Higgs sector renormalization of Supersymmetric models

Melina Gómez Bock

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Motivation

The dinamical breaking of the EW symmetry still requires to be proven $\longrightarrow h^0$

For Supersymmetric models more than one Higgs bosons are predicted:

$$\text{MSSM} \rightarrow h^0,\, H^0, A^0,\, H^\pm$$

$$\text{NMSSM} \rightarrow h_1^0,\, h_2^0,\, h_3^0,\, a_1^0,\, a_2^0,\, H^\pm$$

The world major effort to search for these particles is been carried out at present times at the LHC combined with results on the Tevatron and LEP. In order to achieve the presicion level of the experiments it is necessary to implement the quantum effects on the masses of the Higgs bosons and the renormalization procedure to obtain finite and physically meaningful results.

We present the explicit procedure to obtain *renormalized 1-loop corrected Higgs masses* for two supersymmetric models: MSSM and NMSSM.

Outline

- **Standard Model Higgs mechanism**
  - $\rightsquigarrow$ and UV divergent higher corrections to the Higgs mass
- **Supersymmetry as the extension of SM**
  - $\rightsquigarrow$ MSSM
  - $\rightsquigarrow$ NMSSM $\longrightarrow$ solving the MSSM μ- **problem**
- **Radiative corrections and Renormalization schemes**
  - $\rightsquigarrow$ Minimal substraction (MS) renormalization $\rightarrow$ **RGE**
  - $\rightsquigarrow$ On-shell (OS) renormalization $\rightarrow$ **Feynman Diagramatic Approach**
- **FDA Renormalized supersymmetric Higgs masses**

Standard Model Higgs mechanism

SM Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Higgs Lagrangian

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \tag{1}$$

with the covariant derivative

$$D_\mu = \partial_\mu - i g_2 I_a W_\mu^a + i \frac{g_1}{2} B_\mu \tag{2}$$

Once a vev is chosen $\langle 0|\Phi|0\rangle = v \neq 0$ the EW symmetry is broken:

✯ Gauge bosons acquire mass from the kinetic term.

✯ Fermions acquire mass through the Yukawa couplings.

✯ The Higgs mass is obtained from the Higgs potential after EW SSB.

Radiative corrections to SM Higgs mass

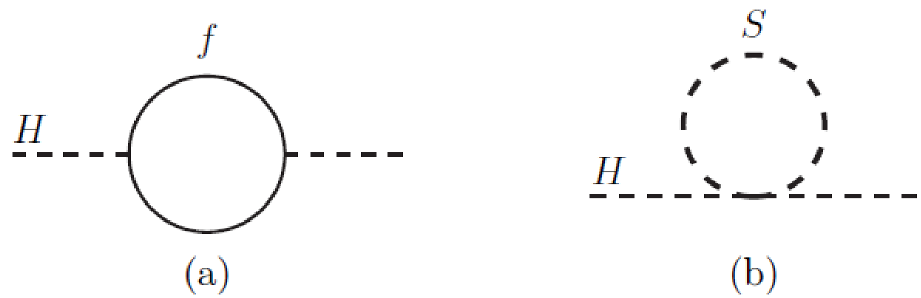


Figure 1: *(a) fermionic and (b) any scalar correction to the Higgs mass*

✧Fermionic contribution to the Higgs mass

$$\Delta m_h^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \cdots \quad (3)$$

✧ Contribution to the Higgs mass from a scalar

$$\Delta m_h^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \log\frac{\Lambda_{UV}}{m_S} + \cdots\right] \quad (4)$$

Supersymmetry as the extension of SM

If each of quarks and leptons of the SM is accompanied by two complex scalars with $\lambda_S = |\lambda_f|^2$ then the Λ_{UV}^2 in (3) and (4) will neatly cancel

[S.P. Martin 08]

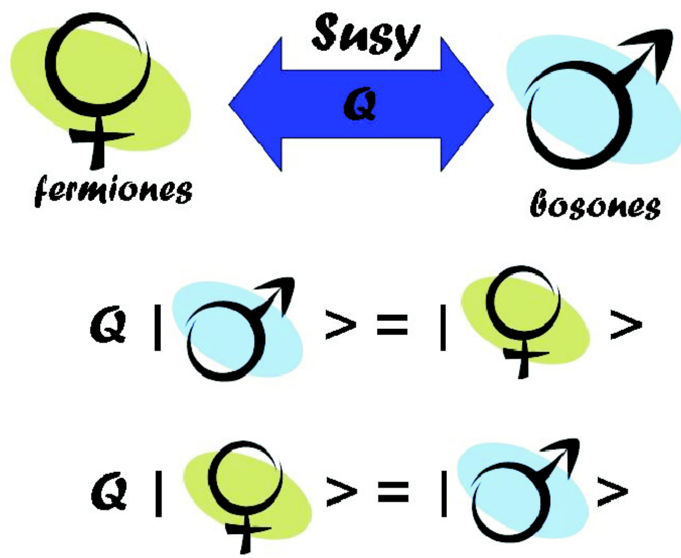


Figure 2: *Symmetry which relates fermions with bosons*

NMSSM solves MSSM μ-problem

Scale μ, the Higgs-higgsino mass parameter in the superpotencial: $\mu H_u H_d$ mixing the two higgs doublets. "Naturaly" expected values of μ

$\mu = 0$ $\Rightarrow$ • By RGE no mixing at any scale, the minimum of Higgs potential at $\langle H_1 \rangle = 0$ causing massless d-quarks and l^- after the SB.

• Would generate a PQ-symmetry and an unacceptable massless axion[1].

$\mu \simeq M_{Pl}$ or $\mu \simeq M_{GUT}$ $\Rightarrow$ • Huge contribution to Higgs scalars.

• in order to generate a desired instability for the Higgs potential at the origin, it should be provided that $|\mu| \lesssim M_{SUSY}$.

$\Downarrow$

μ should be at EW or at M_{SUSY} scale.

Why this scale?

The introduction of a new singlet Higgs field will provide a solution to the $\mu - problem$.

$\rightarrow \langle S \rangle \neq 0$, *i.e.* $\mu_{eff} = \lambda \langle S \rangle$ which will be linked dinamically to the EW-scale.

