _a	$L^i L^j Q^k d^c \bar{L}_i \bar{e}^c H^l H^m \epsilon_{jl} \epsilon_{km}$	$\frac{y_\ell y_d}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	40	$\beta\beta\nu$
26 _b	$L^i L^j Q^k d^c \bar{L}_k \bar{e}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	40	$\beta\beta\nu$
27 _a	$L^i L^j Q^k d^c \bar{Q}_i \bar{d}^c H^l H^m \epsilon_{jl} \epsilon_{km}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
27 _b	$L^i L^j Q^k d^c \bar{Q}_k \bar{d}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
28 _a	$L^i L^j Q^k d^c \bar{Q}_j \bar{u}^c H^l \bar{H}_i \epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
28 _b	$L^i L^j Q^k d^c \bar{Q}_k \bar{u}^c H^l \bar{H}_i \epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
28 _c	$L^i L^j Q^k d^c \bar{Q}_l \bar{u}^c H^l \bar{H}_i \epsilon_{jk}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
29 _a	$L^i L^j Q^k u^c \bar{Q}_k \bar{u}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_u^2}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^5	$\beta\beta\nu$
29 _b	$L^i L^j Q^k u^c \bar{Q}_l \bar{u}^c H^l H^m \epsilon_{ik} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
30 _a	$L^i L^j \bar{L}_i \bar{e}^c \bar{Q}_k \bar{u}^c H^k H^l \epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	2×10^3	$\beta\beta\nu$
30 _b	$L^i L^j \bar{L}_m e^c \bar{Q}_n u^c H^k H^l \epsilon_{ik} \epsilon_{jl} \epsilon^{mn}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	Mass and Mix $\beta\beta\nu$
31 _a	$L^i L^j \bar{Q}_i \bar{d}^c \bar{Q}_l \bar{u}^c H^k H^l \epsilon_{ij}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	4×10^3	$\beta\beta\nu$

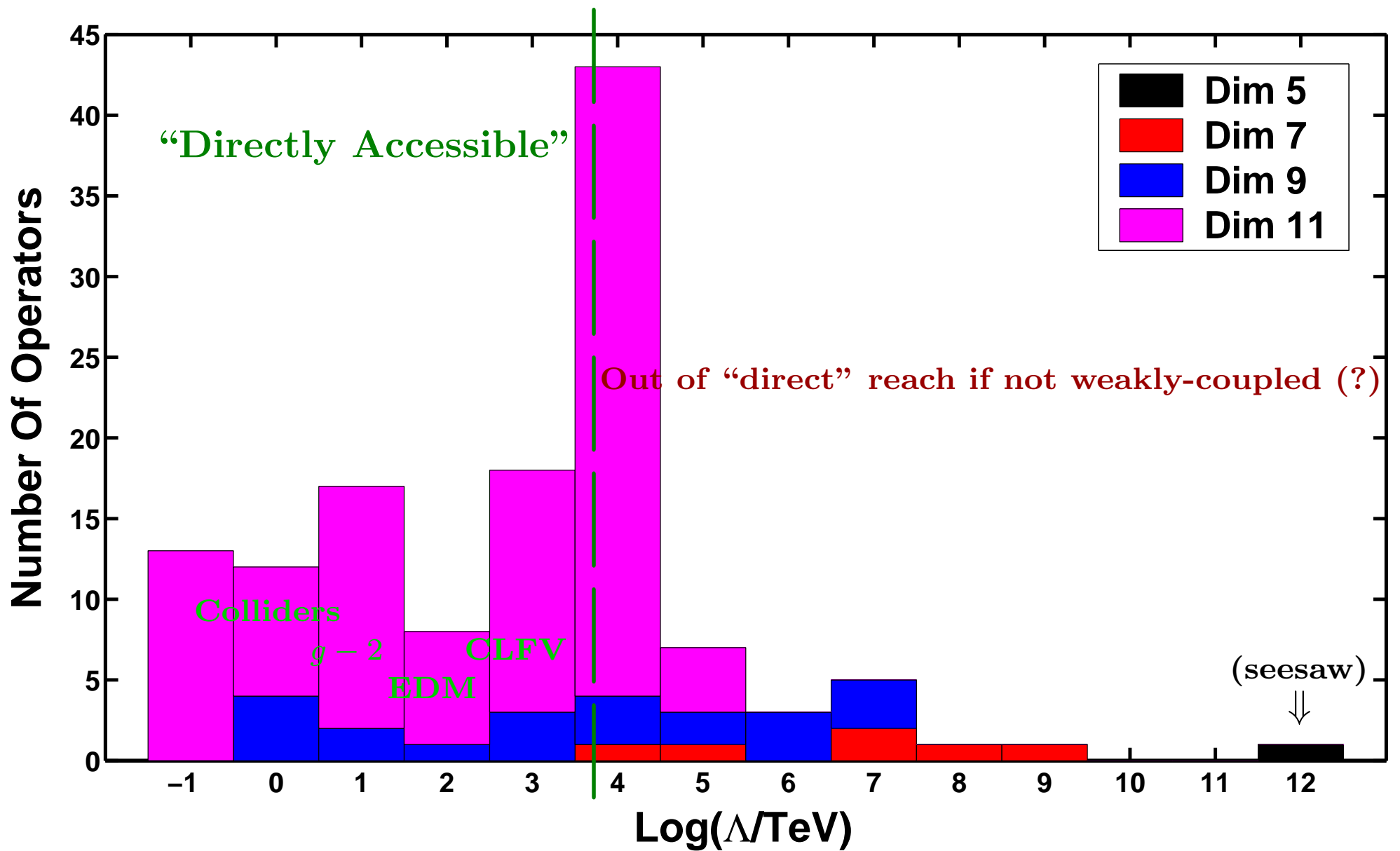
Assumptions:

