Twenty-first Century Lattice Gauge Theory:

Consequences of the QCD Lagrangian

Andreas S. Kronfeld
Fermilab
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Fermilab and Mexico

- Many contributions in experimental physics to the Fermilab program.
- Theoretical Physics Department offers opportunity for aspiring young theorists: “Latin American Graduate Students”.
    - Six month visit to Fermilab to work with one of us, coordinated by Marcela Carena y López.
    - See http:theory.fnal.gov for details.

Aim of this talk

- Provide a survey of results about QCD, obtained using numerical lattice gauge theory, that are both
  - quantitatively impressive;
  - qualitatively noteworthy.
- Some quoted results have replaced ignorance, guesses, and beliefs with scientific knowledge.
- Others aid the interpretation of experiments or observations in particle physics, nuclear physics, and astrophysics.

Quantum Chromodynamics—QCD

- Modern theory of the strong force: quarks+gluons → hadrons → nuclei.

- A gauge theory, mathematically similar to quantum electrodynamics:

$$\begin{aligned}\mathcal{L}_{\mathrm{QED}} &= -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} \\ &- \sum_{\text{charged } l} \bar{\psi}_l(\not{D}_l + m_l)\psi_l \\ \not{D}_l &= \gamma^\mu\left(\partial_\mu + q_l e_0 A_\mu\right)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{QCD}} &= -\frac{1}{2g_0^2}F^a_{\mu\nu}F^{\mu\nu a} \\ &- \sum_{\text{colored } f} \bar{\psi}^i_f(\not{D} + m_f)_{ij}\psi^i_f \\ \not{D}_{ij} &= \gamma^\mu\left(\partial_\mu\delta_{ij} + A^a_\mu t^a_{ij}\right)\end{aligned}$$

- Now the gauged quantum number is not electric charge, but color.

- SU(3) gauge symmetry: gauge boson “gluon” carries color.

- Laws of Nature.

Color *vs*. colour

- With SU(3), states with equal amounts of colors red, green, and blue (or equal amounts of cyan, magenta, and yellow) are neutral.

- In vision, light (ink) with equal amounts of colours red, green, and blue (equal amounts of cyan, magenta, and yellow) are white (black) or gray.

- For QCD, I will follow spelling of physicists Greenberg, Gell-Mann, Nambu, …, rather than francophile administrators and secretaries at CERN.

The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] && r_1 \text{ or } m_\Omega \text{ or } \Upsilon(2S\text{-}1S), \ldots. \\ &- \sum_f \bar{\psi}_f(\not{D} + m_f)\psi_f && m_\pi, m_K, m_{J/\psi}, m_\Upsilon, \ldots. \\ &+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[F_{\mu\nu}F_{\rho\sigma}] && \theta = 0. \end{aligned}$$

- Observable CP violation $\propto \vartheta = \theta - \arg\det m_f$ (if all masses nonvanishing):
  - neutron electric-dipole moment sets limit $\vartheta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

Quantum Chromodynamics

- The most perfect theory—asymptotic freedom.
- Triumph of reductionism: quark model ⊕ parton model ⊕ color = QCD.
- Multi-scale problem: m_u, m_s, m_π, m_K, Λ_QCD, m_c, m_b, m_t; Q^2; a^{-1}; L^{-1}.
- Rich in symmetry: C, P, T; chiral symmetry, heavy-quark symmetry.
- Rich in emergent phenomena: hadron masses, chiral symmetry breaking, phase transitions, atomic nuclei ...
  - ... requiring nonperturbative methods (lattice gauge theory) and a full exploitation of symmetries, asymptotic freedom.

Asymptotic Freedom

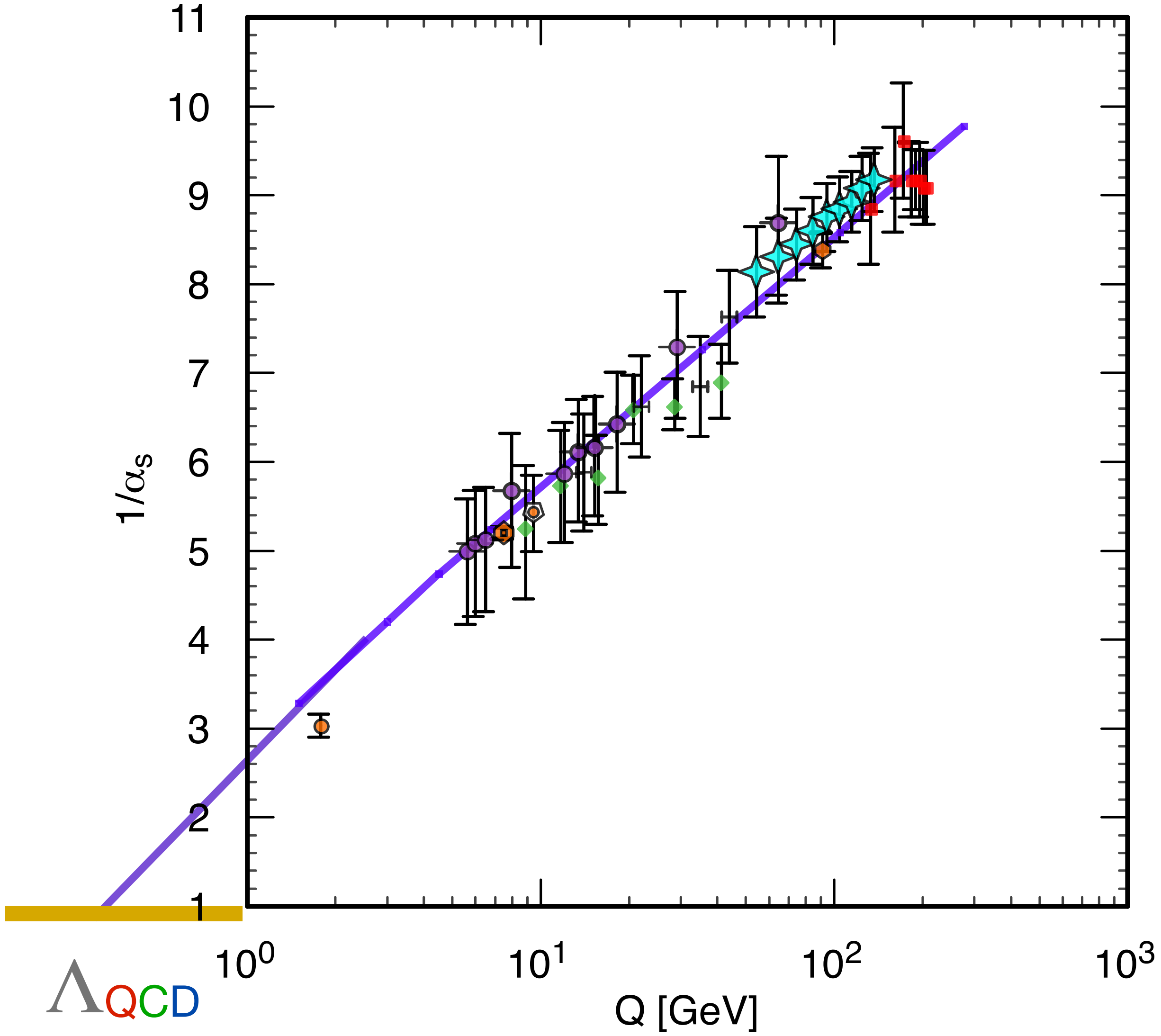
Politzer, *PRL* **30** (1973) 1346;
Gross, Wilczek, *PRL* **30** (1973) 1343

- At short-distances, the force in QCD looks similar to QED:

$$F(r) = -\frac{4}{3}\frac{\alpha_s(1/r)}{r^2}$$

where the 4/3 is a color factor.

- The key difference is that virtual gluons reduce the effective α_s at short distances.
- Verified in experiment.
- Relates α_s to a physical scale, Λ_{QCD}.



ASK & Quigg, arXiv:1002.5032

Lattice Gauge Theory

K. Wilson, *PRD* **10** (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].
- Gauge symmetry on a spacetime lattice:
  - mathematically rigorous definition of QCD functional integrals;

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U\, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

  - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.
- Lowest-order strong coupling expansion demonstrates confinement.

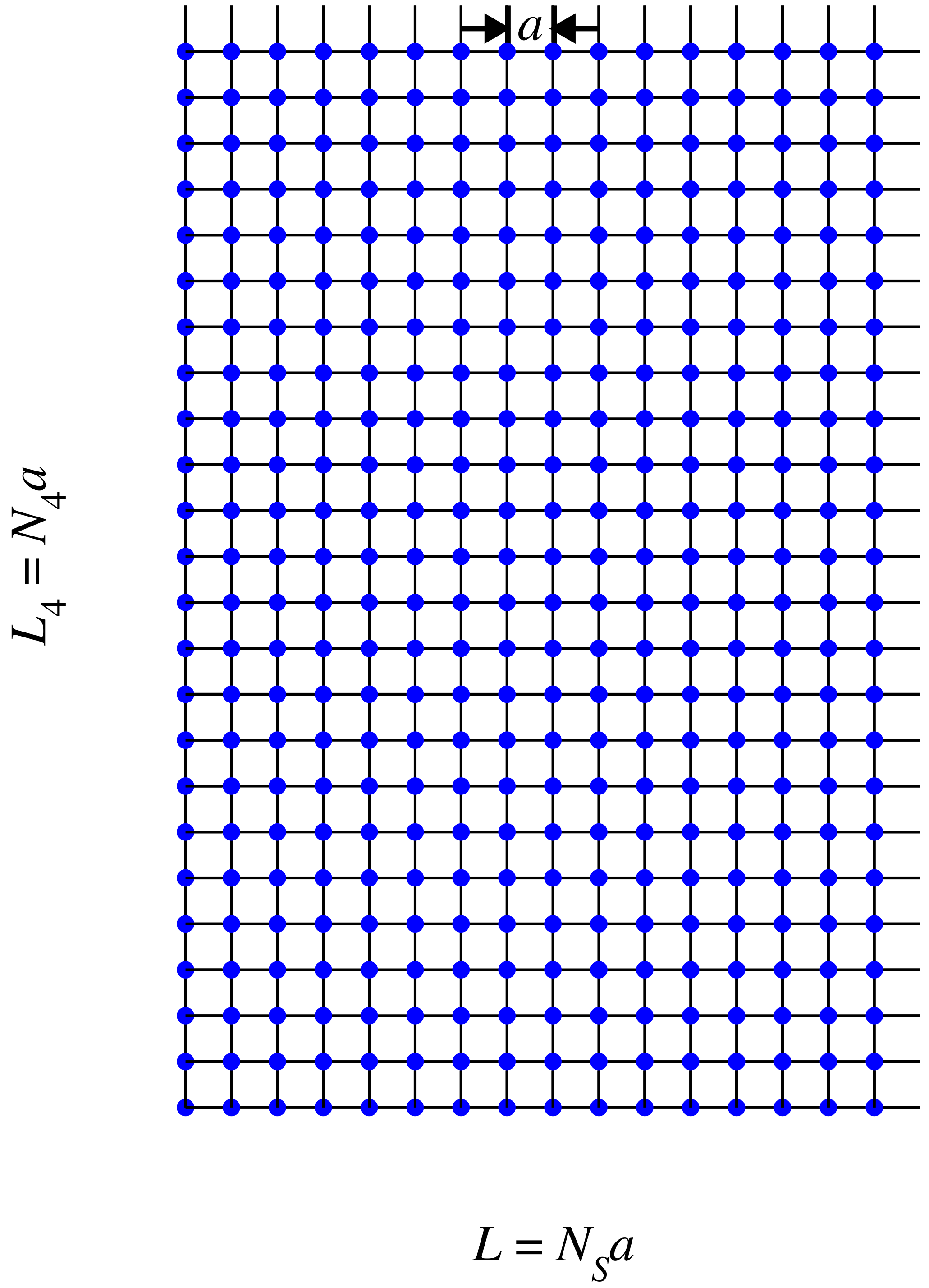
Numerical Lattice QCD

- Nowadays “lattice QCD” usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.
- A big computer.
- Some compromises:
    - finite human lifetime ⇒ Wick rotate to Euclidean time: $x^4 = ix^0$;
    - finite memory ⇒ finite space volume & finite time extent;
    - finite CPU power ⇒ light quarks heavier than up and down.

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z}\int \underset{\text{MC}}{\boxed{\mathcal{D}U}}\,\underset{\text{hand}}{\boxed{\mathcal{D}\psi\mathcal{D}\bar{\psi}}}\exp\left(-S\right)\left[\bullet\right]$$

- Infinite continuum: uncountably many d.o.f.
- Infinite lattice: countably many; used to define QFT
- Finite lattice: can evaluate integrals on a computer; dimension ~ 10^8



Some Jargon

- QCD observables (quark integrals by hand):

$$\langle\bullet\rangle = \frac{1}{Z}\int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D}+m_f)\exp\left(-S_{\text{gauge}}\right)[\bullet]$$

- *Quenched* means replace $\det$ with 1. (Obsolete.)
- *Un*quenched means not to do that.
- *Partially* quenched (usually) doesn't mean "n_f too small" but $m_{\text{val}} \neq m_{\text{sea}}$, or even $\not{D}_{\text{val}} \neq \not{D}_{\text{sea}}$ ("mixed action").