Supersymmetryc Potential and Soft-terms of MSSM and NMSSM

MSSM Superpotential

[for instance S.P. Martin 07]

$$W_{MSSM} = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k \tag{5}$$

MSSM Higgs Superpotential

NMSSM Superpotential of the Higgs sector invariant under a discrete $\mathbb{Z}_3$, which will be motivated to solve μ-**problem**, will be obtained by **if** $\mu = \mu' = \xi_F = 0$, then is given as

[Miller, Nevzorov, Zerwas 03], [Maniatis 09], [Ellwanger, Hugonie, Teixeira 09]

$$W^{Z3-inv}_{Higgs} = \lambda \hat{S}\hat{H}_u \cdot \hat{H}_d + \frac{\kappa}{3}\hat{S}^3 \tag{6}$$

Soft SUSY Lagrangian

$$\mathcal{L}^{Z3-inv}_{soft} = m^2_S|S|^2 + \lambda A_\lambda S H_u H_d + \frac{1}{3}\kappa A_\kappa S^3 + \mathcal{L}^{MSSM}_{soft}. \tag{7}$$

Supersymmetric models differences

	MSSM	**NMSSM**	**NMSSM-Z3inv**
Higgs mix couplings	μ	μ, λ, ξ_F, μ', κ	λ , κ
Soft SUSY Higgs mass terms	m_1^2, m_2^2	m_1^2, m_2^2, m_S^2	m_1^2, m_2^2, m_S^2
Soft SUSY couplings	μB	A_λ, A_κ, m_3^2, $m_S'^2$, ξ_S	A_λ, A_κ
CP-even scalars	h^0, H^0	h_1^0, h_2^0, h_3^0	h_1^0, h_2^0, h_3^0
CP-odd scalars	A^0	a_1^0, a_2^0	a_1^0, a_2^0
neutralinos	$\tilde{\chi}^0_{1,2,3,4}$	$\tilde{\chi}^0_{1,2,3,4,5}$	$\tilde{\chi}^0_{1,2,3,4,5}$
free parameters at tree level	$\tan\beta$, m_A		$\tan\beta$, $\langle S\rangle$, λ , κ, A_λ, A_κ

Table 1: *Additional couplings and particle spectrum for NMSSM and NMSSM-Z3inv, compared to the MSSM. (Other sparticle masses and couplings are considered as in the MSSM).*

Higgs sector of the MSSM

Two complex SU(2) Higgs doublets:

$$\Phi_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

physical Higgs particle spectrum 5:

ϕ_i , $CP = 1 \rightarrow$ two scalar fields: h^0, H^0,

χ_i , $CP = -1 \rightarrow$ one pseudoscalar fields: A^0.

and

$\phi^{\pm}$, $\rightarrow$ two charged fields: $H^{\pm}$

SSB: Assuming the scalar fields to develop nonzero vacuum expectation values that break $SU(2)_L$

$$< \Phi_1 >= v_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad < \Phi_2 >= v_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

Defining: $\tan\beta = \frac{v_2}{v_1}$ and $v = (v_1^2 + v_2^2)^{1/2} \approx 173\ GeV.$

Higgs sector of the NMSSM

Two complex SU(2) Higgs doublets and one scalar singlet:

$$\Phi_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}, \quad S = v_S + \frac{1}{\sqrt{2}}(\phi_S - i\chi_S)$$

physical Higgs particle spectrum 7:

ϕ_i , $CP = 1$ $\rightarrow$ three scalar fields: h_1^0, h_2^0, h_3^0,

χ_i , $CP = -1$ $\rightarrow$ two pseudoscalar fields: a_1^0, a_2^0.

and

$\phi^\pm$, $\rightarrow$ two charged fields: $H^\pm$

SSB: Assuming the scalar fields to develop nonzero vacuum expectation values that break $SU(2)_L$

$$< \Phi_1 >= v_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad < \Phi_2 >= v_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad < S >= v_S \qquad (9)$$

Defining: $\tan\beta = \frac{v_2}{v_1}$ and the bilinear term is genereted by SSB as $\mu = \lambda v_S$

Tree level Higgs mass MSSM

At leading order we have the CP-even neutral Higgs masses releted using m_{A^0} as free parameter:

$$\begin{aligned} m^2_{h,H} &= \frac{1}{2}(m^2_A + m^2_Z) \\ &\mp \frac{1}{2}\sqrt{(m^2_A + m^2_Z)^2 - 4m^2_A m^2_Z \cos^2 2\beta} \\ m^{\pm}_H &= m^2_A + \cos^2\theta_w m^2_Z \end{aligned} \tag{10}$$

☆ The relations within MSSM parameters imposse, at tree level, a strong hierarchical structure on mass spectrum:

$m_h < m_Z$, $m_A < m_H$ and $m_W < m_{H^\pm}$, $\rightarrow$ which is broken by radiative corrections.

1-loop level renormalized Higgs mass

The elements of the mass matrix M_h^2 is constructed explicitly from self-energies contributions diagrams: Σ_h And the renormalized self-energies are given by

$$\hat{\Sigma}_{h_{ij}} = \Sigma_{h_{ij}} + \delta\mathcal{Z}_{h_{ij}}(p^2 - m_h^2) - \delta m^2_{h_{ij}}$$

$$\Sigma_{h_{ij}} \quad \rightarrow \quad \textbf{self-energy diagrams}$$

$$\delta\mathcal{Z}_{h_{ij}}(p^2 - m_h^2) \quad \rightarrow \quad \textbf{filed renormalization kinetic term}$$