- Only consider $\Delta L = 2$ operators;
- Operators made up of only standard model fermions and the Higgs doublet (no gauge bosons);
- Electroweak symmetry breaking characterized by SM Higgs doublet field;
- Effective operator couplings assumed to be “flavor indifferent”;
- Operators “turned on” one at a time, assumed to be leading order (tree-level) contribution of new lepton number violating physics.
- We can use the effective operator to estimate the coefficient of all other lepton-number violating lower-dimensional effective operators (loop effects, computed with a hard cutoff).

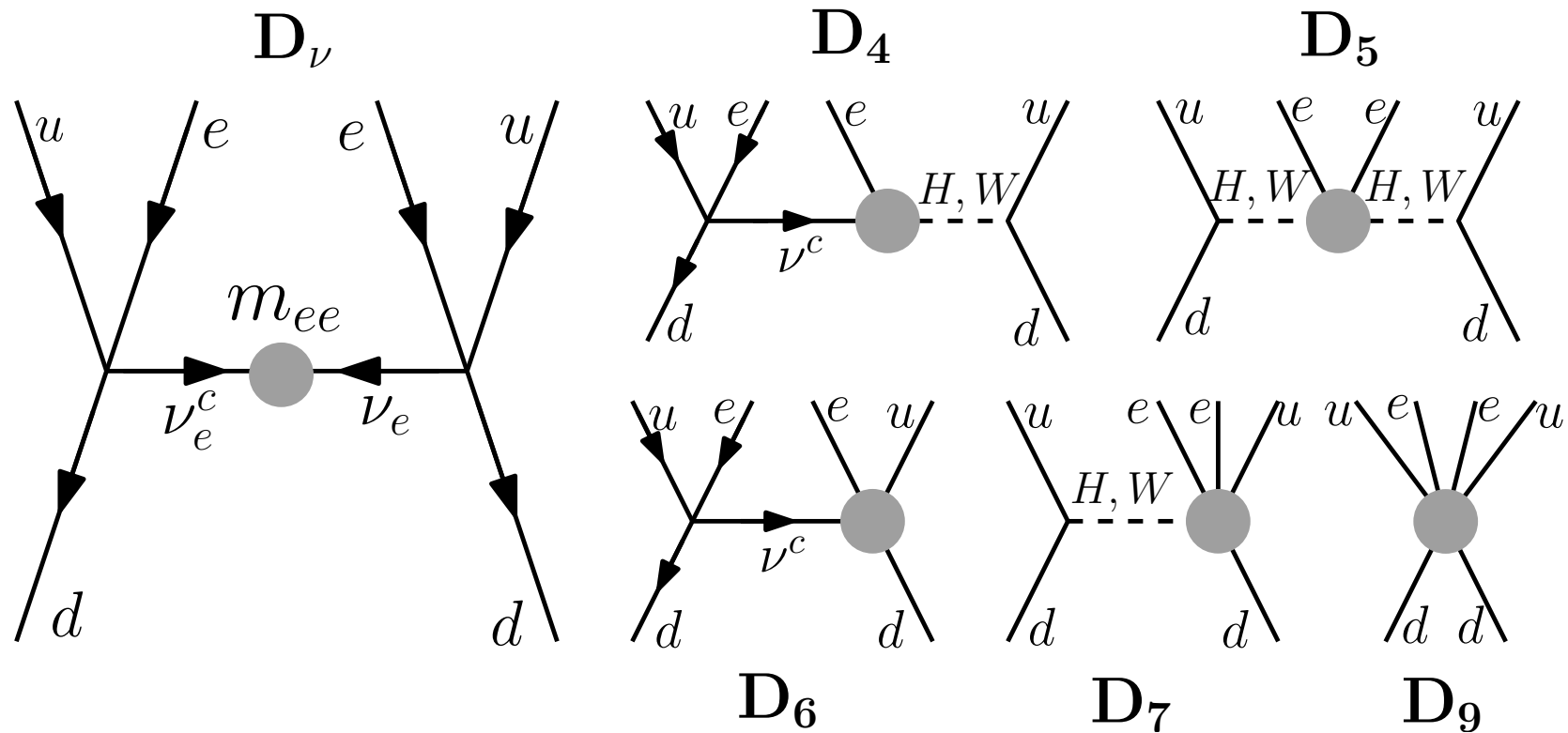
All results presented are order of magnitude *estimates*, not precise quantitative results.