Some algorithmic issues

e.g., ASK, hep-lat/0205021

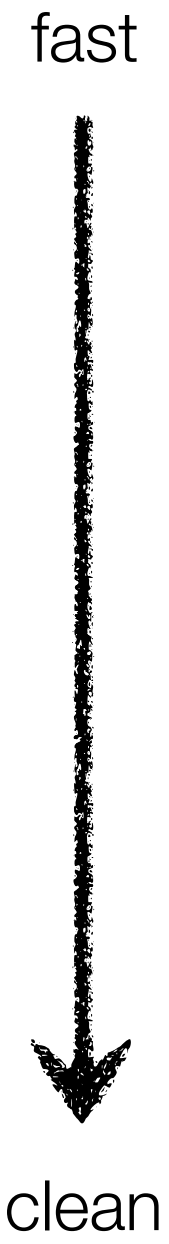
- lattice $N_S^3 \times N_4$, spacing a
- memory $\propto N_S^3 N_4 = L_S^3 L_4 / a^4$
- $\tau_\mathrm{g} \propto a^{-(4+z)}$, $z = 1$ or 2.
- $\tau_\mathrm{q} \propto (m_\mathrm{q} a)^{-p}$, $p = 1$ or 2.
- Imaginary time:
  - static quantities

- size $L_S = N_S a$, $L_4 = N_4 a$;
- dimension of spacetime = 4
- critical slowing down
- especially dire with sea quarks
- thermodynamics: $T = 1/N_4 a$

$$\begin{aligned} \langle \bullet \rangle &= \frac{1}{Z} \int \mathcal{D}U\, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) \left[\bullet\right] \\ &= \mathrm{Tr}\{\bullet\, e^{-\hat{H}/T}\} / \mathrm{Tr}\{e^{-\hat{H}/T}\} \end{aligned}$$

Sea Quarks

- Staggered quarks, with rooted determinant, $\mathrm{O}(a^2)$.
- Wilson quarks, $\mathrm{O}(a)$:
  - tree or nonperturbatively $\mathrm{O}(a)$ improved $\Rightarrow$ $\mathrm{O}(a^2)$;
  - twisted mass term—auto $\mathrm{O}(a)$ improvement $\Rightarrow$ $\mathrm{O}(a^2)$.
- Ginsparg-Wilson (domain wall or overlap), $\mathrm{O}(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.



- Many numerical simulations with sea quarks are called (perhaps misleadingly) “full QCD.”
  - $n_f = 2$: with same mass, omitting strange sea;
  - $n_f = 3$: may (or may not) imply 3 of same mass;
  - $n_f = 2{+}1$: strange sea + 2 as light as possible for up and down;
  - $n_f = 2{+}1{+}1$: add charmed sea to $2{+}1$.
- “Full QCD” can also mean $m_{\mathrm{val}} = m_{\mathrm{sea}}$, or $\not{D}_{\mathrm{val}} = \not{D}_{\mathrm{sea}}$.

Correlators Yield Masses & Matrix Elements

- Two-point functions for masses $\pi(t) = \bar{\psi}_u \gamma_5 S \psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-\boxed{m_{\pi_n}} t)$$

- Two-point functions for decay constants:

$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \boxed{\langle 0 | \hat{J} | \pi_n \rangle} \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

- Three-point functions for form factors, mixing:

$$\langle \pi(t) J(u) B^\dagger(0) \rangle \; = \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \boxed{\langle \pi_n | \hat{J} | B_m \rangle} \langle B_m | \hat{B}^\dagger | 0 \rangle$$
$$\times \exp[-m_{\pi_n}(t-u) - m_{B_m} u]$$

Standard Model: 19 Parameters or 28

- Gauge couplings: α_s, α_{QED}, $\alpha_W = (m_W/v)^2/\pi$;
- Lepton masses: m_e, m_μ, m_τ; $m_{\nu1}$, $m_{\nu2}$, $m_{\nu3}$;
- Quark masses: $m_u e^{i\theta}$, m_d, m_s, m_c, m_b, m_t;

“Instability” → “renormalization.”

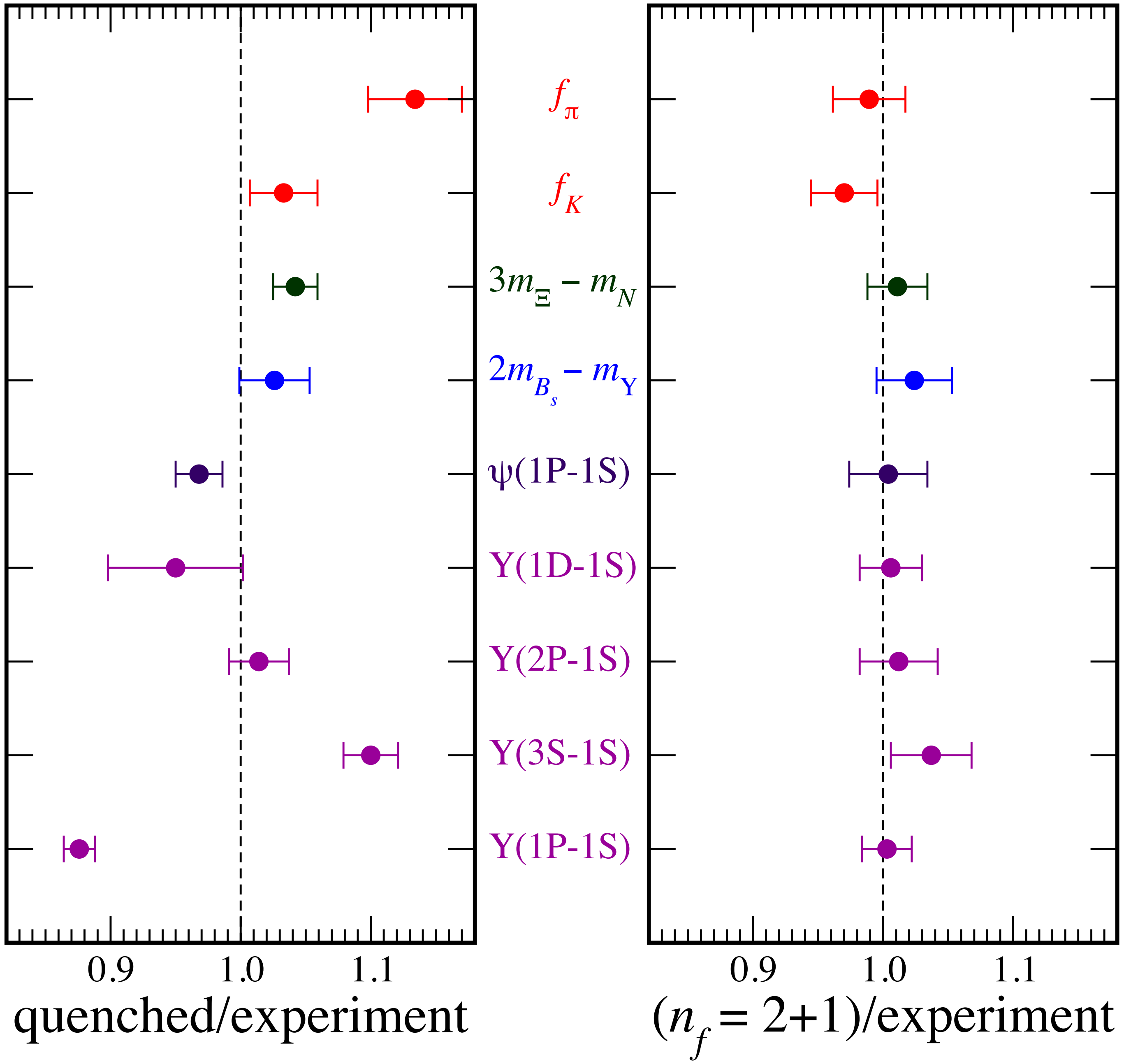
- CKM: $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\exp(i\delta_{KM})$; θ_{12}, θ_{23}, θ_{13}, δ_{PMNS}, ϕ_1, ϕ_2;
- EWSB: $v = 246$ GeV, $\lambda = (m_H/v)^2/2$.

“Infinite λ” → “triviality.”

- Need lattice QCD, lattice Yukawa.

2+1 Sea Quarks!

HPQCD, MILC, Fermilab Lattice, hep-lat/0304004

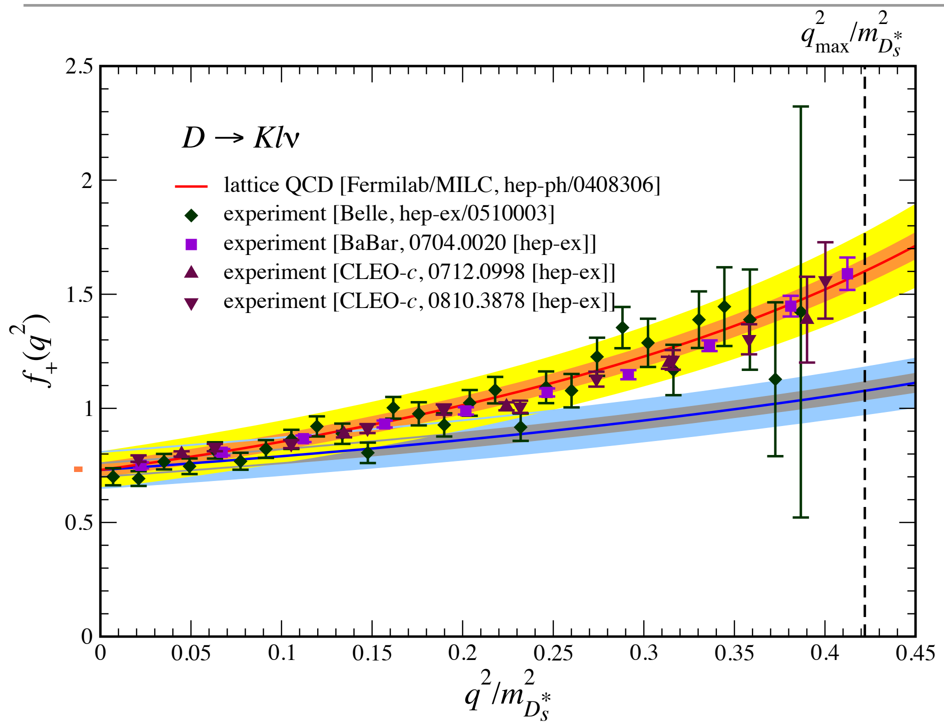


- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

Predictions

Fermilab Lattice, MILC, HPQCD,
hep-ph/0408306, hep-lat/0411027, hep-lat/0506030

- Semileptonic form factor for $D \to K l \nu$
- Mass of B_c meson
- Charmed-meson decay constants



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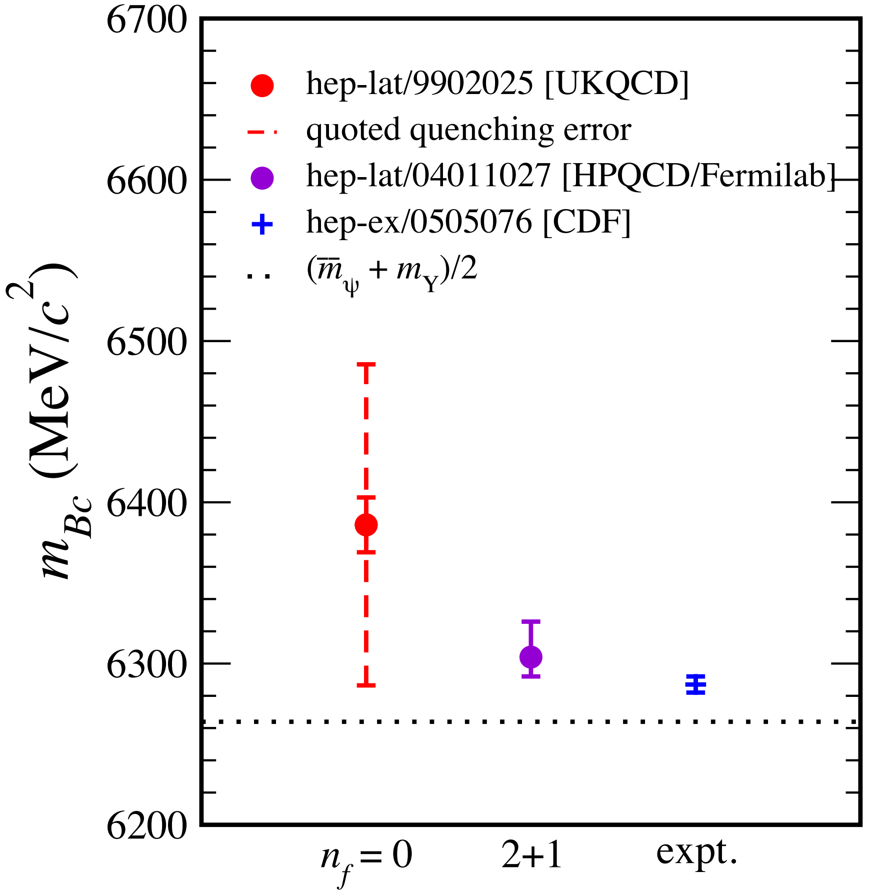
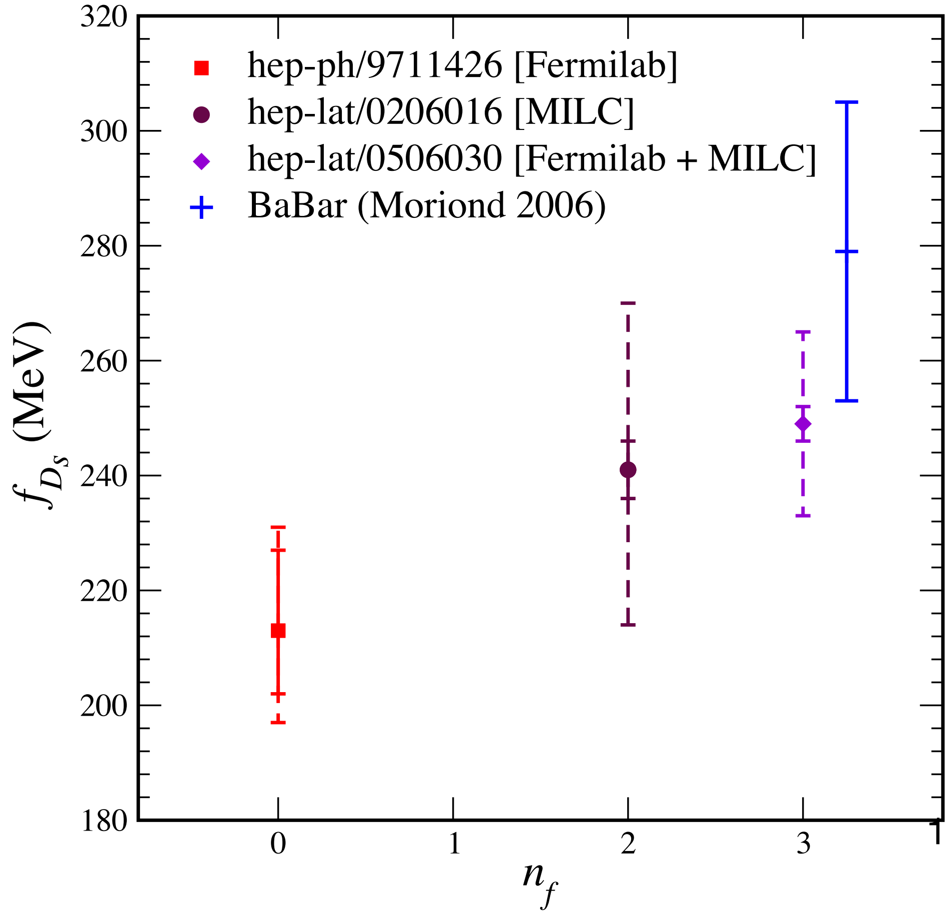