$$\delta m^2_{h_{ij}} \quad \rightarrow \quad \textbf{Higgs mass counterterms}$$

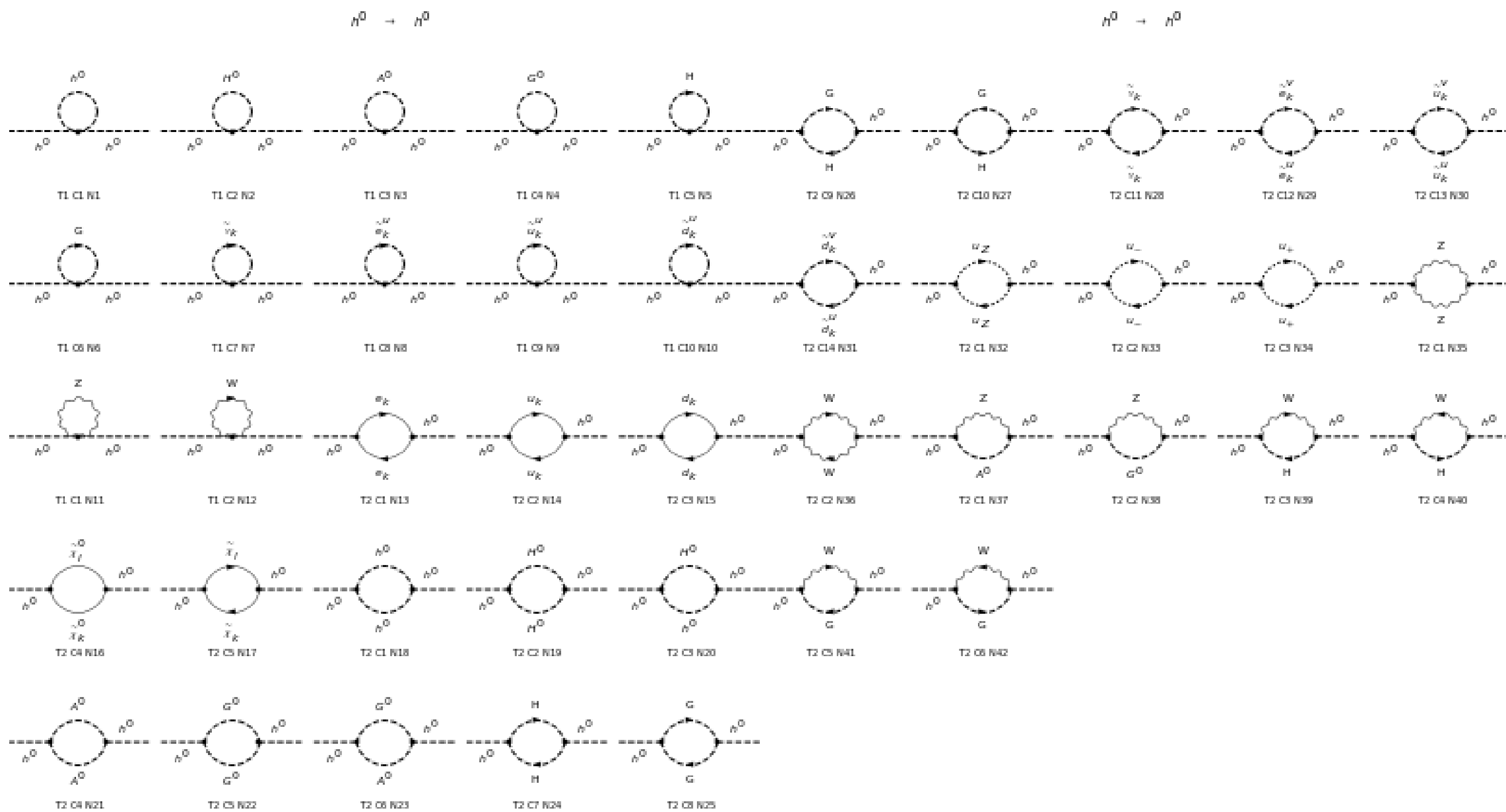
The Higgs mass is defined as the poles of the propagator, and the propagator is given as

$$\hat{\Delta}^{-1}_{h_{ij}}(s) = s - \mathcal{M}^2_{h_{ij}}(s)$$

with the mass matrix elements

$$\mathcal{M}^2_{h_{ij}}(s) = m^2_{h_{ii}} - \hat{\Sigma}_{h_{ij}}(s)$$

MSSM radiative corrections to h^0 mass



Renormalization and radiative corrections

The inclusion of higher order corrections in the calculation of Green functions and S-Matrix elements beyond the tree-level approximation has two main complications.

- Tree-level relations between the Lagrangian parameters and observables are no longer valid. Thus, the Lagrangian parameters depend on a certain set of definitions.

- The sum over all possible intermediate states prescribed by perturbation theory diverges

$$\sum_{i,j} \langle i|H|j\rangle_{NLO} = infinity;$$

  the calculation of higher order Feynman diagrams represented by closed loops leads to the integration over indefinite momenta leading to UV-divergencies for high momenta.

Introduction of an energy cut-off Λ on the 4-momentum integration of the amplitud. The integral becomes $\Lambda - dependent$ and UV-divergent for $\Lambda \rightarrow infinity$, thus breaking Lorentz invariance.

[Eberl 2010]

Renormalization

General Procedure

❶ Expressing phyisical predictions in terms of only physically measurable parameters.

❷ Dimensional Regularization: UV divergences have to be separated in a consistent way, the Lorentz invariant dimensional regularisation is based on analytic continuation of all four-vectors, momenta and vector fields, to $D = 4 - \epsilon$ dimensions.

[t'Hooft, Veltman 72]

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For SUSY the regularization is called Dimensional Reduction (DR or $\overline{\mathrm{DR}}$) is a regularization that preserve supersymmetry: all momenta live in $D = 4 - 2\epsilon$ dimensions, but the vector fields keep all 4 components. Physically, such a reduced $4D$ vector field comprises one species of a $D - dimensional$ vector plus 2ϵ species of the scalar of the superfield.

❸ Introducing renormalization counter terms (CTs) for the Lagrangian parameters and fields. These CTs cancel all UV divergences and we get UV finite amplitudes.

❹ Field renormalization ensures that we end up with finite Green functions.

Renormalization schemes

A renormalization scheme is a choice of definite procedures for dealing with the parameters of the theory together with the infinitness of the loop amplitudes in terms of measureable physical quantities.

[Hollik 93, Dabelstein 94]

- $\overline{\mathrm{MS}}$, $\overline{\mathrm{DR}}$ $\rightarrow$ **Renormalization Group Equations** The running couplings $g_i(E)$ depend on the energy scale E and also on the specific rules we use to fix the finite parts of counterterms $\delta g_i(E)$. Is a scale dependent scheme and takes only into account the $\log$ finite terms.

$$\frac{2}{\epsilon} - \gamma_E + \log(4\pi) + \log\mu^2 \rightarrow \log\mu^2_{\overline{\mathrm{MS}}}$$

- **OS** $\rightarrow$ **Feynman Diagrammatic Approach**. A complete one-loop calculation to masses and couplings accommodates all SUSY particles and mass parameters (or soft breaking parameters, respectively) in the radiatively corrected version of masses (2-point functions) and mixing angles. No scale dependance. All finite parts of counterterms are taken into account.