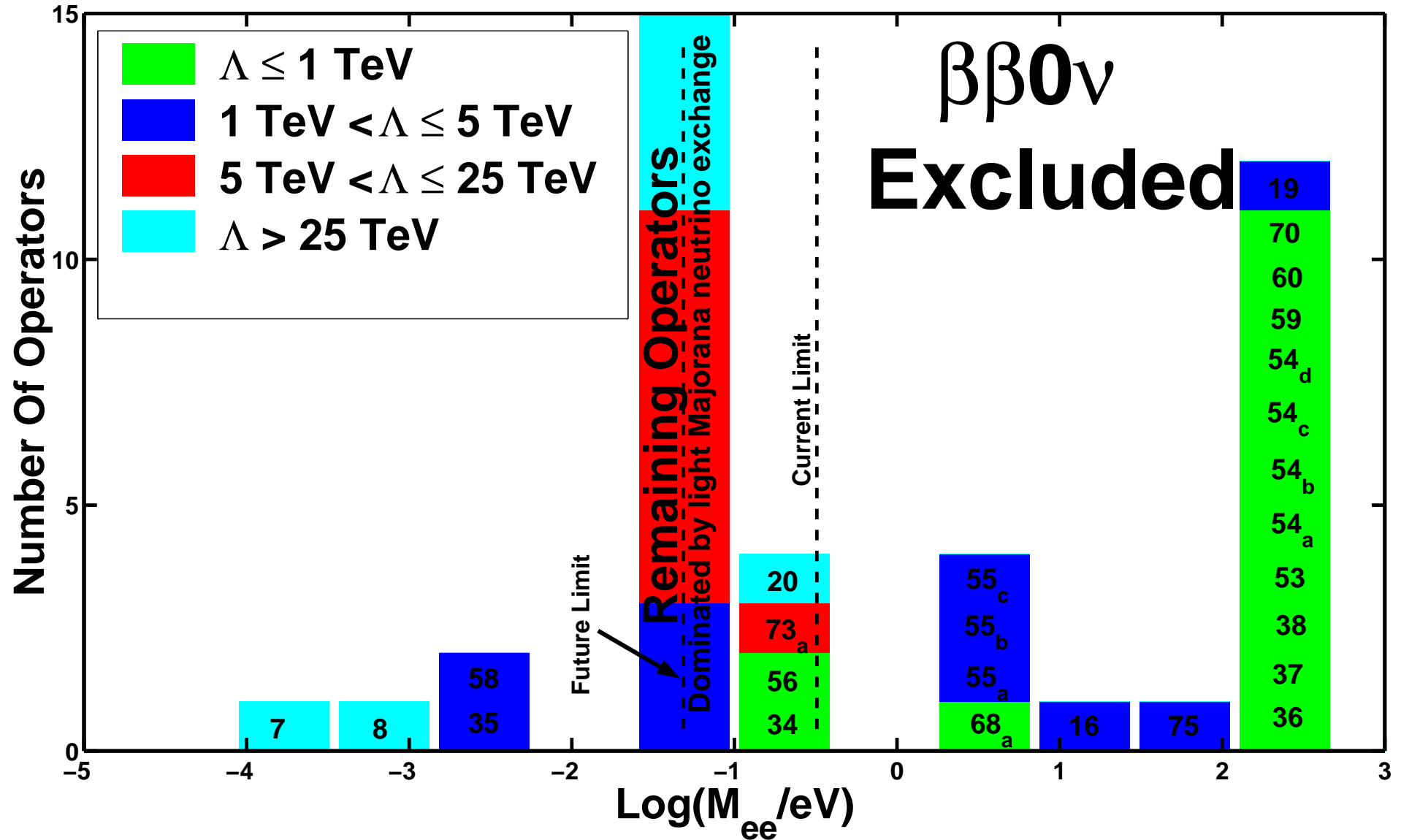




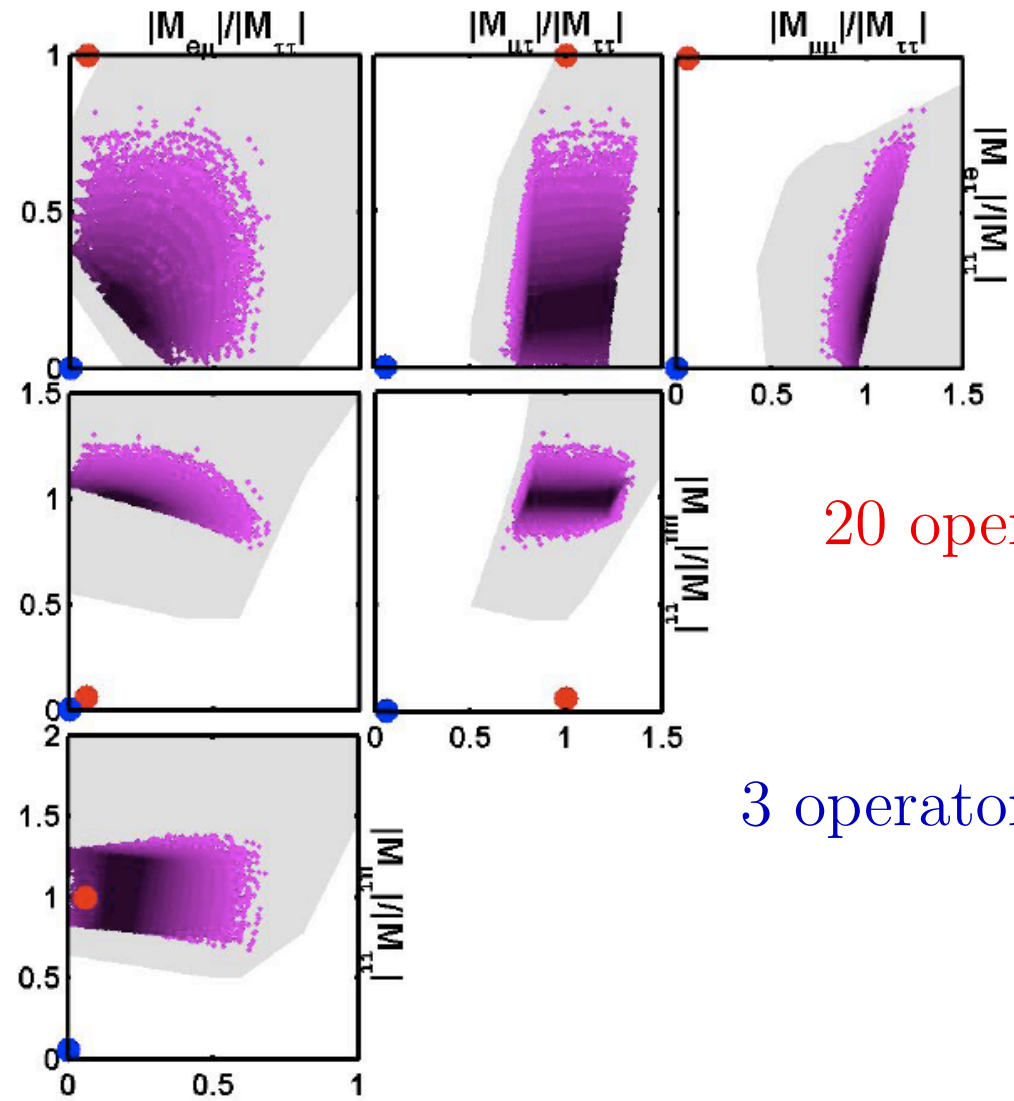
Other Experimental Consequences: LNV Observables



Neutrinoless Double-beta Decay ($0\nu\beta\beta$)



Implied neutrino mass textures (numerical results)



- Require $m_{ee} < 10^{-4}$ eV
- all phases vary freely