Outline

- Introduction
- Chiral Symmetry Breaking
- Hadron Spectrum
- QCD Parameters
- Flavor Physics
- Thermodynamics
- Summary & Challenges

Chiral Symmetry Breaking

Chiral Symmetry

Y. Nambu, *PRL* **4** (1960) 380

- The hadron spectrum has a striking feature:
  - $m_\pi = 135$ MeV but $m_\varrho = 770$ MeV, $m_\mathrm{p} = 938$ MeV, *etc*.
- Nambu applied lessons from superconductivity, noting (4 years before quarks) that the pion's small mass could be arise from a *spontaneously* broken *axial* symmetry (moderated with a small amount of explicit breaking).
- QCD explained the origin: if up and down quark masses are neglected, the Lagrangian has an $SU_L(2)\times SU_R(2)$ chiral symmetry, which provides candidate axial symmetry.
- (If so, pions break $SU_L(2)\times U_Y(1)$: without terascale EWSB, $W^\pm$ and Z would have masses around 100 MeV.)

- In the 20th century, we were already confident that QCD was a good theory of the strong interactions, based on, *e.g.*, its explanation of the SLAC deep-inelastic scattering experiments.

- Because QCD was (considered) right, and since Nambu's picture was (considered) right, it was believed that QCD must drive spontaneous chiral symmetry breaking.

- But does it?

- Goldstone formula: $m_\pi^2\langle\bar{\psi}\psi\rangle = 0$, if $\langle\bar{\psi}\psi\rangle \neq 0$, then $m_\pi = 0$.

- What is it?

Chiral Condensate

e.g., H. Fukaya *et al*. [JLQCD], arXiv:0911.5555

$m_{\mathrm{u}}, m_{\mathrm{d}} \to 0,\ \ m_{\mathrm{s}}$ physical

$$\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \left[242 \pm 4_{\mathrm{stat}}{}^{+19}_{-18}{}_{\mathrm{syst}}\ \mathrm{MeV}\right]^3$$

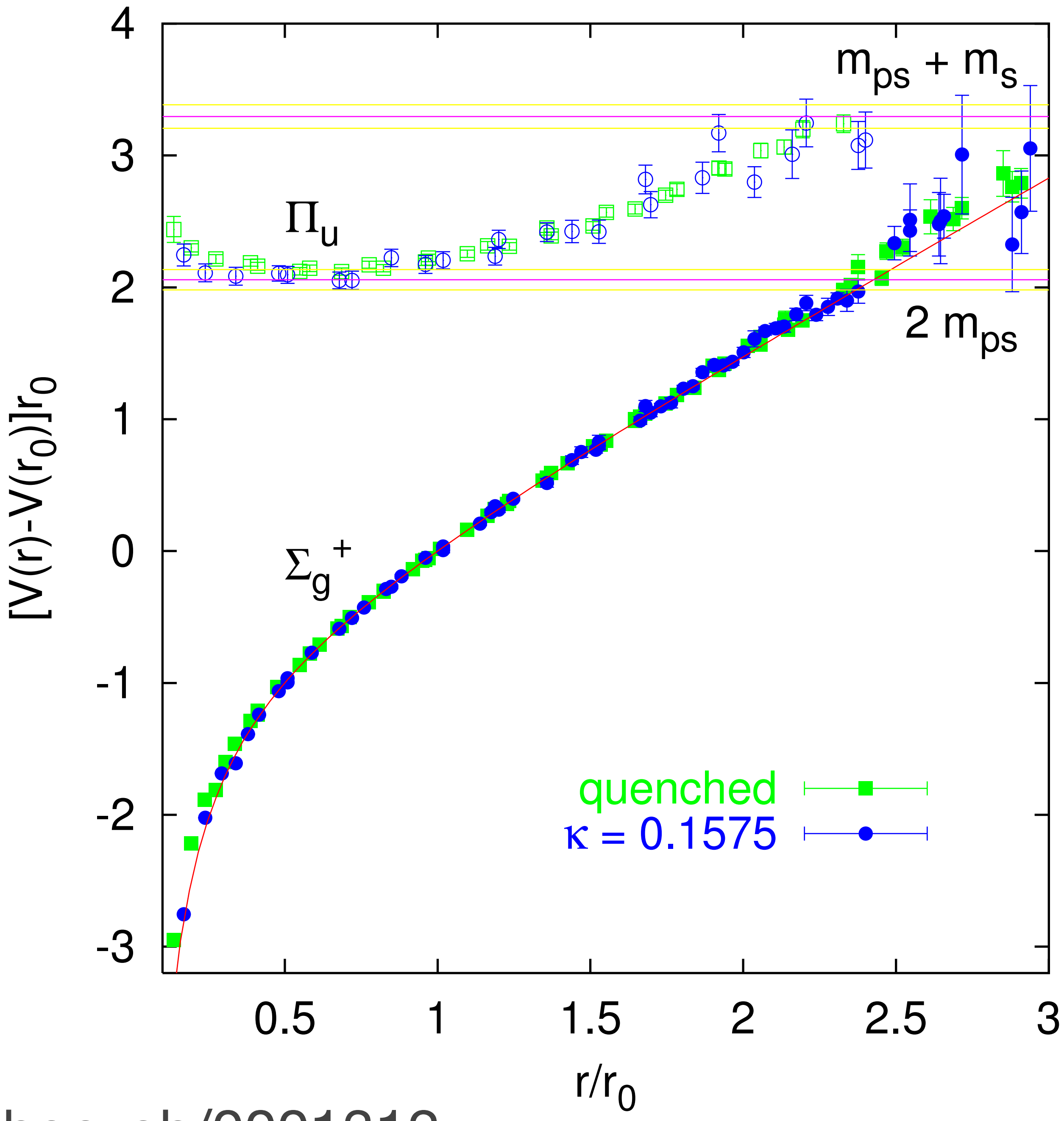
- At the hadronic level, the spontaneous breaking of chiral symmetry allows the nucleon mass to be nonzero [Nambu], even when $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$.

- In nature, m_{u} & m_{d} are small, so the physical picture of chiral symmetry is:
  - dominantly spontaneously broken (Nambu’s mechanism);
  - small corrections from explicit breaking (chiral perturbation theory).

Hadron Spectrum

Why Compute Hadron Masses?

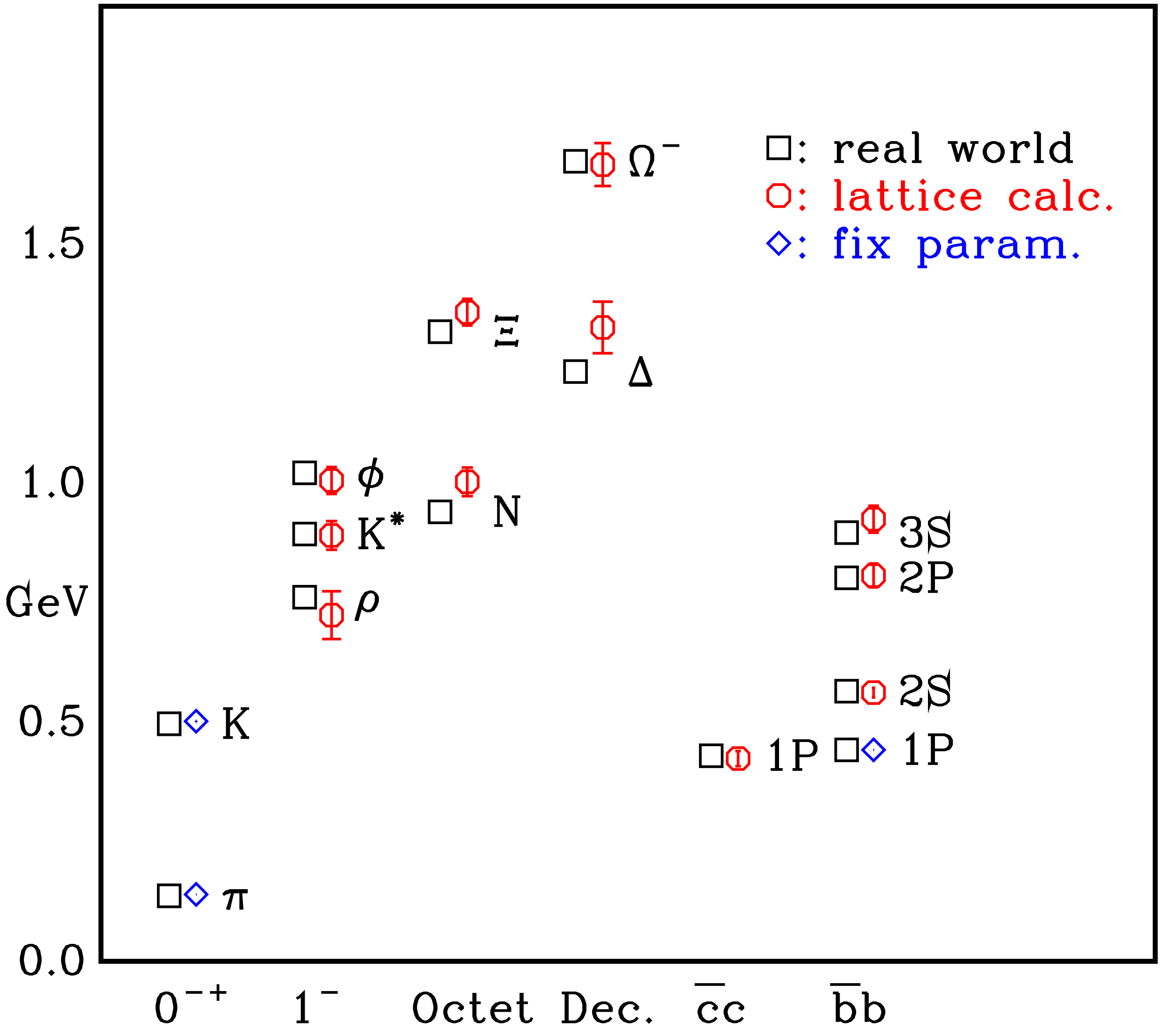
- Show that the QCD Lagrangian generates hadron masses.
- Understand more deeply Nature's only known mechanism for generating masses.
- At short distances, the potential (force) is Coulombic.
- At large distances, the potential (force) rises linearly (flattens at a positive value).



G. Bali, hep-ph/0001312

Hadron Spectrum 1

MILC Collaboration, *PRD* **70** (2004) 094505; arXiv:0903.3598



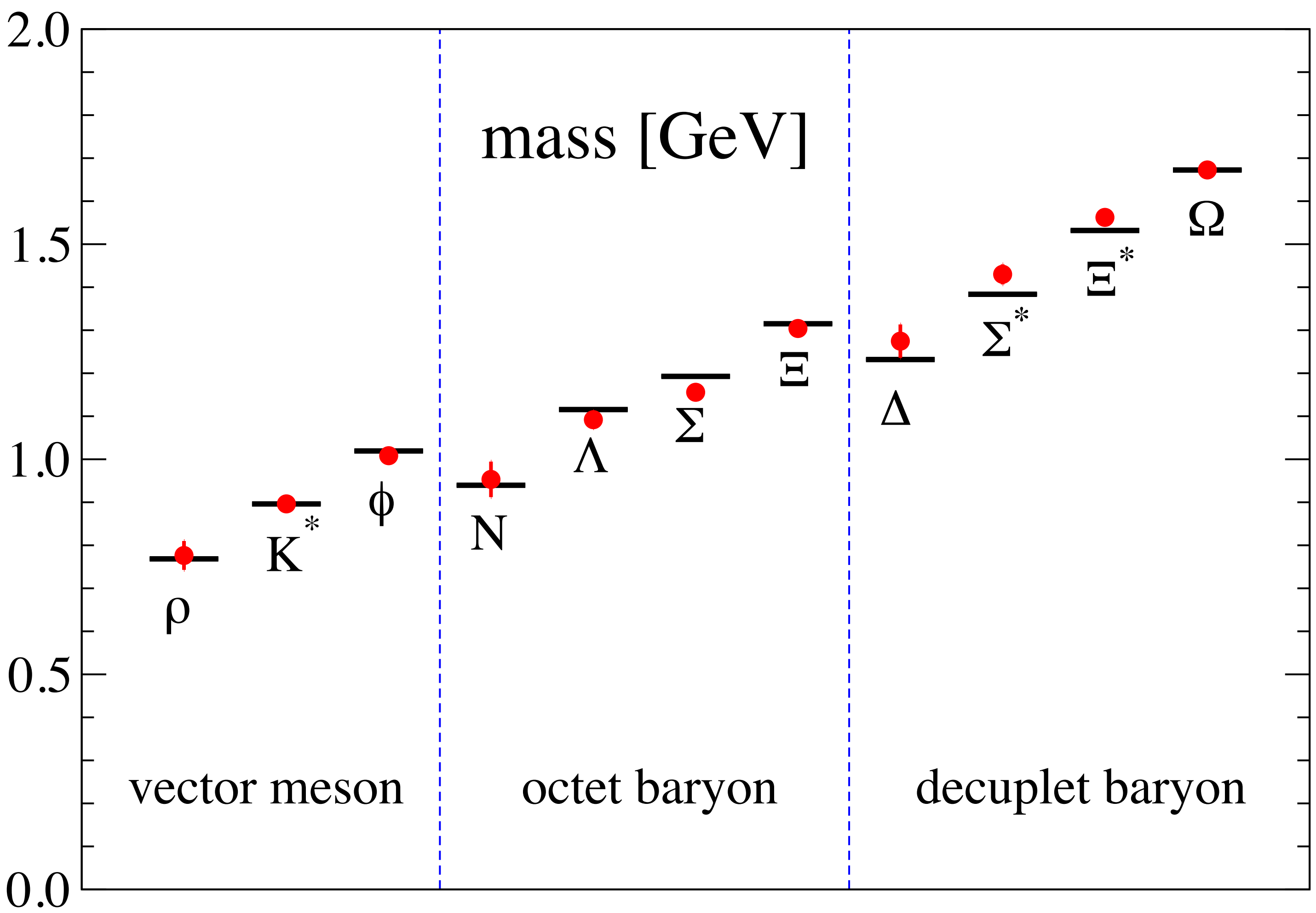
- $a = 0.12$ & 0.09 fm;
- O(a^2) improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 2