Renormalized supersymmetric Higgs masses (NMSSM)

$$\begin{aligned}
V_H &= m_1^2 H_{1i}^\dagger H_{1i} + m_2^2 H_{2i}^\dagger H_{2i} + m_S^2 |S|^2 \\
&\quad + |\lambda \varepsilon_{ab}(H_{2a}H_{1b}) + \kappa S^2|^2 + |\lambda|^2 |S|^2 (H_{2i}^\dagger H_{2i} + H_{1i}^\dagger H_{1i}) \\
&\quad + \left(\lambda A_\lambda \varepsilon_{ab}(H_{2a}H_{1b})S + \frac{1}{3}\kappa A_\kappa S^3 + \text{c.c.}\right) \\
&\quad + \frac{1}{8}(g_1^2 + g_2^2)(H_{1i}^\dagger H_{1i} - H_{2i}^\dagger H_{2i})^2 + \frac{1}{2} g_2^2 |H_{1i}^* H_{2i}|^2 .
\end{aligned} \tag{11}$$

The supersymmetric Higgs potential can be expressed as a decomposition in powers of fields

$$\begin{aligned}
V_H &= \cdots - T_{\phi_1}\phi_1 - T_{\phi_2}\phi_2 - T_{\phi_S}\phi_S \\
&\quad + \tfrac{1}{2}\left(\phi_1, \phi_2, \phi_S\right) \mathbf{M}_{\phi\phi\phi} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_S \end{pmatrix} + \tfrac{1}{2}\left(\chi_1, \chi_2, \chi_S\right) \mathbf{M}_{\chi\chi\chi} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_S \end{pmatrix} + \\
&\quad + \left(\phi_1^-, \phi_2^-\right) \mathbf{M}_{\phi^\pm\phi^\pm} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} + \cdots ,
\end{aligned} \tag{12}$$

[Benbrik,Gomez-Bock,Heinemeyer,Stal and Weiglein in preparation]

Set of physical parameters

In order to perform the renormalization procedure in a transparent way, we express the parameters in V_H in terms of physical parameters.

MSSM

In total, V_H^{MSSM} contains 8 independent real parameters:
v_1, v_2, v_s, g_1^2, g_2^2, m_1^2, m_2^2, m_{12}^2 which can be replaced by the parameters
M_Z, M_W, e, $m_{H^\pm}$, $\tan\beta$, T_h, T_H, μ .

NMSSM

In total, V_H^{NMSSM} contains 12 independent real parameters:
v_1, v_2, v_s, g_1^2, g_2^2, m_1^2, m_2^2, m_S^2, λ (with $\mu_{\text{eff}} = \lambda v_s$), κ, A_λ, A_κ
which can be replaced by the parameters
M_Z, M_W, e, $m_{H^\pm}$, $\tan\beta$, μ_{eff}, T_h, T_H, T_S as well as λ, κ, A_κ.

Thereby, the coupling constants g_1 and $g_2 \Rightarrow e$ and $\theta_W \Rightarrow$
$c_\text{w} \equiv \cos\theta_\text{w} = M_W/M_Z, s_\text{w} = \sqrt{1 - c_\text{w}^2}$.

Renormalization of physical parameters

In order to derive the counterterms entering the one-loop corrections to the Higgs-boson masses and effective couplings we renormalize the parameters appearing in the linear and bilinear terms of the Higgs potential,

$$
\begin{aligned}
M_Z^2 &\to M_Z^2 + \delta M_Z^2, & T_{h_1} &\to T_{h_1} + \delta T_{h_1}, \\
M_W^2 &\to M_W^2 + \delta M_W^2, & T_{h_2} &\to T_{h_2} + \delta T_{h_2}, \\
\mathbf{M}_{\phi\phi\phi} &\to \mathbf{M}_{\phi\phi\phi} + \delta\mathbf{M}_{\phi\phi\phi}, & T_{h_3} &\to T_{h_3} + \delta T_{h_3}, \\
\mathbf{M}_{\chi\chi\chi} &\to \mathbf{M}_{\chi\chi\chi} + \delta\mathbf{M}_{\chi\chi\chi}, & \tan\beta &\to \tan\beta + \delta\tan\beta\,, \\
\mathbf{M}_{\phi^\pm\phi^\pm} &\to \mathbf{M}_{\phi^\pm\phi^\pm} + \delta\mathbf{M}_{\phi^\pm\phi^\pm}, & \mu_{\rm eff} &\to \mu_{\rm eff} + \delta\mu_{\rm eff}, \\
\lambda &\to \lambda + \delta\lambda & \kappa &\to \kappa + \delta\kappa \\
& & A_\kappa &\to A_\kappa + \delta A_\kappa\,.
\end{aligned}
\tag{13}
$$

Field Renormalization

For the field renormalization, which is necessary in order to obtain finite Higgs self-energies for arbitrary values of the external momentum, we choose to give each Higgs doublet one renormalization constant,

$$\mathcal{H}_1 \to (1 + \tfrac{1}{2}\delta Z_{\mathcal{H}_1})\mathcal{H}_1, \quad \mathcal{H}_2 \to (1 + \tfrac{1}{2}\delta Z_{\mathcal{H}_2})\mathcal{H}_2, \quad S \to (1 + \tfrac{1}{2}\delta Z_S)S \ . \tag{14}$$

In the mass eigenstate basis, the field renormalization matrices read

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{h_1h_1} & \frac{1}{2}\delta Z_{h_1h_2} & \frac{1}{2}\delta Z_{h_1h_3} \\ \frac{1}{2}\delta Z_{h_2h_1} & 1 + \frac{1}{2}\delta Z_{h_2h_2} & \frac{1}{2}\delta Z_{h_2h_3} \\ \frac{1}{2}\delta Z_{h_3h_1} & \frac{1}{2}\delta Z_{h_3h_2} & 1 + \frac{1}{2}\delta Z_{h_3h_3} \end{pmatrix} \cdot \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \ , \tag{15}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ G \end{pmatrix} \to \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{a_1a_1} & \frac{1}{2}\delta Z_{a_1a_2} & \frac{1}{2}\delta Z_{a_1G} \\ \frac{1}{2}\delta Z_{a_2a_1} & 1 + \frac{1}{2}\delta Z_{a_2a_2} & \frac{1}{2}\delta Z_{a_2G} \\ \frac{1}{2}\delta Z_{Ga_1} & \frac{1}{2}\delta Z_{Ga_2} & 1 + \frac{1}{2}\delta Z_{GG} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ G \end{pmatrix} \ , \tag{16}$$

$\overline{\rm DR}$

☆ The $\overline{\rm DR}$ renormalization of the parameter $\tan\beta$, which is manifestly process-independent, is convenient since there is no obvious relation of this parameter to a specific physical observable that would favor a particular on-shell definition.