- 95% confidence limits
- θ_{13} varies from $0^\circ - 14^\circ$

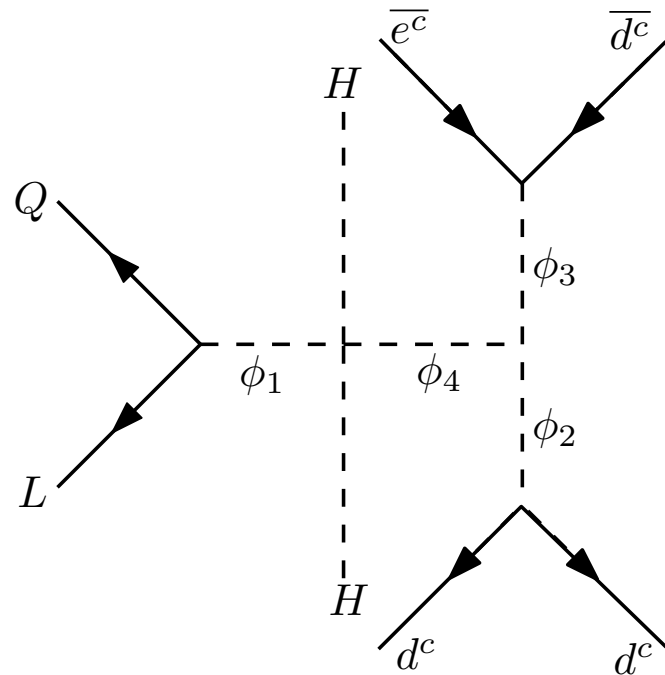
20 operators: $M \propto \begin{pmatrix} \lambda_e & \lambda_\mu & \lambda_\tau \\ \lambda_\mu & \lambda_\mu & \lambda_\tau \\ \lambda_\tau & \lambda_\tau & \lambda_\tau \end{pmatrix}$