PACS-CS Collaboration, *PRD* **79** (2009) 034503

cf. earlier work by CP-PACS

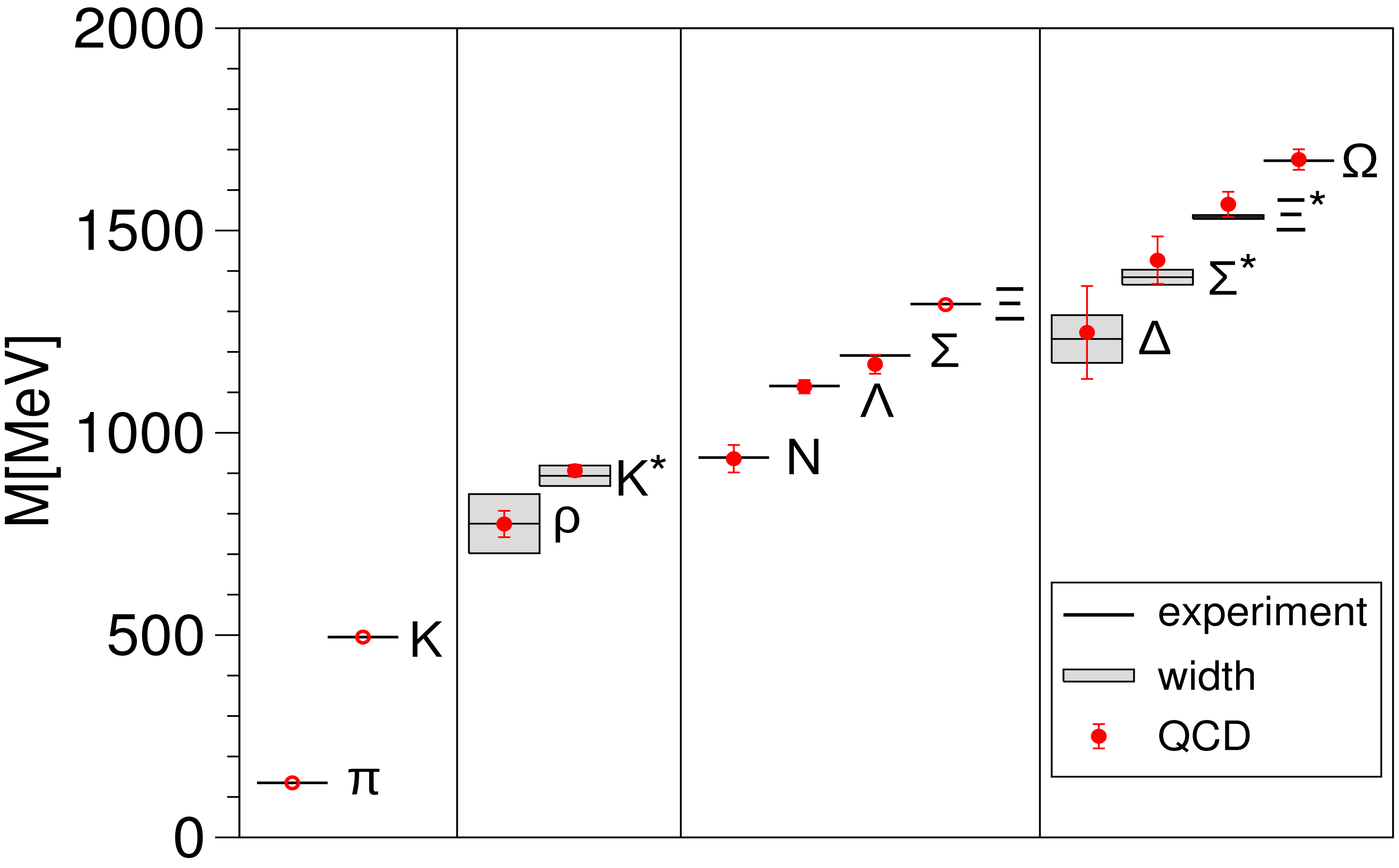


- $a = 0.091$ fm;
- NP O(a) Wilson;
- no smearing;
- $m_q \approx 1.3 m_l$;
- π, K, Ω input

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 3

BMW Collaboration: *Science* **322** (2008) 1224



- $a = 0.125, 0.085, \&$ 0.065 fm;
- tree $O(a)$ Wilson;
- 6× stout smearing;
- $2m_l < m_q < 1.7m_s$;
- π, K, Ξ input.

QCD postdicts the low-lying hadron masses!

Now, quark masses are MeV not GeV!

$$m = E_0/c^2$$

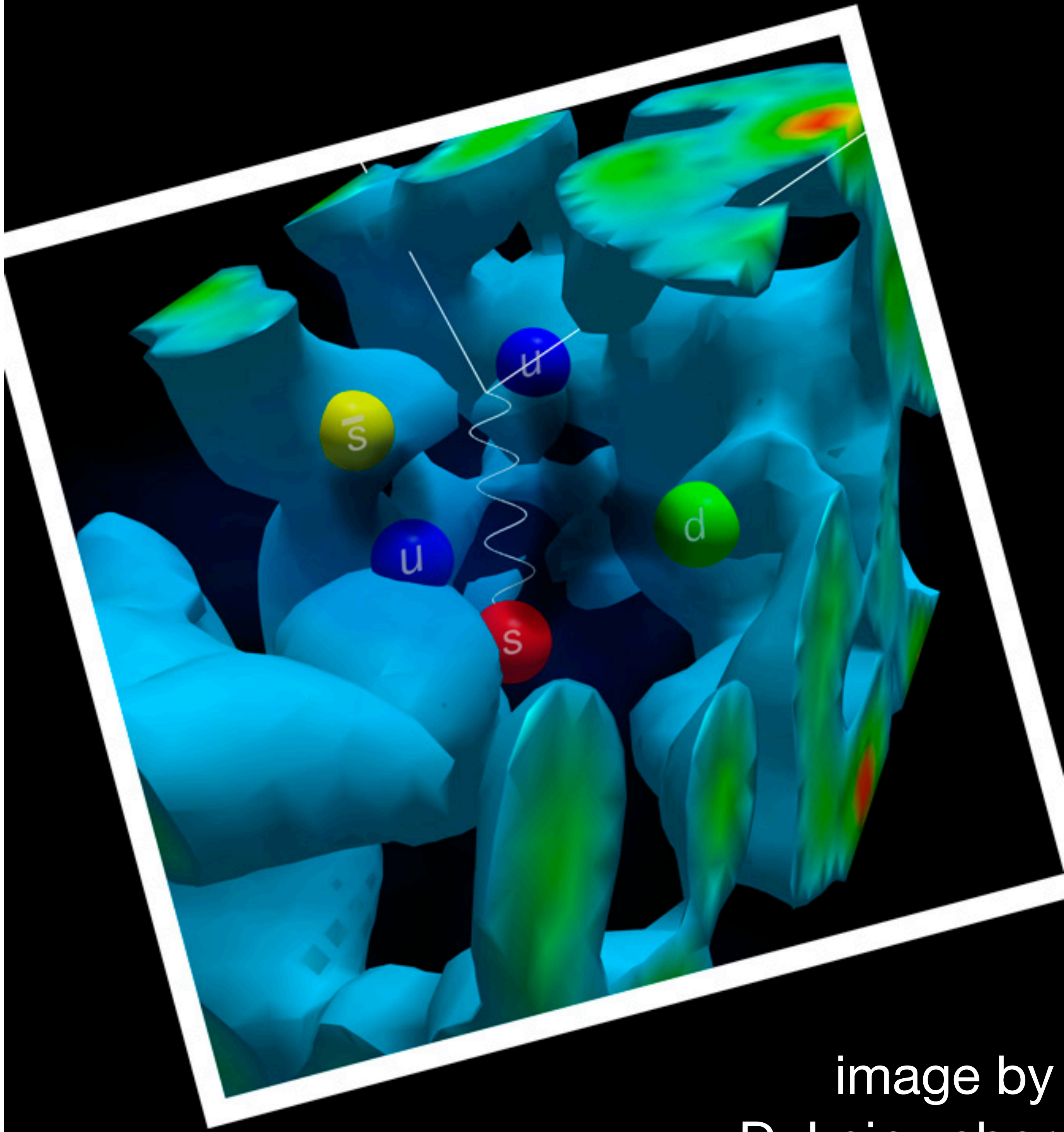


image by
D. Leinweber

QCD Parameters: α_s and Quark Masses

Light Quark Masses

- The nonzero pion (kaon) mass is very sensitive to the light (strange) masses.

- Chiral perturbation theory predicts ratios of masses, but not the overall scale.

Lattice QCD	**MILC**	**RBC**	**BMW**	**HPQCD**
$\overline{m}_u(2\ \mathrm{MeV})$	1.9 ± 0.2	2.24 ± 0.35	2.15 ± 0.11	
$\overline{m}_d(2\ \mathrm{MeV})$	4.6 ± 0.3	4.65 ± 0.35	4.79 ± 0.14	
$\overline{m}_s(2\ \mathrm{MeV})$	88 ± 5	97.6 ± 6.2	95.5 ± 1.9	92.4 ± 1.5

- These are small—up & down masses are 4 & 9 times electron mass.

- The up mass is far from 0: the strong CP problem is indeed a problem.

Strong *CP* Problem

- Quark masses arise from Yukawa couplings, $\boldsymbol{m} = v\boldsymbol{y}/\sqrt{2}$, and from low-energy QCD instantons (tunneling between classical vacua):
    - observable *CP* violation $\propto \vartheta = \theta_{\mathrm{QCD}} - \arg\det \boldsymbol{y} < 10^{-11}$.
- If $\boldsymbol{y}$ has a zero mode, then its phase can be anything and, thus, chosen so that $\vartheta = 0$; no *CP* violation arises.
- Though m_u is small, lattice QCD calculations show no evidence for an instanton effect big enough to allow a zero mode in $\boldsymbol{y}$.
- A non-Standard symmetry (Peccei-Quinn) is then the least implausible explanation for the cancellation. Consequence: weird particles called axions.

Heavy Quark Masses

- The charmonium correlator also yields impressive precision on the charm mass [Bochkarev & de Forcrand, hep-lat/9505025]; analogous to determination from e^+e^- by Chetyrkin *et al.*:
  - lattice + PT: $m_c(m_c) = 1.268(9)$ GeV [arXiv:0807.1687];
  - e^+e^- + PT: $m_c(m_c) = 1.279(13)$ GeV [arXiv:0907.2110].
- Similarly, $m_b(m_b) = 4.164(23)$ GeV [HPQCD, arXiv:1004.4285], *cf*. $m_b(m_b) = 4.163(16)$ GeV [Chetyrkin *et al.* e^+e^-, arXiv:0907.2110].
- Even more stunning [HPQCD, arXiv:0910.3102]: $m_c/m_s = 11.85(16)$, whence $m_s(2 \text{ GeV}) = 92.4(1.5)$ MeV.