☆ The $\overline{\rm DR}$ scheme for the field renormalization constants is convenient in order to avoid the possible occurrence of unphysical threshold effects. Higgs bosons appearing as external particles in a physical process of course have to obey proper on-shell conditions.

$$\delta Z_{\mathcal{H}_1} = \delta Z_{\mathcal{H}_1}^{\overline{\rm DR}} = -\left[\mathrm{Re}\,\Sigma'_{\phi_1\phi_1}\right]^{\rm div}, \tag{17}$$

$$\delta Z_{\mathcal{H}_2} = \delta Z_{\mathcal{H}_2}^{\overline{\rm DR}} = -\left[\mathrm{Re}\,\Sigma'_{\phi_2\phi_2}\right]^{\rm div}, \tag{18}$$

$$\delta Z_S = \delta Z_S^{\overline{\rm DR}} = -\left[\mathrm{Re}\,\Sigma'_{\phi_S\phi_S}\right]^{\rm div}, \tag{19}$$

$$\delta\tan\beta = \frac{1}{2}\left(\delta Z_{\mathcal{H}_2} - \delta Z_{\mathcal{H}_1}\right) = \delta\tan\beta^{\,\overline{\rm DR}} \tag{20}$$

OS renormalization conditions

We determine the one-loop counterterms by requiring the following renormalization conditions. The SM gauge bosons and the charged Higgs boson are renormalized on-shell,

$$\mathrm{Re}\,\hat{\Sigma}_{ZZ}(M_Z^2) = 0, \quad \mathrm{Re}\,\hat{\Sigma}_{WW}(M_W^2) = 0, \quad \mathrm{Re}\,\hat{\Sigma}_{H^+H^-}(M_{H^\pm}^2) = 0 \ , \tag{21}$$

where the gauge-boson self-energies are to be understood as the transverse parts of the full self-energies. For the mass counterterms, Eq. (21) yields

$$\delta M_Z^2 = \mathrm{Re}\,\Sigma_{ZZ}(M_Z^2), \quad \delta M_W^2 = \mathrm{Re}\,\Sigma_{WW}(M_W^2), \quad \delta m_{H^\pm}^2 = \mathrm{Re}\,\Sigma_{H^+H^-}(M_{H^\pm}^2) \ . \tag{22}$$

The renormalization of s_w and c_w can be expressed through δM_Z^2 and δM_W^2,

$$\delta s_\mathrm{w} = -\frac{c_\mathrm{w}}{s_\mathrm{w}}\delta c_\mathrm{w}, \quad \delta c_\mathrm{w} = \frac{c_\mathrm{w}}{2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) . \tag{23}$$

Explicit form of the mass counterterms δm_h

As the tadpole coefficients are required to vanish, their counterterms follow from

$$T_{\{h,H,S\}(1)} + \delta T_{\{h,H,S\}} = 0 \ , \tag{24}$$

where $T_{\{h,H,S\}(1)}$ denote the one-loop contributions to the respective Higgs tadpole graphs:

$$\delta T_{h_1} = -T_{h_{1(1)}}, \quad \delta T_{h_2} = -T_{h_{2(1)}}, \quad \delta T_{h_3} = -T_{h_{3(1)}}. \tag{25}$$

Inserting the ronormalization introduced in Eq. (13) and applying the zeroth order relations for tadpoles $T_{\{h,H,A\}} = 0$.

Explicit form of the NMSSM Higgs mass counter terms δm_h

The coefficients of the first-order expressions yields for the $\mathcal{CP}$-even part of the Higgs sector