3 operators: $M \propto \begin{pmatrix} \lambda_e \lambda_e & \lambda_e \lambda_\mu & \lambda_e \lambda_\tau \\ \lambda_e \lambda_\mu & \lambda_\mu \lambda_\mu & \lambda_\mu \lambda_\tau \\ \lambda_e \lambda_\tau & \lambda_\mu \lambda_\tau & \lambda_\tau \lambda_\tau \end{pmatrix}$

Rest are “anarchical”



[arXiv:0708.1344 [hep-ph]]



Order-One Coupled, Weak Scale Physics

Can Also Explain Naturally Small

Majorana Neutrino Masses:

Multi-loop neutrino masses from lepton number violating new physics.

$$-\mathcal{L}_{\nu\text{SM}} \supset \sum_{i=1}^4 M_i \phi_i \bar{\phi}_i + iy_1 QL\phi_1 + y_2 d^c d^c \phi_2 + y_3 e^c d^c \phi_3 + \lambda_{14} \bar{\phi}_1 \phi_4 HH + \lambda_{234} M \phi_2 \bar{\phi}_3 \phi_4 + h.c.$$

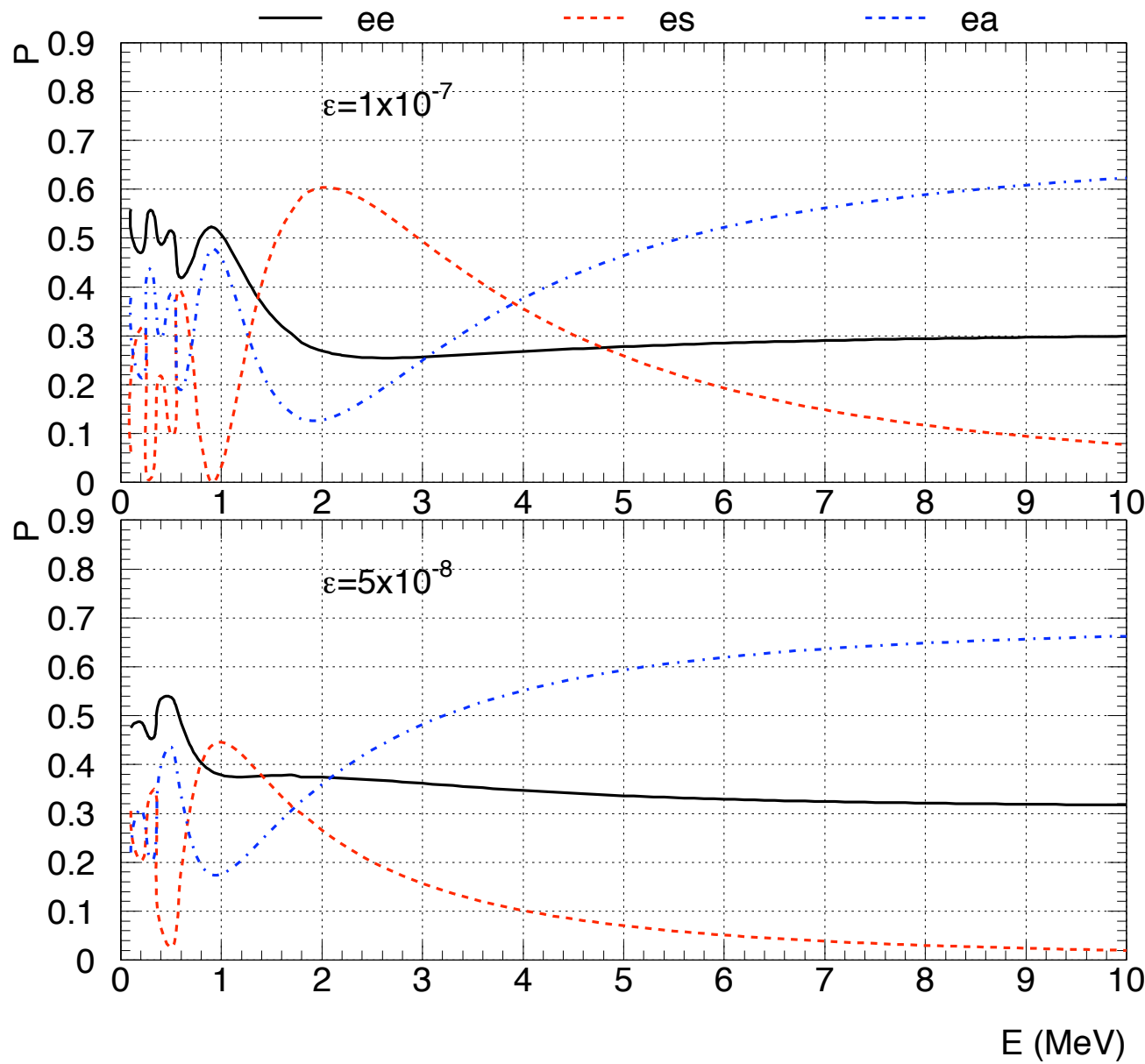
$$m_\nu \propto (y_1 y_2 y_3 \lambda_{234}) \lambda_{14} / (16\pi)^4 \rightarrow \text{neutrino masses at 4 loops, requires } M_i \sim 100 \text{ GeV!}$$