Strong Coupling α_s

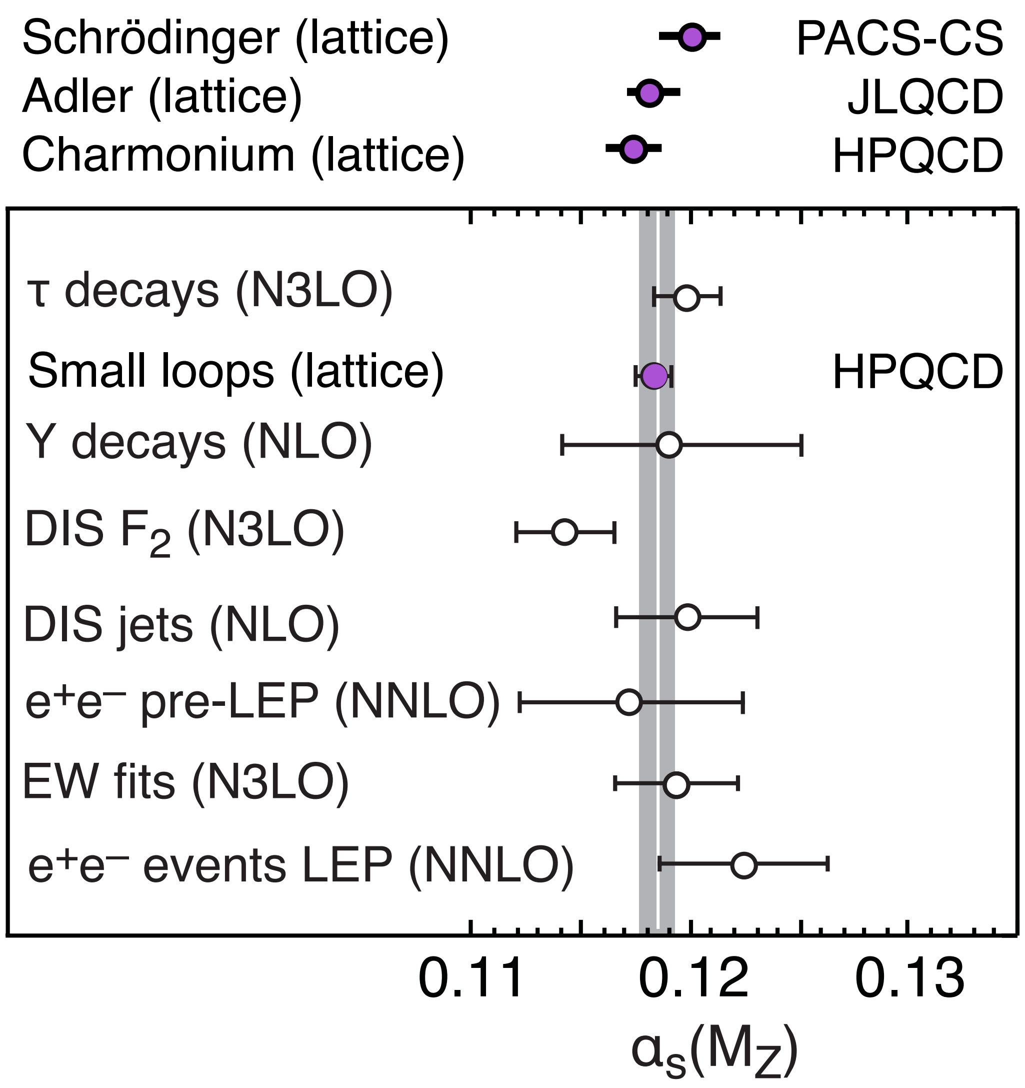
- In lattice gauge theory, the bare coupling—$g_0^2(1/a)$— is an input. Aim is to relate this to $\alpha_s(m_Z) = g_s^2(m_Z)/4\pi$. Alas, conversion in PT does not converge.

- Two main strategies:
  - compute a short-distance lattice quantity (*e.g.*, small Wilson loop, Creutz ratio of small Wilson loops, ...); compare MC with (lattice) PT → $\alpha_s(1/a)$;
  - compute short-distance continuum quantity (*e.g.*, Schrödinger functional, quarkonium correlator, Adler function); compare MC with PT → $\alpha_s(2m_Q)$.

Results for α_s (all $n_f = 2+1$):

- Schrödinger [PACS-CS, arXiv:0906.3906] | Adler [JLQCD, arXiv:1002.0371]:
  - $\alpha_s(m_Z) = 0.1205(8)(5)(+0/-17)$ | $\alpha_s(m_Z) = 0.1181(3)(+13/-4)$;
- Wilson Creutz, *etc.* [HPQCD, arXiv:0807.1687 | Maltman, arXiv:0807.2020]:
  - $\alpha_s(m_Z) = 0.1183(8) \mid 0.1192(11)$;
- Charmonium correlator [HPQCD + Karlsruhe, arXiv:0805.2999]:
  - $\alpha_s(m_Z) = 0.1174(12)$ [update in arXiv:1004.4285];
- Bethke's world average (without lattice | with HPQCD WL) [arXiv:0908.1135]:
  - $\alpha_s(m_Z) = 0.1186(11) \mid 0.1184(7)$.

QCD of hadrons = QCD of partons



Bethke, arXiv:0908.1135

Flavor Physics

Weak Interactions

- At energies probed by the Tevatron and the LHC, left- and right-handed quarks are different:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad \text{aka } Q_L^i$$

$$u_R \qquad c_R \qquad t_R \qquad \text{aka } U_R^j$$

$$d_R \qquad s_R \qquad b_R \qquad \text{aka } D_R^j$$

9 fields: 3 doublets and 6 singlets under $SU_L(2)\times U_Y(1)$.

- The electroweak interactions treat all three “generations” same, and the fields can be transformed so the $SU_L(2)\times U_Y(1)$ gauge fields don’t couple generations to each other—“weak eigenstate basis.”

Identity from Higgs and Yukawa

- Whatever breaks electroweak symmetry has a weak-SU(2) doublet, Φ, so it can have Yukawa interactions

$$y^u_{ij}\bar{Q}^i_L\Phi U^j_R + y^d_{ij}\bar{Q}^i_L\tilde{\Phi}^* D^j_R + \text{h.c.} =$$

$$y^u_{ij}\,(\bar{U} \quad \bar{D})^i_L \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix} U^j_R + y^d_{ij}\,(\bar{U} \quad \bar{D})^i_L \begin{pmatrix} \Phi^+ \\ \bar{\Phi}^0 \end{pmatrix} D^j_R + \text{h.c.}$$

where indices label generations.

- Spontaneous symmetry breaking driven by $\Phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$:
  - generates masses for the quarks (as noted above on strong *CP* problem).

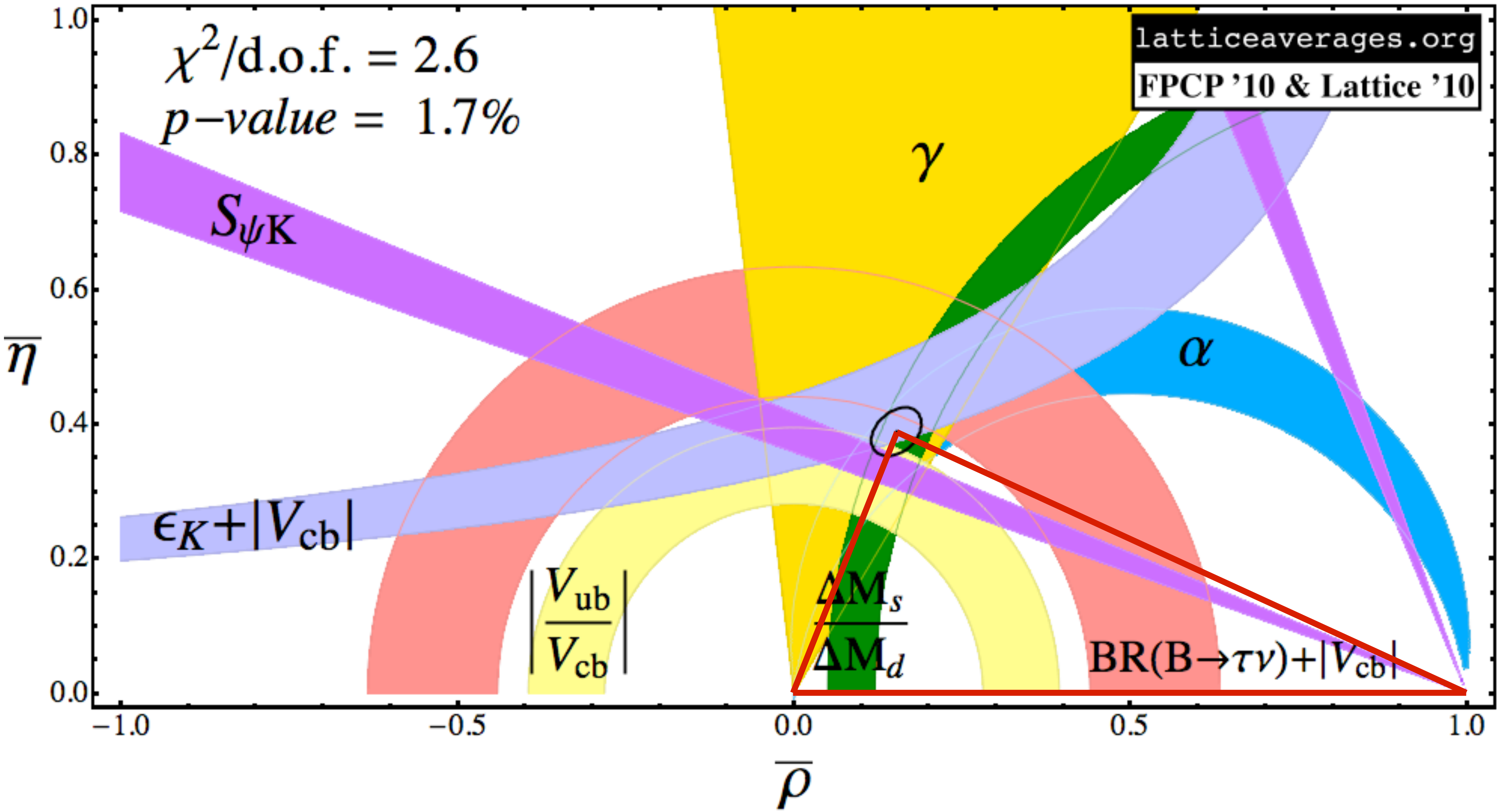
CKM Matrix

Cabibbo, *PRL* **10** (1963) 531;
Kobayashi, Maskawa, *Prog. Theor. Phys.* **49** (1973) 652

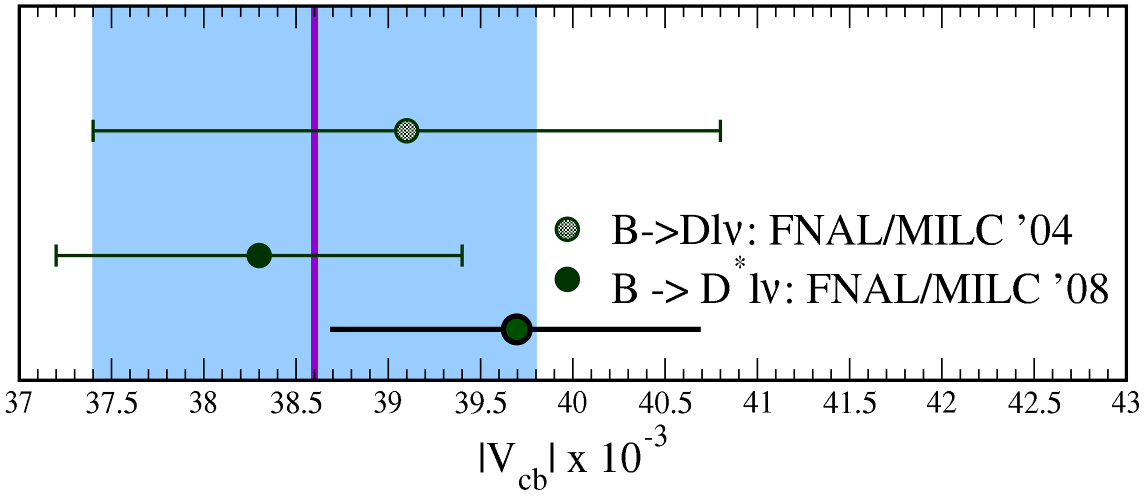
- Mass and weak eigenstates of quarks are related by unitary transformations.
- Observable part of these rotations is the CKM matrix.
- 4 parameters: $|V_{\mathrm{us}}|$, $|V_{\mathrm{cb}}|$, $|V_{\mathrm{ub}}|$, $i\delta_{\mathrm{KM}}$—as fundamental as electron mass.
- Unitarity relations, *e.g.*, $V_{ud}^{*}V_{ub} + V_{cd}^{*}V_{cb} + V_{td}^{*}V_{tb} = 0$:
  - triangle in the complex plane.
- Probed by many measurements + corresponding QCD.

Unitarity Triangle

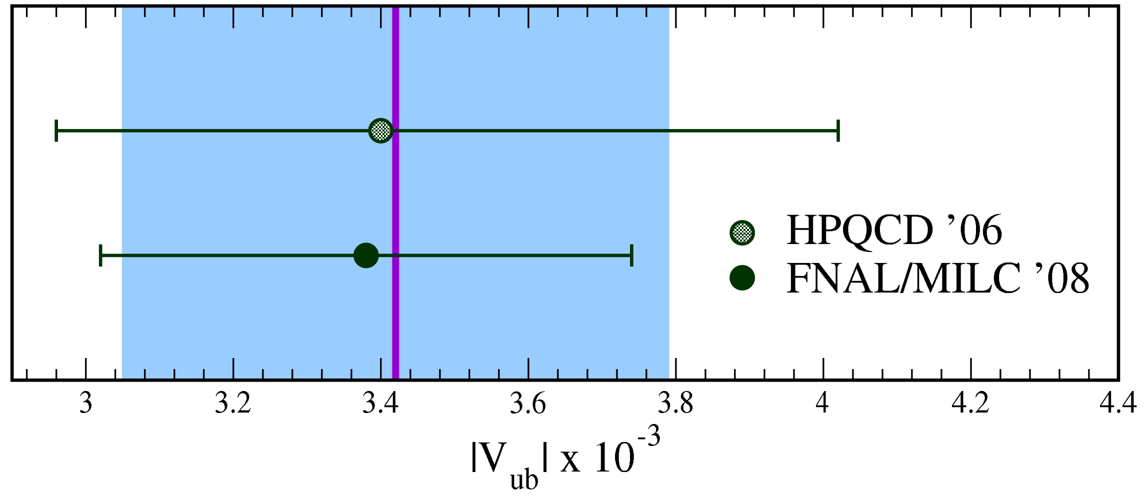
c.f., Laiho, Lunghi, Van de Water, arXiv:0910.2928