$$
\begin{aligned}
\delta m^2_{\phi_1} \quad = \quad & \left(4\lambda\delta\lambda M^2_W s^2_{\mathrm{w}} + \delta m^2_{H\pm}\right)\sin^2\beta + \left(\cos^2\beta + 2\lambda^2\frac{c^4_{\mathrm{w}}}{e^2}\sin^2\beta\right)\delta M^2_Z - \frac{e\delta T_{\phi_1}}{2s_{\mathrm{w}}M_W}\sec\beta \\
& - \left(2\lambda^2\frac{c_{2\mathrm{w}}}{e^2} + 1\right)\sin^2\beta\delta M^2_W + \left(\hat{m}^2_A - M^2_W - M^2_Z s^2_{\mathrm{w}}\right)\sin 2\beta\cos^2\beta\delta T\beta, \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\delta m^2_{\phi_2} \quad = \quad & \left(4\lambda\delta\lambda M^2_W s^2_{\mathrm{w}} + \delta m^2_{H\pm}\right)\cos^2\beta + \left(\sin^2\beta + 2\lambda^2\frac{c^4_{\mathrm{w}}}{e^2}\cos^2\beta\right)\delta M^2_Z - \frac{e\delta T_{\phi_2}}{2s_{\mathrm{w}}M_W}\csc\beta \\
& - \left(2\lambda^2\frac{c_{2\mathrm{w}}}{e^2} + 1\right)\cos^2\beta\delta M^2_W - \left(\hat{m}^2_A - M^2_W - M^2_Z s^2_{\mathrm{w}}\right)\sin 2\beta\cos^2\beta\delta T\beta, \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
\delta m^2_{\phi_S} \quad = \quad & \frac{(\lambda A_\kappa + 8\kappa\mu_{\mathrm{eff}})}{\lambda^3}(\mu_{\mathrm{eff}}(\lambda\delta\kappa - \kappa\delta\lambda) + \lambda\kappa\delta\mu_{\mathrm{eff}}) - \frac{s^2_{\mathrm{w}}M^2_W}{e^2}\sin 2\beta(\lambda\delta\kappa + \kappa\delta\lambda) \\
& + \frac{s^2_{\mathrm{w}}M^2_W}{e^2}\frac{\lambda}{\mu^3_{\mathrm{eff}}}\sin^2 2\beta\left[\hat{m}^2_A(2\mu_{\mathrm{eff}}\delta\lambda - \lambda\delta\mu_{\mathrm{eff}}) - (m^2_{H\pm} - M^2_W)\mu_{\mathrm{eff}}\delta\lambda\right] + \kappa\frac{\mu_{\mathrm{eff}}}{\lambda}\delta A_\kappa \\
& + \frac{\lambda^2}{2\mu^2_{\mathrm{eff}}}\sin^2 2\beta\left[\frac{(c^4_{\mathrm{w}}\delta M^2_Z - c_{2\mathrm{w}}\delta M^2_W)}{e^2}\left(2\hat{m}^2_A - m^2_{H\pm} + M^2_W\right) + \frac{s^2_{\mathrm{w}}M^2_W}{e^2}(\delta m^2_{H\pm} - \delta M^2_W)\right] \\
& - \frac{(c^4_{\mathrm{w}}\delta M^2_Z - c_{2\mathrm{w}}\delta M^2_W)}{e^2}\sin 2\beta\lambda\kappa - \frac{\lambda}{\mu_{\mathrm{eff}}}\frac{\delta T_{\phi_S}}{\sqrt{2}}, \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\delta m^2_{\phi_1\phi_2} &= \delta m^2_{\phi_2\phi_1} = \sin 2\beta 2\lambda \frac{s_{\rm w}^2 M_W^2}{e^2}\delta\lambda + \cos^2\beta\cos 2\beta\left(\hat{m}_A^2 - 2m^2_{H^\pm} + M_W^2 - s_{\rm w}^2 M_Z^2\right)\delta T\beta \\
&+ \frac{\sin 2\beta}{2}\left[\left(1 - 2\lambda^2\frac{c_{2\rm w}}{e^2}\right)\delta M_W^2 - \left(1 - 2\lambda^2\frac{c_{\rm w}^4}{e^2}\right)\delta M_Z^2 - \delta m^2_{H^\pm}\right] \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\delta m^2_{\phi_1\phi_S} &= \delta m^2_{\phi_S\phi_1} = \frac{(c_{\rm w}^4\delta M_Z^2 - c_{2\rm w}\delta M_W^2)}{e\sqrt{2}M_W s_{\rm w}}\left[\frac{\lambda}{\mu_{\rm eff}}\sin^2\beta\left(2m^2_{H^\pm} - 2M_W^2 - 3\hat{m}_A^2\right) - \mu_{\rm eff}\left(\kappa\sin\beta - 2\lambda\right)\right] \\
&+ \frac{\sqrt{2}s_{\rm w}M_W}{e}\sin\beta\left[\frac{\lambda}{\mu_{\rm eff}}\sin\beta\left(\delta m^2_{H^\pm} - \delta M_W^2\right) - (\mu_{\rm eff}\delta\kappa + \kappa\delta\mu_{\rm eff})\right] \\
&+ \frac{\sqrt{2}s_{\rm w}M_W}{e}\left[\frac{\sin^2\beta}{\mu_{\rm eff}^2}\left(\hat{m}_A^2(\lambda\delta\mu_{\rm eff} - 3\mu_{\rm eff}\delta\lambda) + 2(m^2_{H^\pm} - M_W^2)\mu_{\rm eff}\delta\lambda\right) + 2(\mu_{\rm eff}\delta\lambda + \lambda\delta\mu_{\rm eff})\right] \\
&- \frac{\sqrt{2}s_{\rm w}M_W}{e}\cos^2\beta\left[\mu_{\rm eff}\left(\kappa + 2\lambda\sin\beta\right) + \hat{m}_A^2\frac{\lambda}{\mu_{\rm eff}}\sin\beta(2 - 3\sin^2\beta)\right]\delta T\beta \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
\delta m^2_{\phi_2\phi_S} &= \delta m^2_{\phi_S\phi_2} = \frac{(c_{\rm w}^4\delta M_Z^2 - c_{2\rm w}\delta M_W^2)}{e\sqrt{2}M_W s_{\rm w}}\left[\frac{\lambda}{\mu_{\rm eff}}\cos^2\beta\sin\beta\left(2m^2_{H^\pm} - 2M_W^2 - 3\hat{m}_A^2\right) - \mu_{\rm eff}\left(\kappa - 2\lambda\sin\beta\right)\right] \\
&- \frac{\sqrt{2}s_{\rm w}M_W}{e}\left[\frac{\lambda}{\mu_{\rm eff}}\sin\beta\left(\delta m^2_{H^\pm} - \delta M_W^2\right) + (\mu_{\rm eff}\delta\kappa + \kappa\delta\mu_{\rm eff})\right] \\
&+ \frac{\sqrt{2}s_{\rm w}M_W}{e}\sin\beta\left[\frac{\cos^2\beta}{\mu_{\rm eff}^2}\left(\hat{m}_A^2(\lambda\delta\mu_{\rm eff} - 3\mu_{\rm eff}\delta\lambda) + 2(m^2_{H^\pm} - M_W^2)\mu_{\rm eff}\delta\lambda\right) + 2(\mu_{\rm eff}\delta\lambda + \lambda\delta\mu_{\rm eff})\right] \\
&+ \frac{\sqrt{2}s_{\rm w}M_W}{e}\cos^2\beta\left[\mu_{\rm eff}\left(\kappa\sin\beta + 2\lambda\right) - \hat{m}_A^2\frac{\lambda}{\mu_{\rm eff}}(\cos 2\beta - \sin^2\beta)\right]\delta T\beta
\end{aligned}
$$

more parameters on the NMSSM

The remaining three parameters of NMSSM λ, κ and A_κ are renormalized via the Higgs vertices.

As we need three independent equations we choose the following tree level Higgs vertices, which we named as

$$\begin{aligned} h_1 h_1 h_1 &\rightarrow V_1 = V_1^{(0)} + V_1^{(1)} + \cdots \, , \\ h_1 a_1 a_1 &\rightarrow V_1 = V_1^{(0)} + V_1^{(1)} + \cdots \, , \\ h_1 H^+ H^- &\rightarrow V_1 = V_1^{(0)} + V_1^{(1)} + \cdots \end{aligned}$$

The renormalization of these parameters will be fixed by the following 3 equations from corrected vertex

$$V_i^{(1)div} + f_i^\lambda \delta\lambda + f_i^\kappa \delta\kappa + f_i^{A_\kappa} \delta A_\kappa + \delta V_{Rest} + f_i(\delta Z) V_i^{(0)} = 0, \;\; i = 1, 2, 3$$