WARNING: For illustrative purposes only. Details still to be worked out. Scenario most likely ruled out by charged-lepton flavor-violation, LEP, Tevatron, and HERA.

Going All the Way: What Happens When $M \ll \mu$?

In this case, the six Weyl fermions pair up into three quasi-degenerate states (“quasi-Dirac fermions”).

These states are fifty–fifty active–sterile mixtures. In the limit $M \rightarrow 0$, we end up with Dirac neutrinos, which are clearly allowed by all the data.

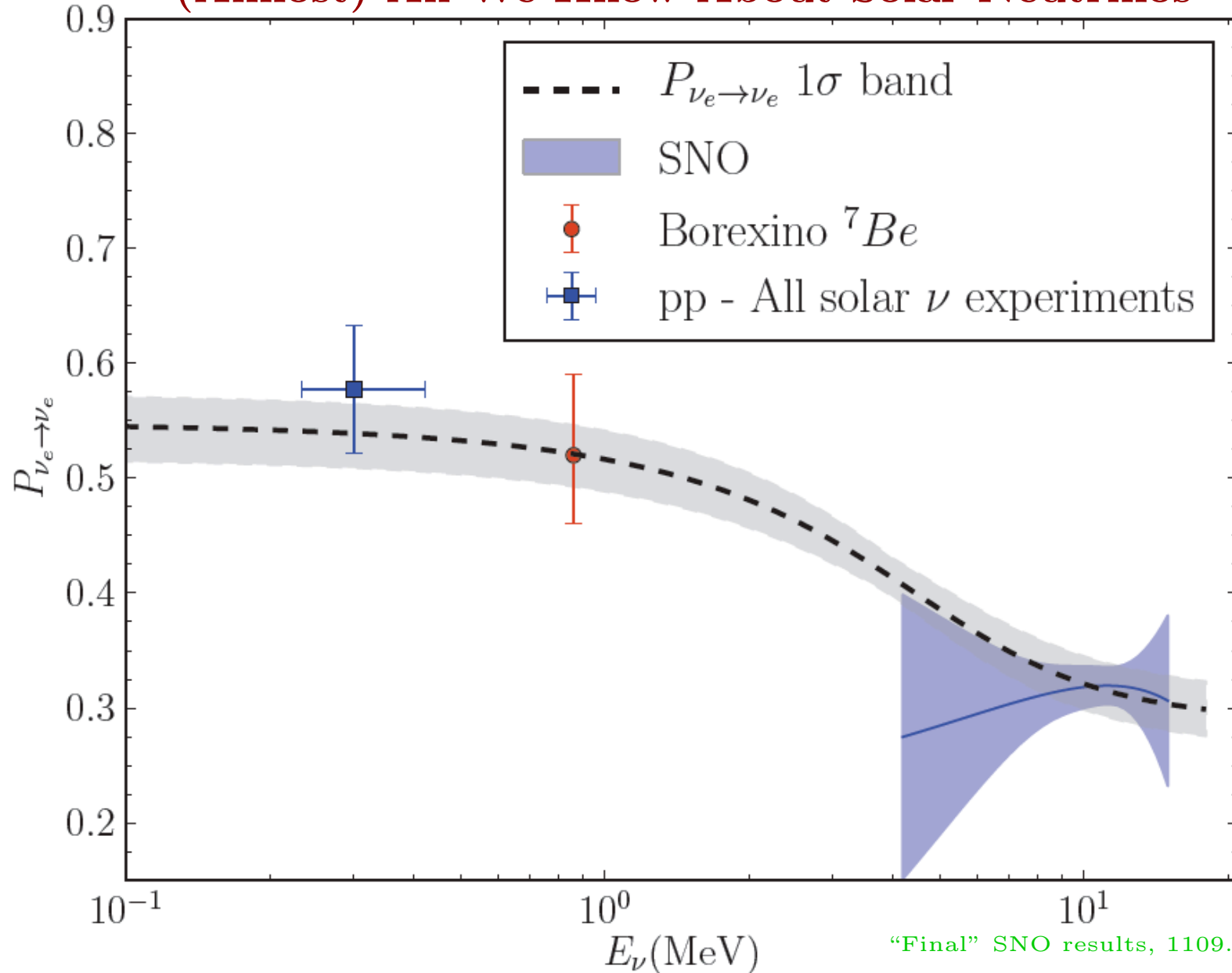


Quasi-Sterile Neutrinos

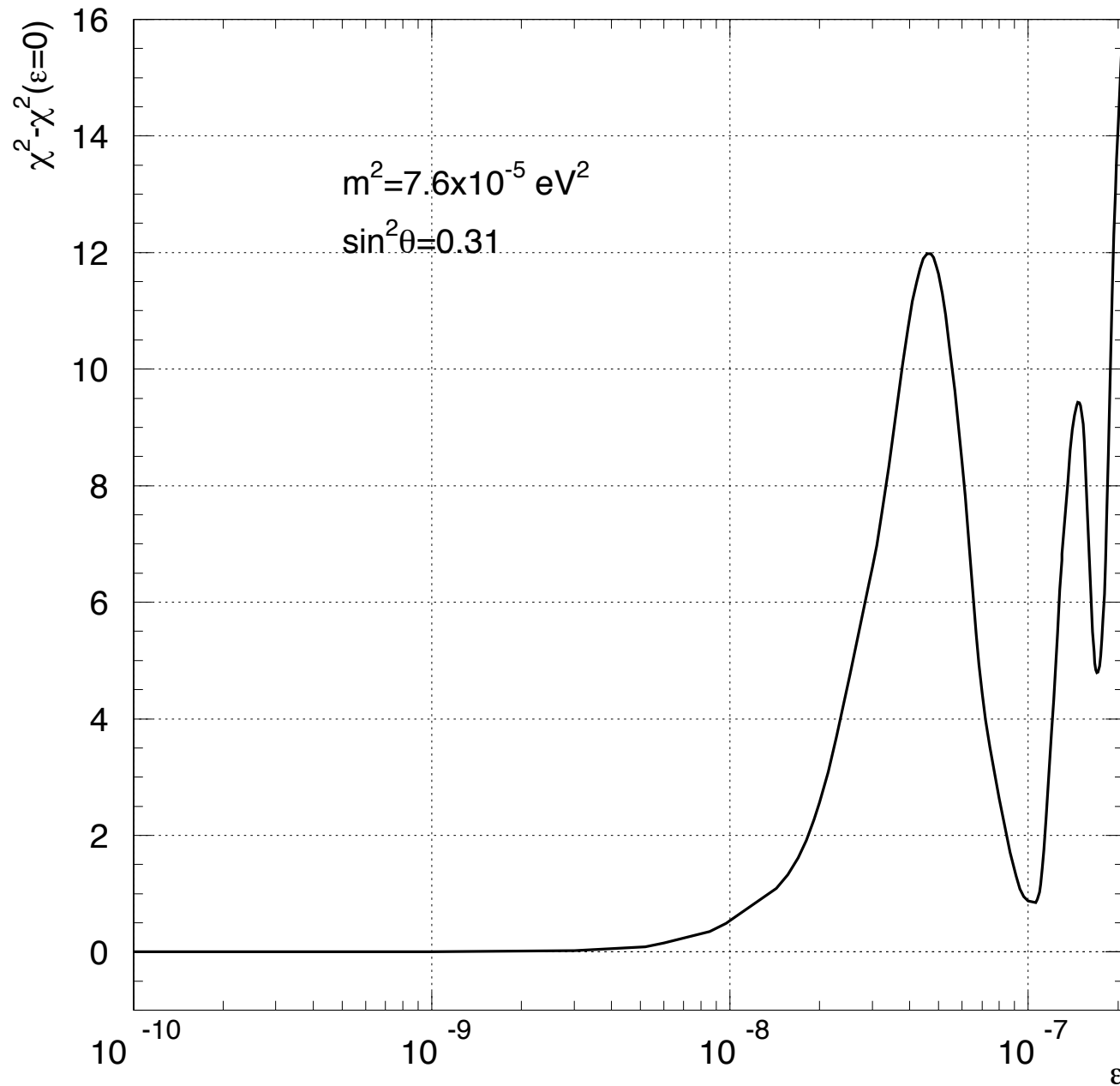
- tiny new $\Delta m^2 = \epsilon \Delta m_{12}^2$,
- maximal mixing!
- Effects in Solar ν_s