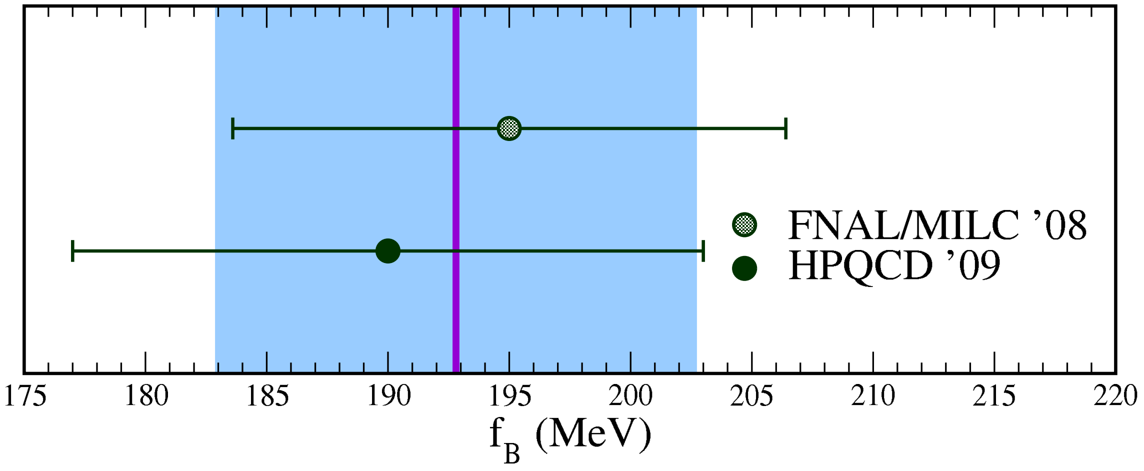
B-Meson Averages from Lattice QCD

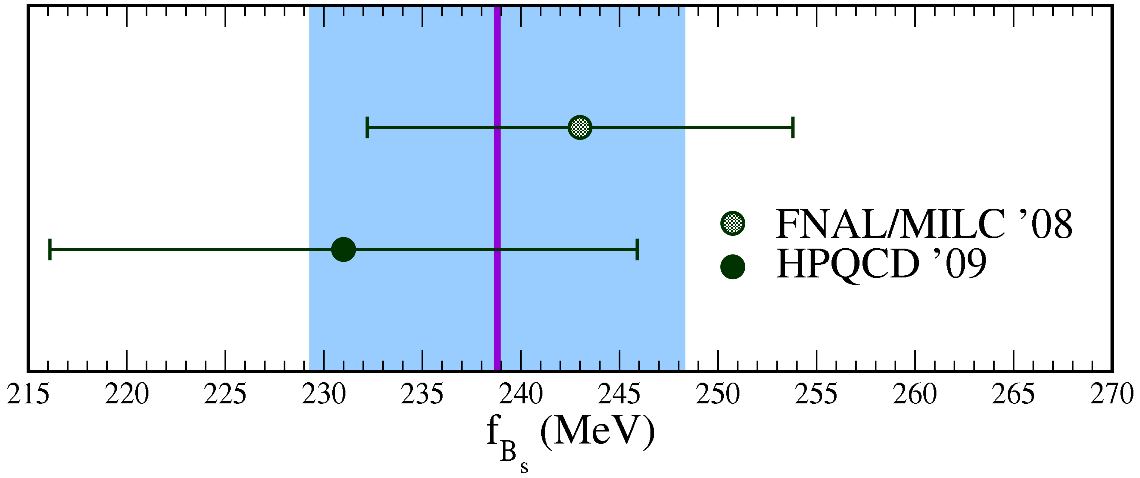


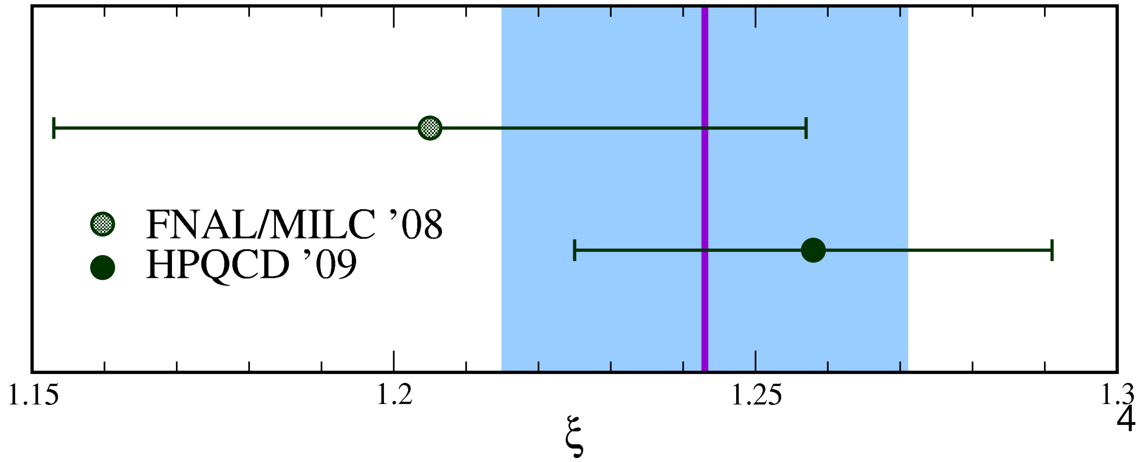
inclusive $|V_{cb}|$ is ~2σ higher



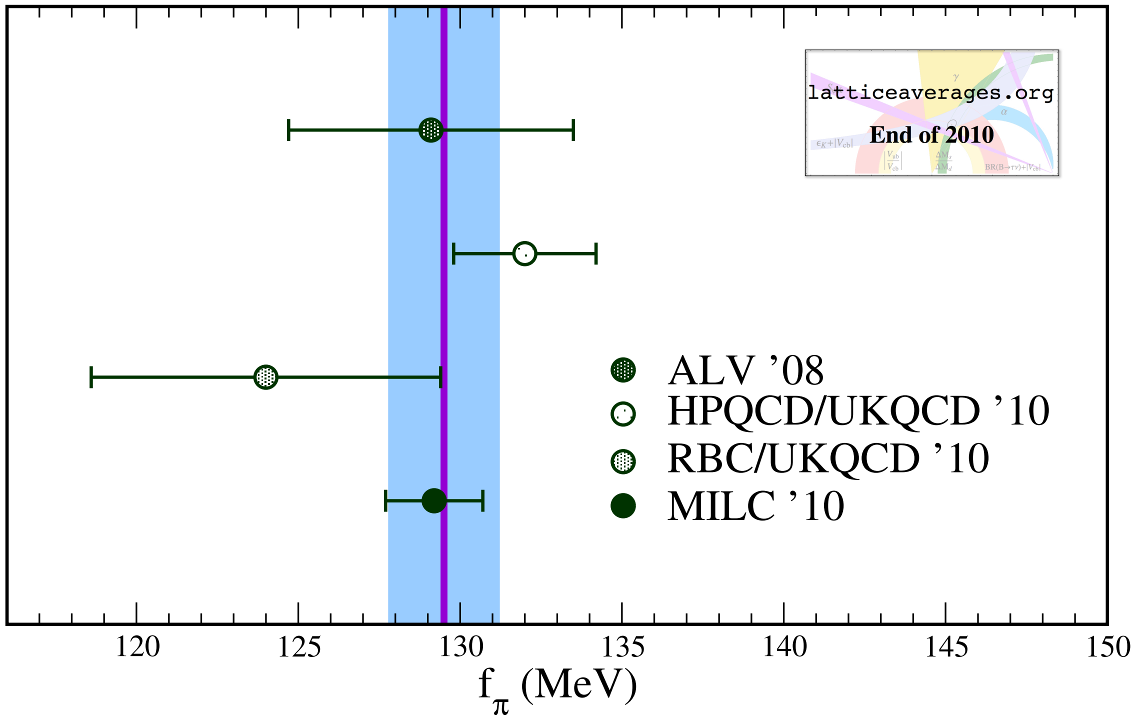
inclusive $|V_{ub}|$ is ~1σ higher

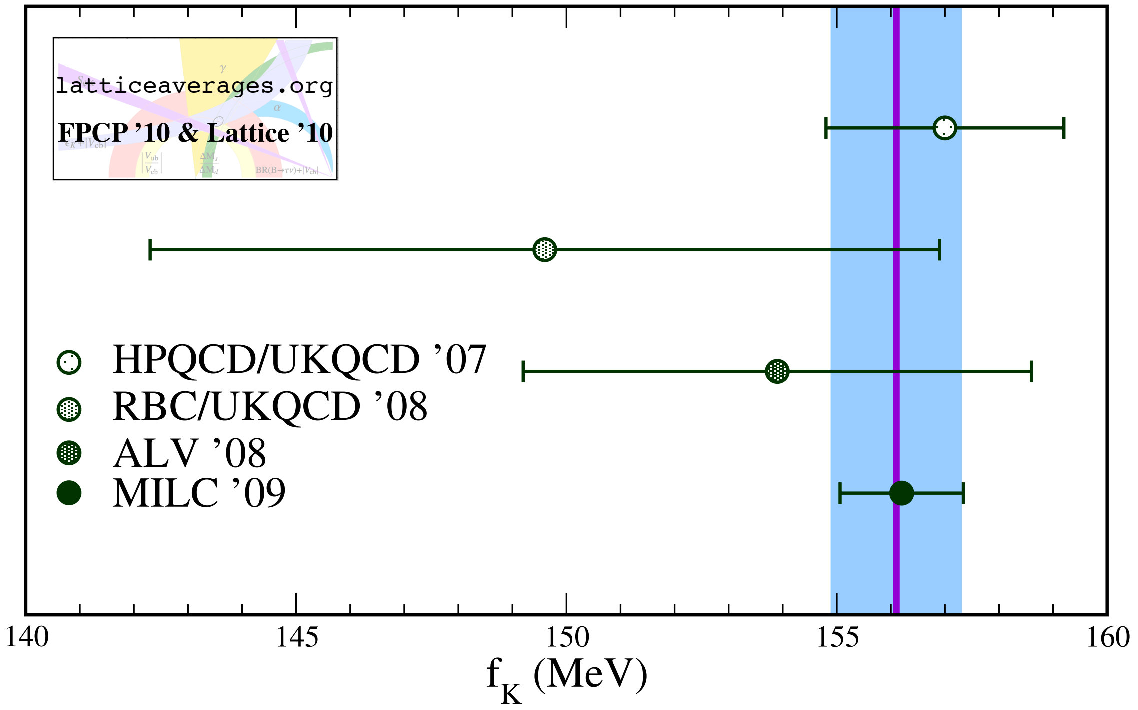




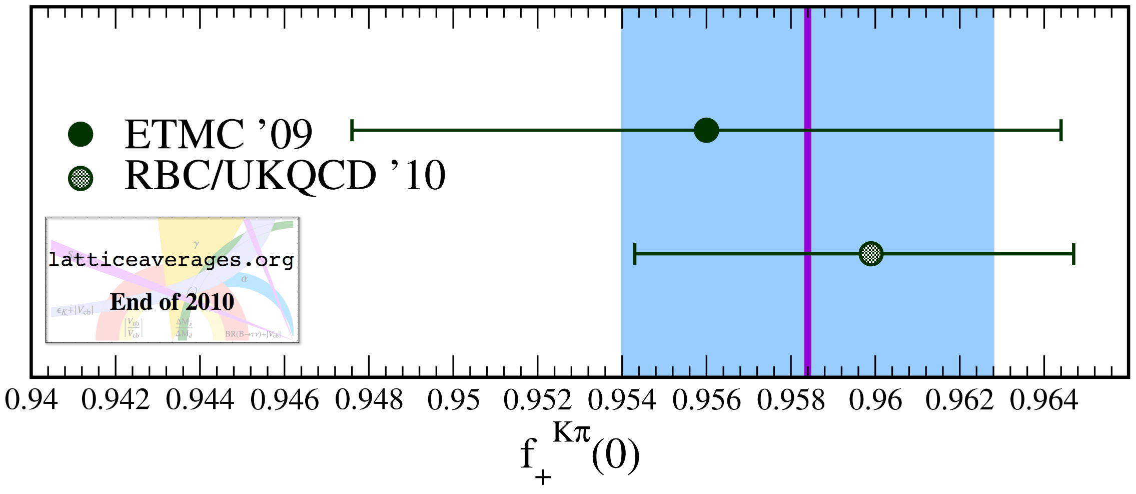


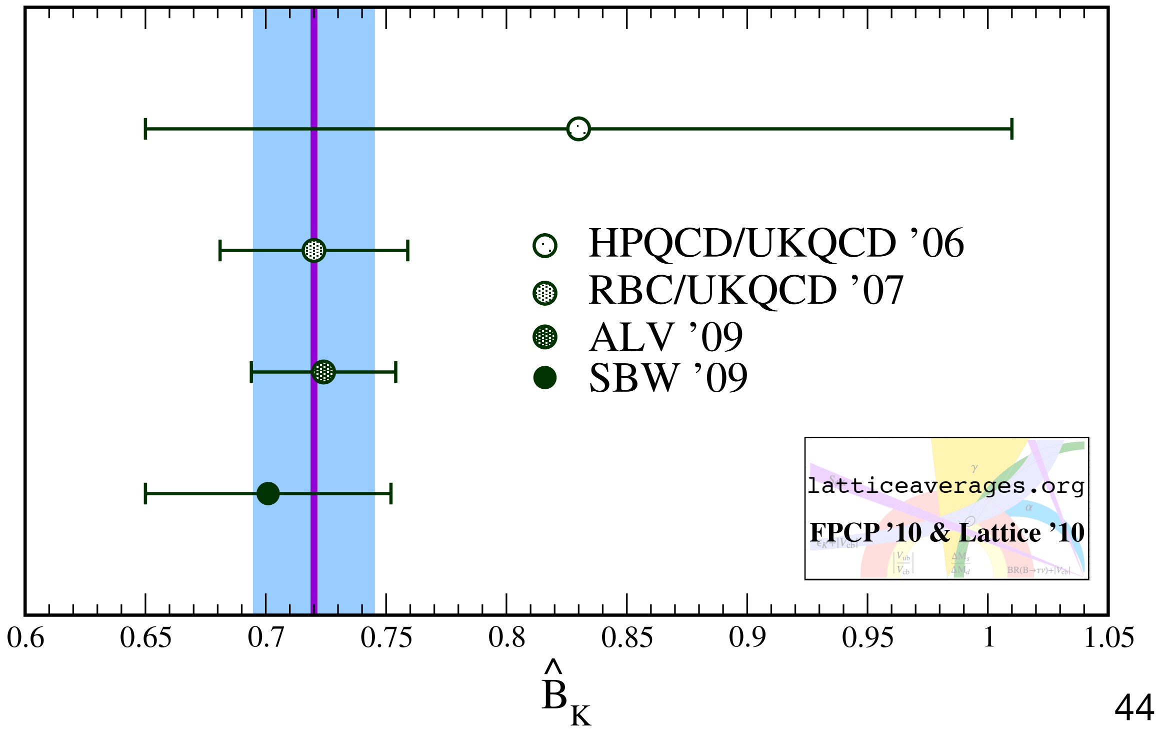
π- & *K*-Meson Averages from Lattice QCD