For the NMSSM tree level vertices we have the following expressions

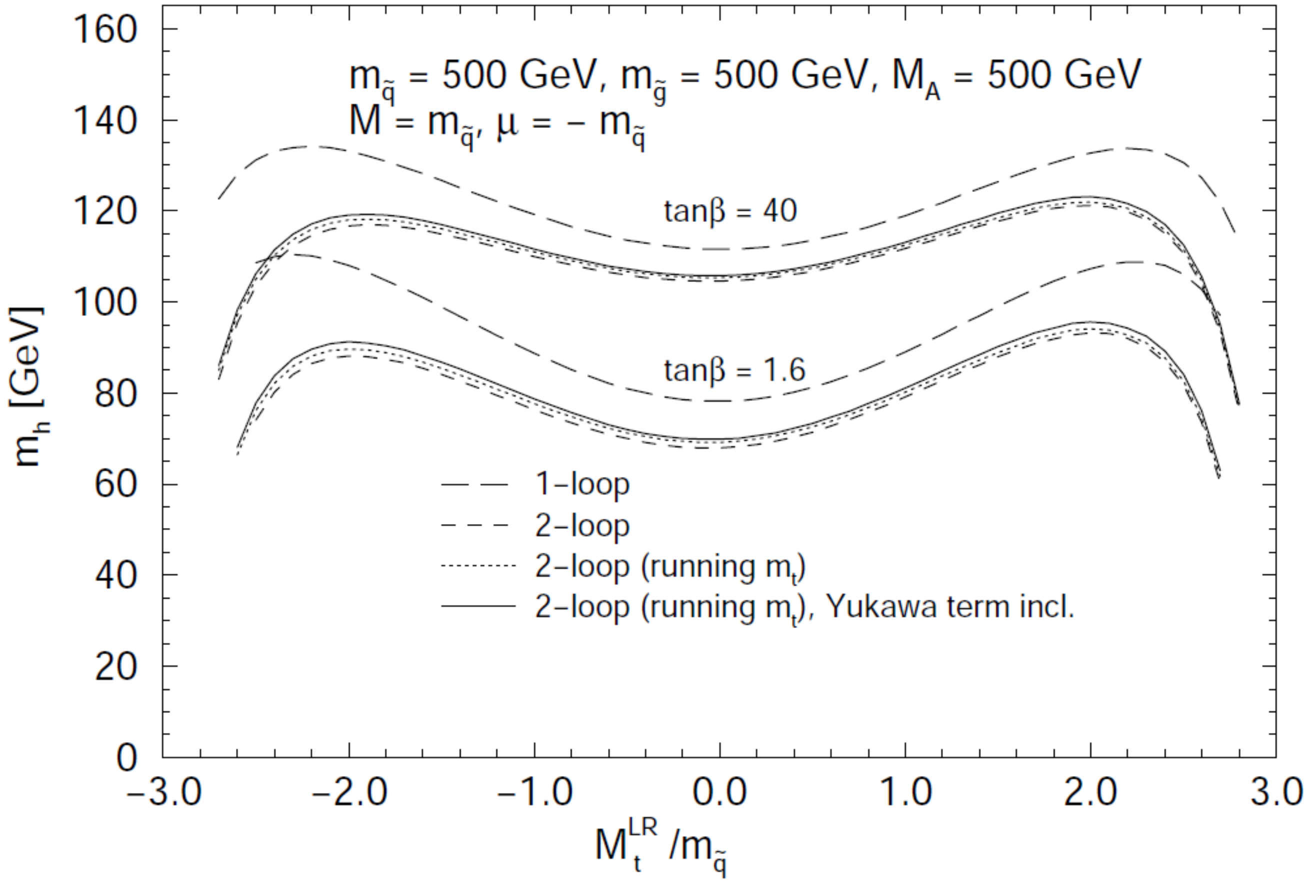
$$\begin{aligned}
V_1^{(0)} &= \sqrt{2}\mathbf{U}_{\mathrm{e}13}^3\left(A_\kappa+6\kappa^2\frac{\mu_{\mathrm{eff}}}{\lambda}\right)+3\mathbf{U}_{\mathrm{e}11}\mathbf{U}_{\mathrm{e}12}\mathbf{U}_{\mathrm{e}13}\left(\sqrt{2}\kappa\mu_{\mathrm{eff}}\left(\frac{1}{\lambda}-2\right)-\sin 2\beta\frac{1}{\sqrt{2}}\frac{\hat{m}_A^2}{\mu_{\mathrm{eff}}}\right)\\
&+\frac{3eM_Z}{8c_{\mathrm{W}}s_{\mathrm{W}}}(\mathbf{U}_{\mathrm{e}11}^2-\mathbf{U}_{\mathrm{e}12}^2)\left(\mathbf{U}_{\mathrm{e}11}-\mathbf{U}_{\mathrm{e}12}\sin\beta\right)\\
&+6\lambda\mathbf{U}_{\mathrm{e}13}\left[\frac{\mu_{\mathrm{eff}}}{\sqrt{2}}\left(\mathbf{U}_{\mathrm{e}11}^2+\mathbf{U}_{\mathrm{e}12}^2\right)-\kappa\mathbf{U}_{\mathrm{e}13}\frac{M_W s_{\mathrm{W}}}{e}\left(\mathbf{U}_{\mathrm{e}11}\sin\beta+\mathbf{U}_{\mathrm{e}12}\right)\right]\\
&+6\lambda^2\frac{M_W s_{\mathrm{W}}}{e}\left[\sin\beta\mathbf{U}_{\mathrm{e}12}\left(\mathbf{U}_{\mathrm{e}11}^2+\mathbf{U}_{\mathrm{e}13}^2\right)+\mathbf{U}_{\mathrm{e}11}\left(\mathbf{U}_{\mathrm{e}12}^2+\mathbf{U}_{\mathrm{e}13}^2\right)\right] \qquad (31)
\end{aligned}$$

$$\begin{aligned}
V_2^{(0)} &= 2\frac{M_W s_{\mathrm{W}}}{e}\lambda^2\left(\mathbf{U}_{\mathrm{e}11}\sin^2\beta_{\mathrm{n}}+\mathbf{U}_{\mathrm{e}12}\sin\beta\cos^2\beta_{\mathrm{n}}\right)+\frac{eM_Z}{8c_{\mathrm{W}}s_{\mathrm{W}}}\cos 2\beta_{\mathrm{n}}\left(\mathbf{U}_{\mathrm{e}11}-\mathbf{U}_{\mathrm{e}12}\sin\beta\right)\\
&+\frac{\mathbf{U}_{\mathrm{e}13}}{2\sqrt{2}\lambda\mu_{\mathrm{eff}}}\left[\sin 2\beta_{\mathrm{n}}\left(\lambda\hat{m}_A^2\sin 2\beta+2\kappa(2\lambda-1)\mu_{\mathrm{eff}}^2\right)+4\lambda^2\mu_{\mathrm{eff}}^2\right] \qquad (32)
\end{aligned}$$