[AdG, Huang, Jenkins, arXiv:0906.1611]

(Almost) All We Know About Solar Neutrinos



“Final” SNO results, 1109.0763



Quasi-Sterile Neutrinos

- tiny new $\Delta m^2 = \epsilon \Delta m_{12}^2$,
- maximal mixing!
- Effects in Solar ν_s

[AdG, Huang, Jenkins, arXiv:0906.1611]

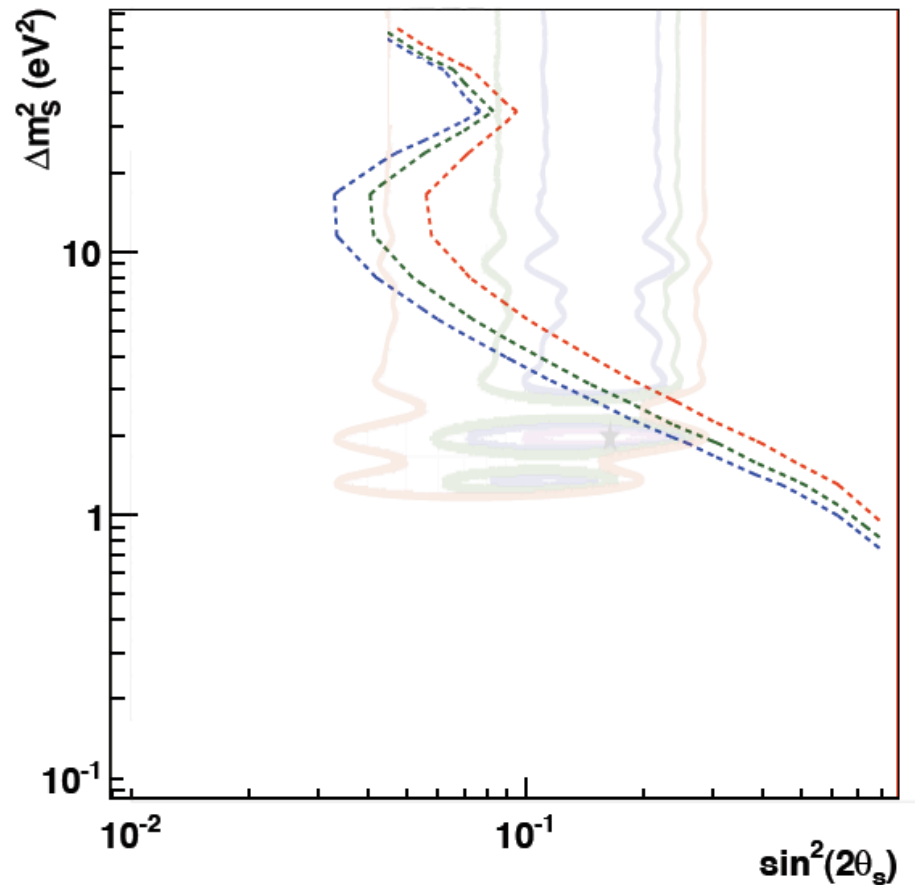
Predictions: **Tritium beta-decay**

Heavy neutrinos participate in tritium β -decay. Their contribution can be parameterized by

$$m_{\beta}^2 = \sum_{i=1}^6 |U_{ei}|^2 m_i^2 \simeq \sum_{i=1}^3 |U_{ei}|^2 m_i^2 + \sum_{i=1}^3 |U_{ei}|^2 m_i M_i,$$

as long as M_i is not too heavy (above tens of eV). For example, in the case of a 3+2 solution to the LSND anomaly, the heaviest sterile state (with mass M_1) contributes the most: $m_{\beta}^2 \simeq 0.7 \text{ eV}^2 \left(\frac{|U_{e1}|^2}{0.7} \right) \left(\frac{m_1}{0.1 \text{ eV}} \right) \left(\frac{M_1}{10 \text{ eV}} \right)$.

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1}) \text{ eV}^2$.



[Barrett, Formaggio, 1105.1326]

FIG. 2: Sensitivity of the KATRIN neutrino mass measurement for a sterile neutrino with relatively large mass splitting (dashed contours). Figure shows exclusion curves of mixing angle $\sin^2(2\theta_s)$ versus mass splitting $|\Delta m_s^2|^2$ for the 90% (blue), 95% (green), and 99% (red) C.L. after three years of data taking. Figure 7 from Ref. [2] show in solid curves in the background.