plots from latticeaverages.org



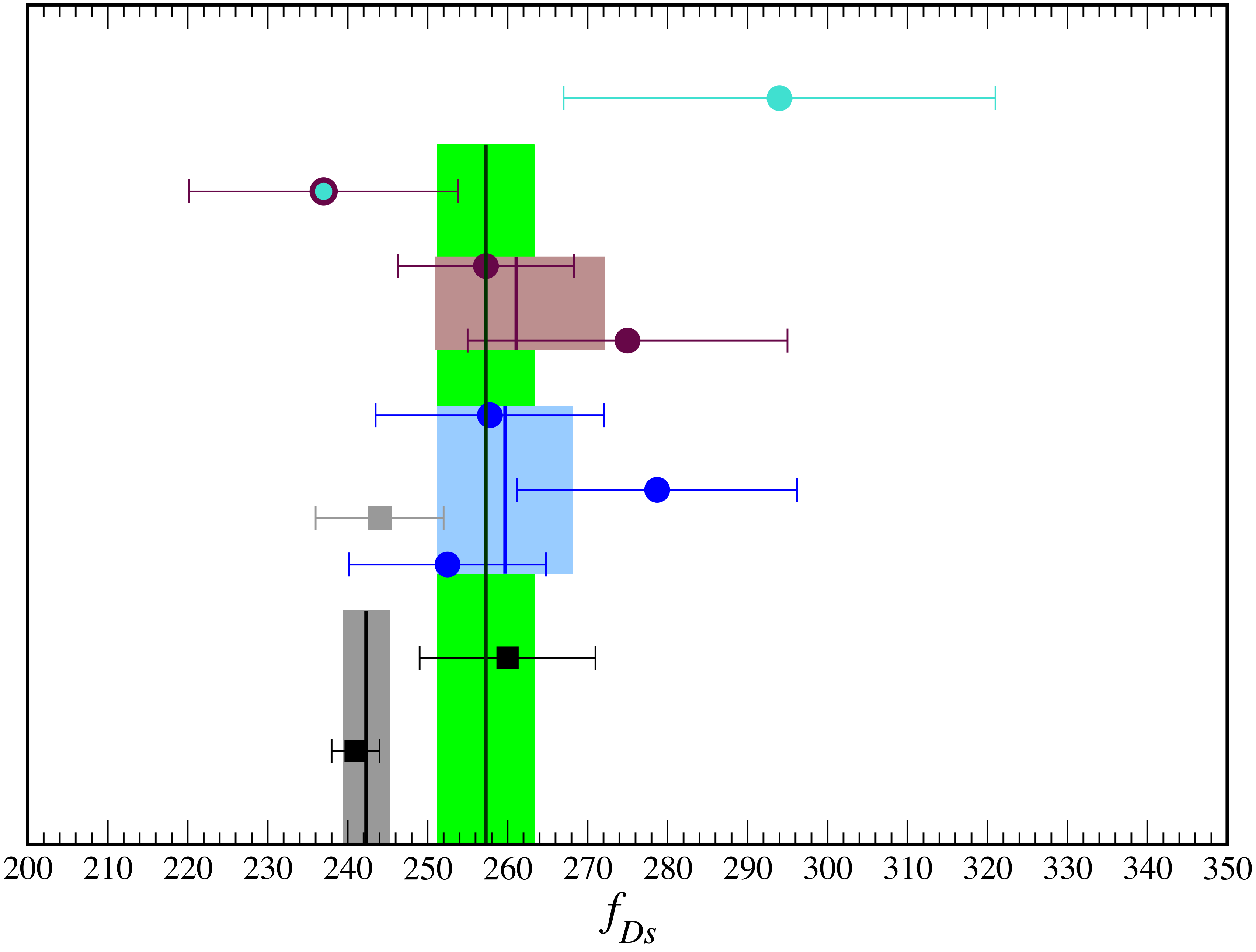


Lessons

- Lattice QCD plays a crucial role for neutral-meson mixing (K, B, B_s).
- Lattice QCD plays a key role in $|V_{\mathrm{us}}|$, $|V_{\mathrm{cs}}|$, $|V_{\mathrm{ub}}/V_{\mathrm{cb}}|$, $|V_{\mathrm{cb}}|$.
- Suite of experiments, pQCD, and lQCD shows that CKM flavor violation and KM CP violation predominates.
- Still room for new physics: tension at 2–3σ level:
  - confidence level of global fit improves more, if NP in kaon mixing [LLV];
  - ε_K band uses corrections of ASK, Ligeti, Nierste [hep-ph/0201071].

Tension in $D_s \to l\nu$?

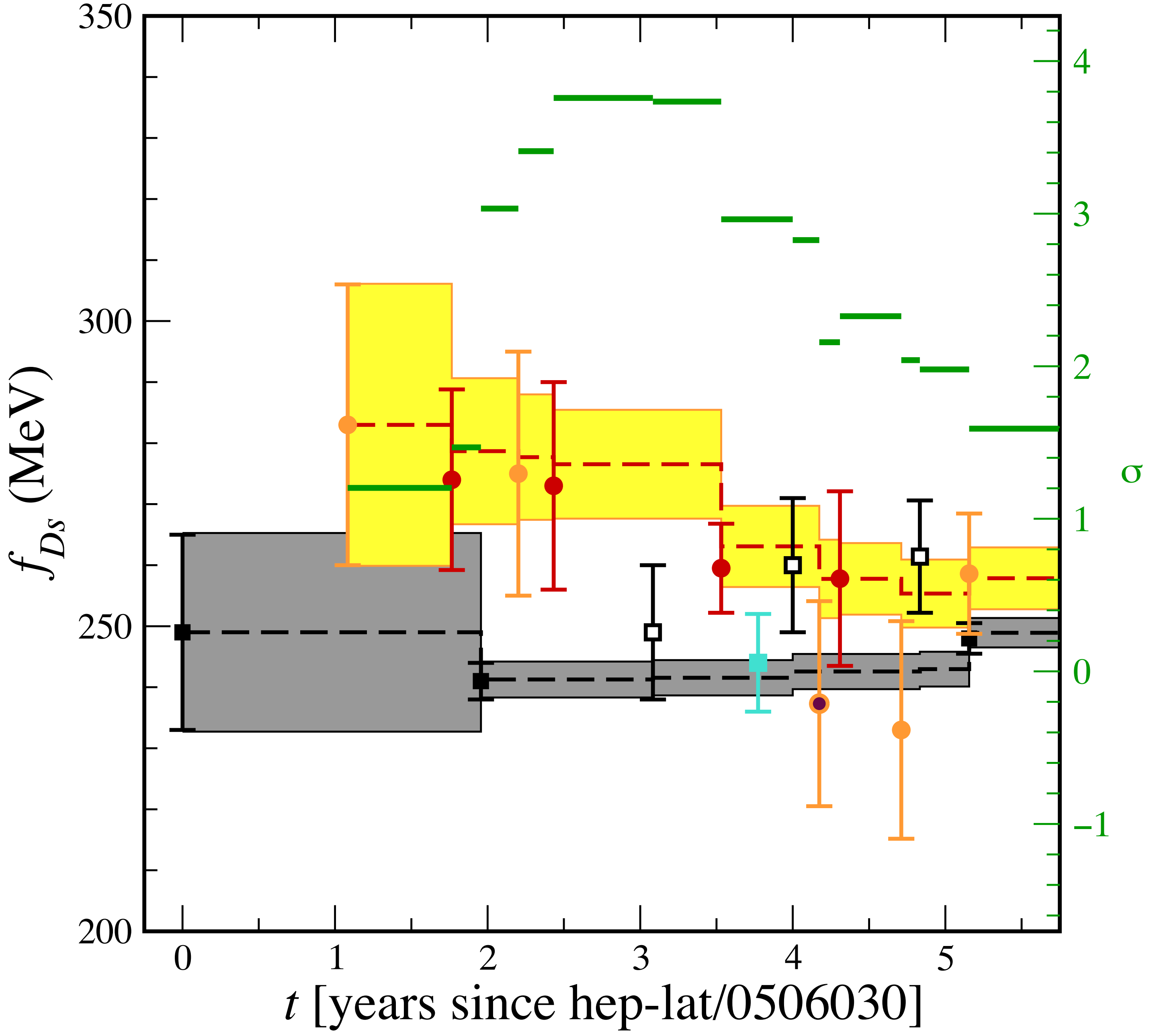
$\mathrm{BR} \propto |f_{Ds} V_{cs}|^2$



- Earlier 3.8σ, now <2σ.
- New physics, *e.g.*, leptoquark?
- w/ $A_{LQ} \sim +0.1 A_{SM}$?
- Dobrescu & ASK arXiv:0803.0512

Tension in $D_s \to l\nu$?

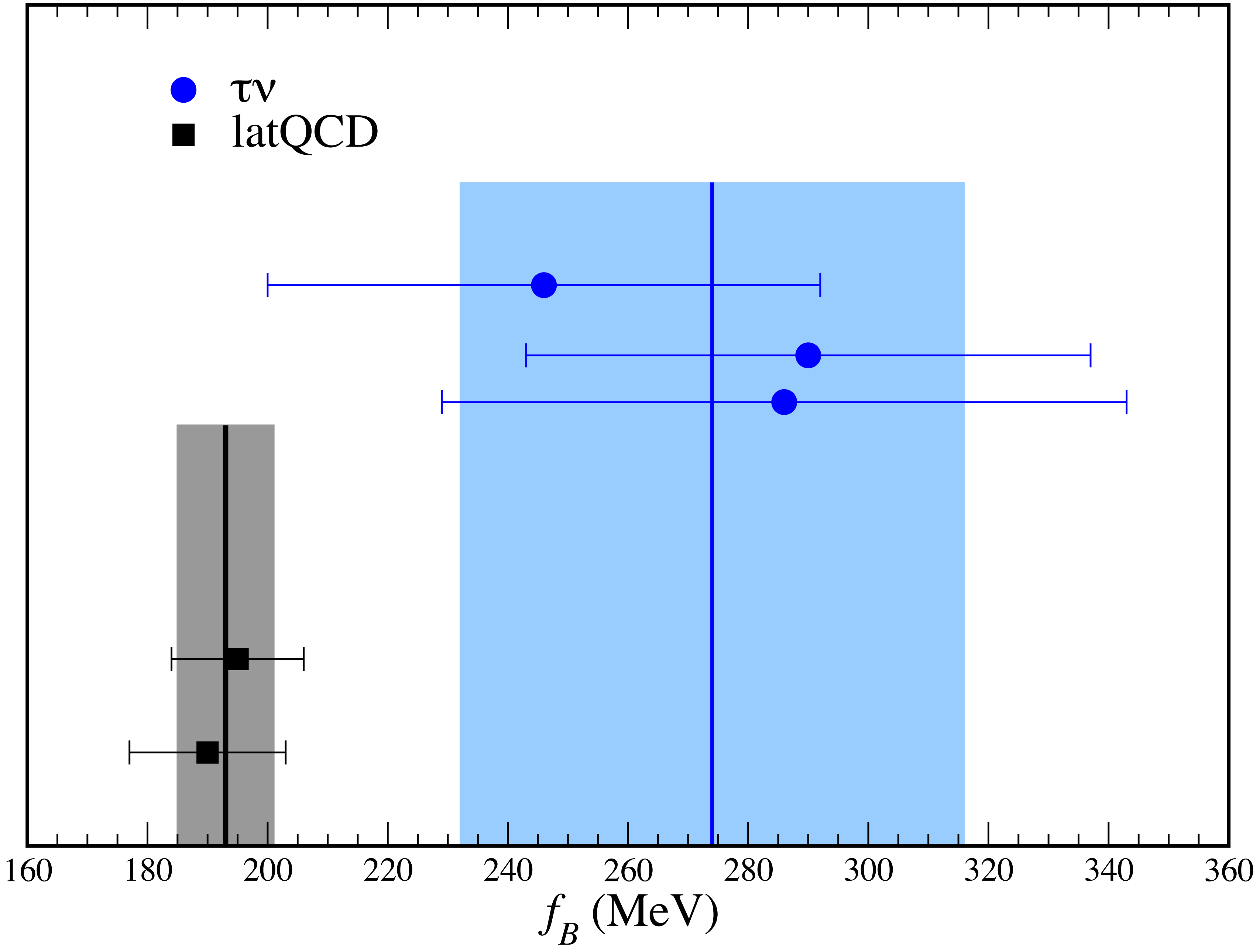
ASK, arXiv:0912.0543



- **Gray**: running lQCD avg.
- **Yellow**: running expt avg.
- **Orange**: BaBar, Belle.
- **Red**: CLEO.
- **Green** (right y axis): running deviation in σ.
- σ is mostly exptl stats.

Tension in $B \to \tau\nu$?