$$\begin{aligned}
V_3^{(0)} &= \frac{e\sin 2\beta_{\mathrm{c}}}{2s_{\mathrm{W}}M_W}\left(\hat{m}_A^2-m_{H^\pm}^2\right)\left(\mathbf{U}_{\mathrm{e}11}\sin\beta+\mathbf{U}_{\mathrm{e}12}\right)+\cos 2\beta_{\mathrm{c}}\frac{eM_Z}{4c_{\mathrm{W}}s_{\mathrm{W}}}\left(c_{\mathrm{W}}^2\mathbf{U}_{\mathrm{e}12}\sin\beta+2s_{\mathrm{W}}^2\mathbf{U}_{\mathrm{e}11}\right)\\
&+\frac{eM_Z}{8c_{\mathrm{W}}s_{\mathrm{W}}}\left[\mathbf{U}_{\mathrm{e}12}\sin\beta\left(\sin^2\beta_{\mathrm{c}}-4s_{\mathrm{W}}^2\cos^2\beta_{\mathrm{c}}\right)+2c_{\mathrm{W}}^2\left(2\mathbf{U}_{\mathrm{e}11}+\mathbf{U}_{\mathrm{e}12}\sin\beta\right)\right]\\
&-\frac{\mathbf{U}_{\mathrm{e}13}}{2\sqrt{2}\lambda\mu_{\mathrm{eff}}}\left[\sin 2\beta_{\mathrm{c}}\left(\lambda\hat{m}_A^2\sin 2\beta+2\kappa(2\lambda-1)\mu_{\mathrm{eff}}^2\right)-4\lambda^2\mu_{\mathrm{eff}}^2\right] \qquad (33)
\end{aligned}$$

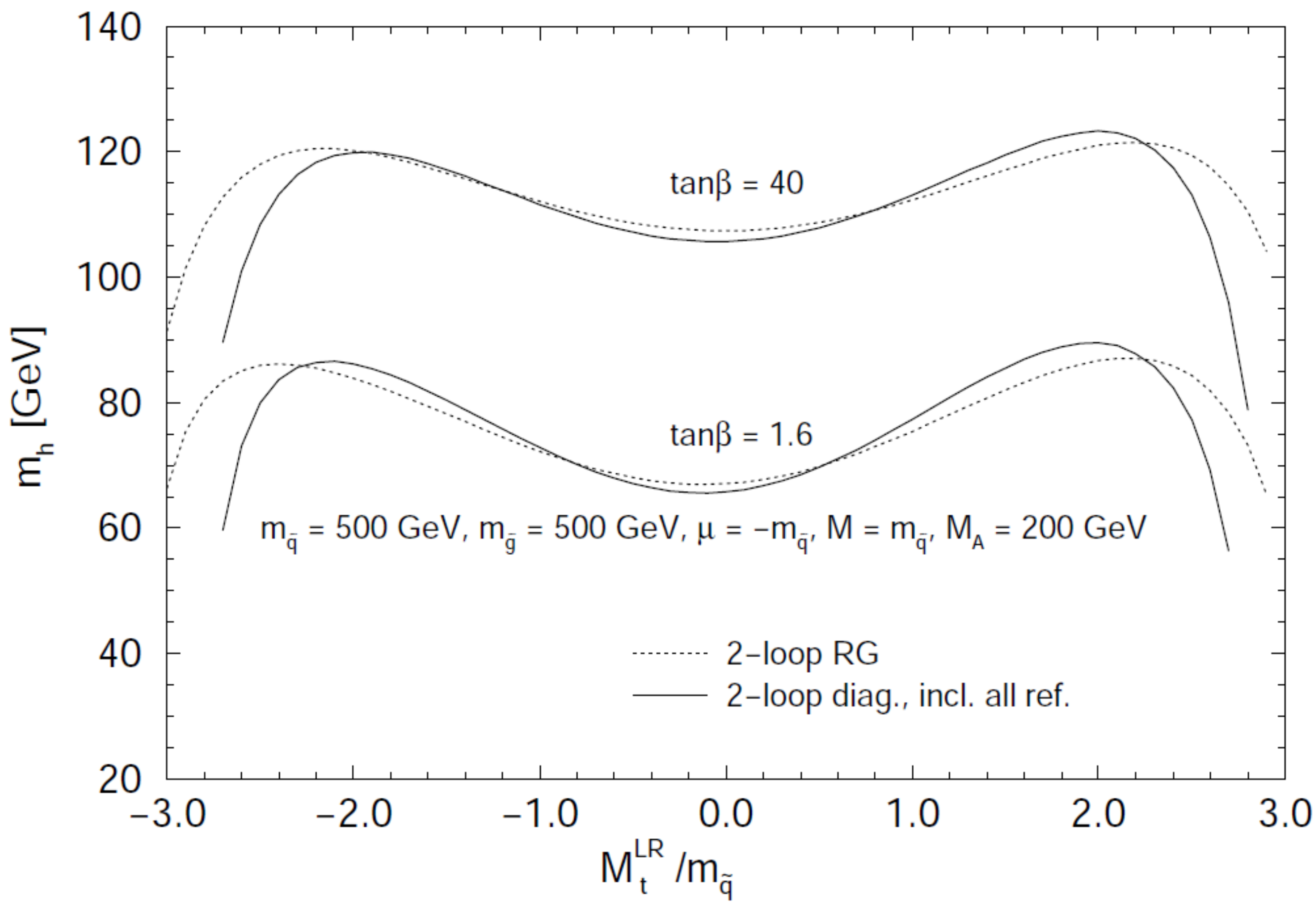
Examples of results on 2-loop renormalized Higgs mass MSSM

2-loop corrected m_h^2 mass in the MSSM



[Heinemeyer, Hollik, Weiglein 98]

m_h^2 *mass in the MSSM, 2 renormalization schemes*



[Heinemeyer, Hollik, Weiglein 98]

thank you