$\mathrm{BR} \propto |f_B V_{\mathrm{ub}}|^2$



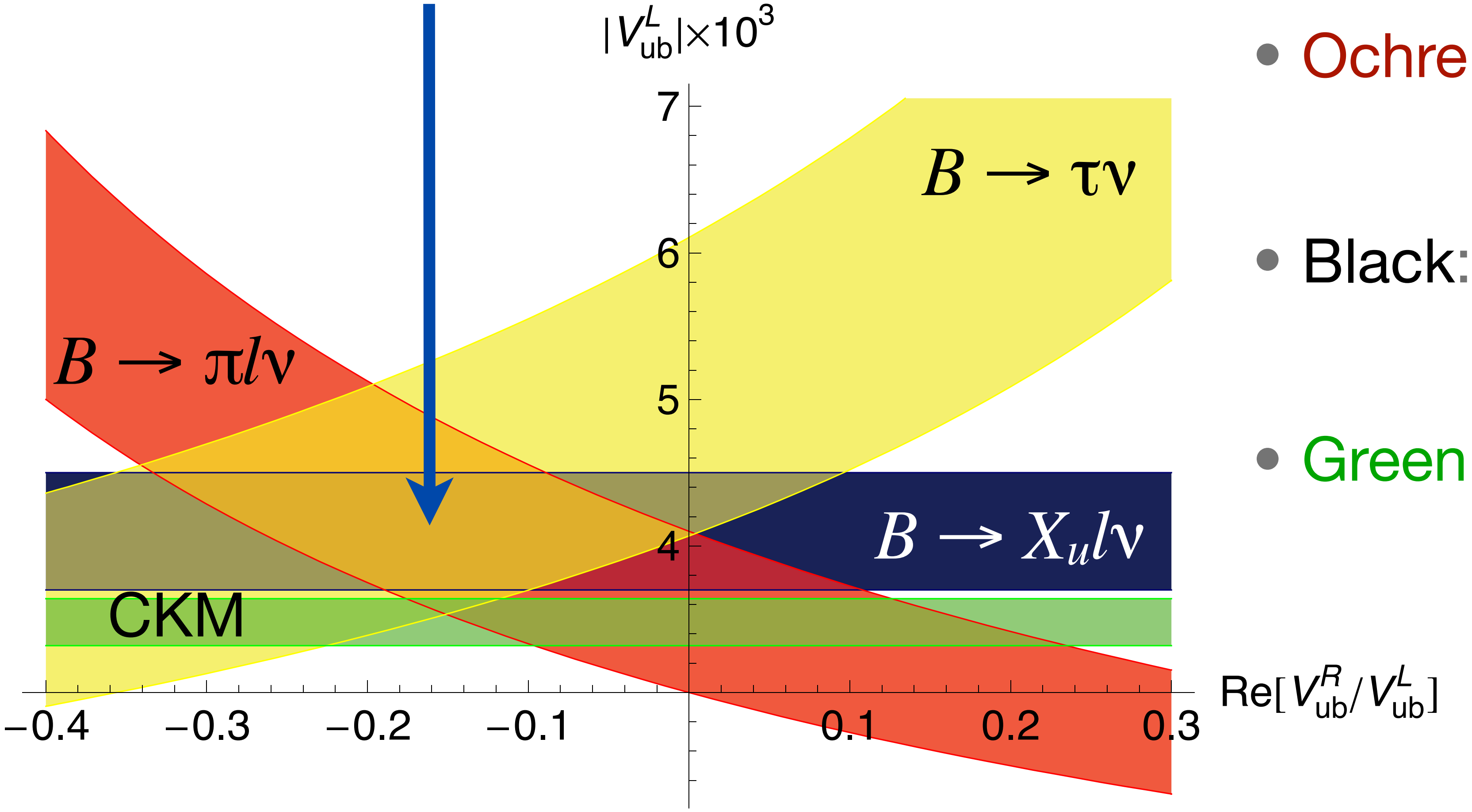
- New physics, *e.g.*, charged Higgs of MSSM?

- w/ $A_{\mathrm{MSSM}} \sim -1.1 A_{\mathrm{SM}}$?

Tension in $b \to u$?

Crivillin, arXiv:0907.2461

- Right-handed currents could explain different "V_{ub}" from exclusive, inclusive, $B \to \tau\nu$.
- Best fit is ~–15% RH current.

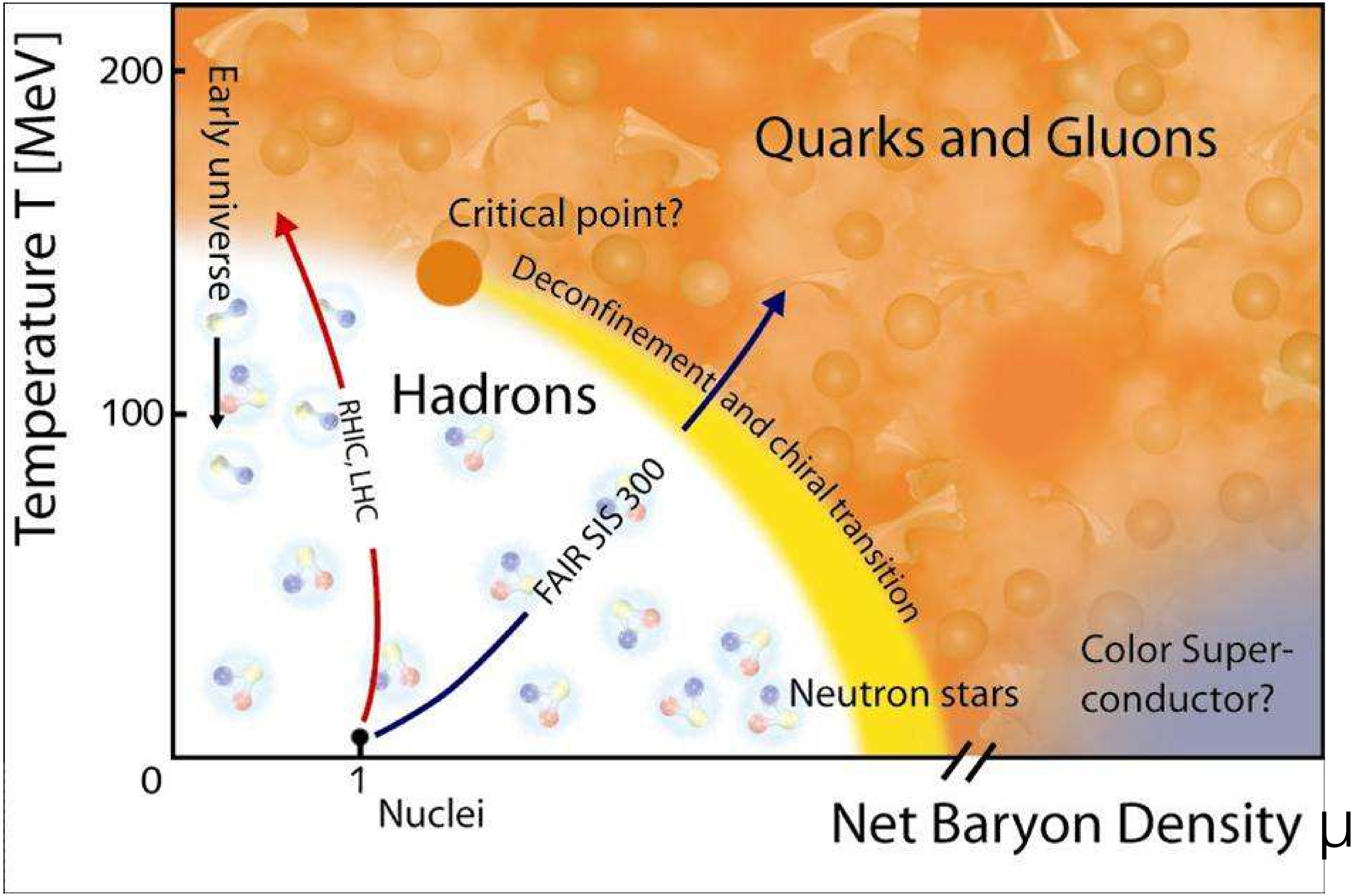


- Denote couplings V_{ubR} & V_{ubL}
- Yellow: $B \to \tau\nu$: $|V_{ubR} - V_{ubL}|^2$
- Ochre: $B \to \pi l\nu$: $|V_{ubR} + V_{ubL}|^2$
- Black: $B \to X_u l\nu$: $|V_{ubR}|^2 + |V_{ubL}|^2$
- Green CKM unitarity.

Thermodynamics

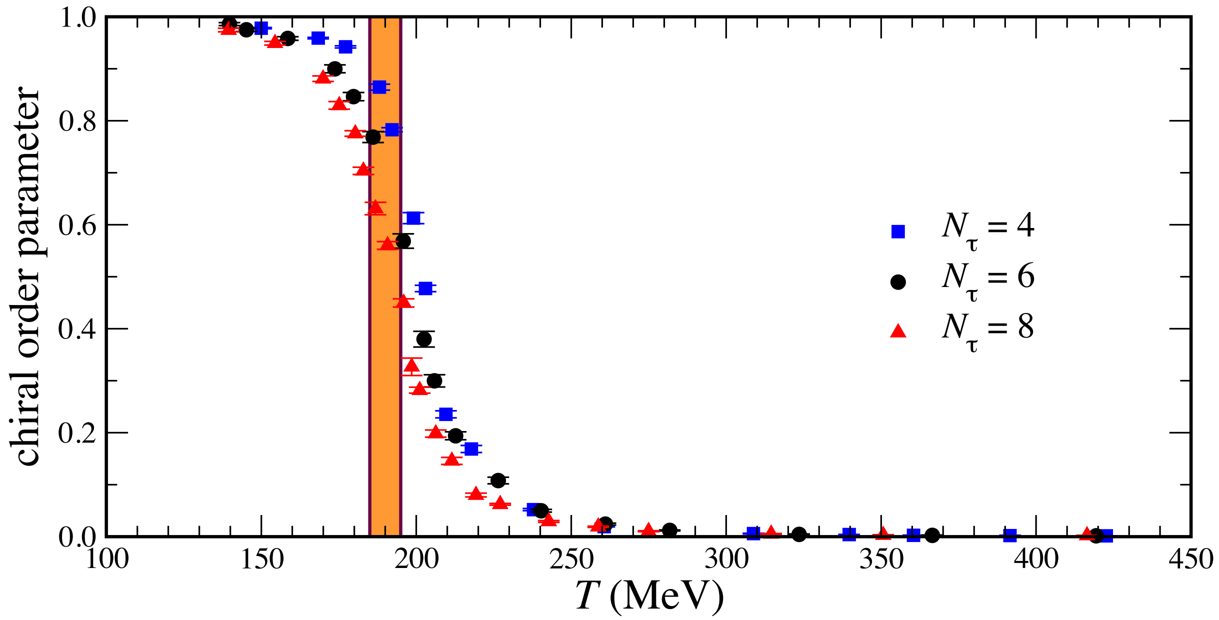
QCD Phase Diagram

CBM Collaboration

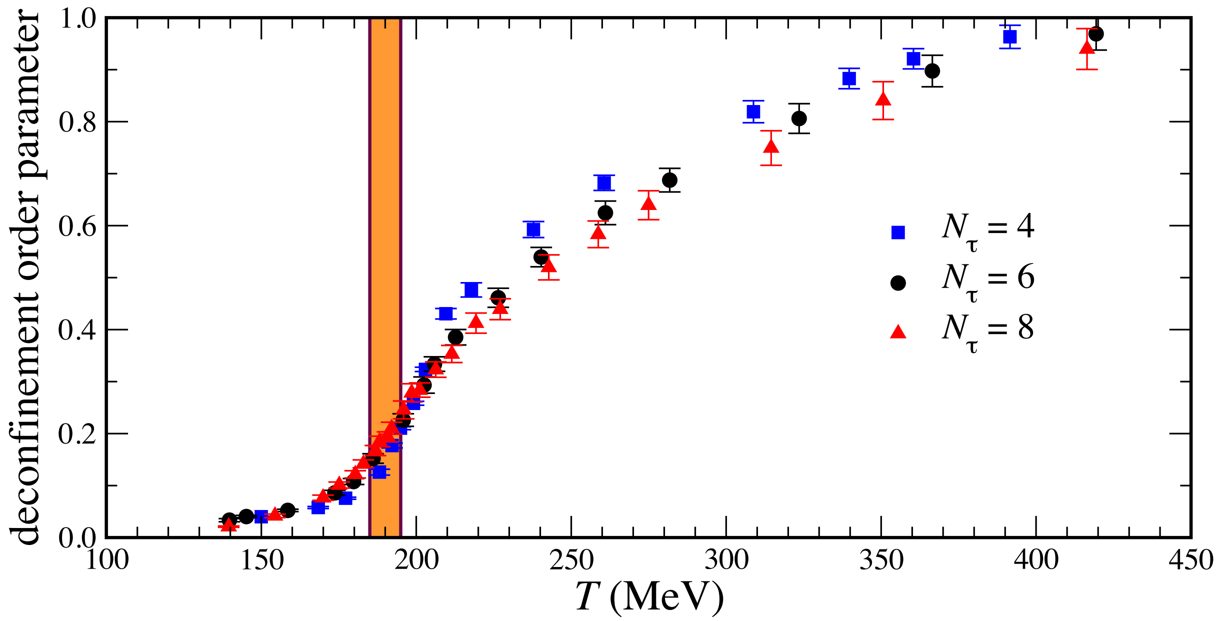


QCD Phase Transition ($\mu = 0$)

Y. Aoki *et al*. *Nature* **443** (2006) 675; A. Bazavov *et al.*, arXiv: 0903.4379



10^{12} K



- Temperature $T = 1/N_\tau a$.
- Smooth crossover to phase, in which the grand canonical average becomes:
    - chirally symmetric;
    - deconfined.
- Same transition temperature: also for other observables, *e.g.,* susceptibilities peak.

Quarks and gluons vs. hadrons

- The thermal average is

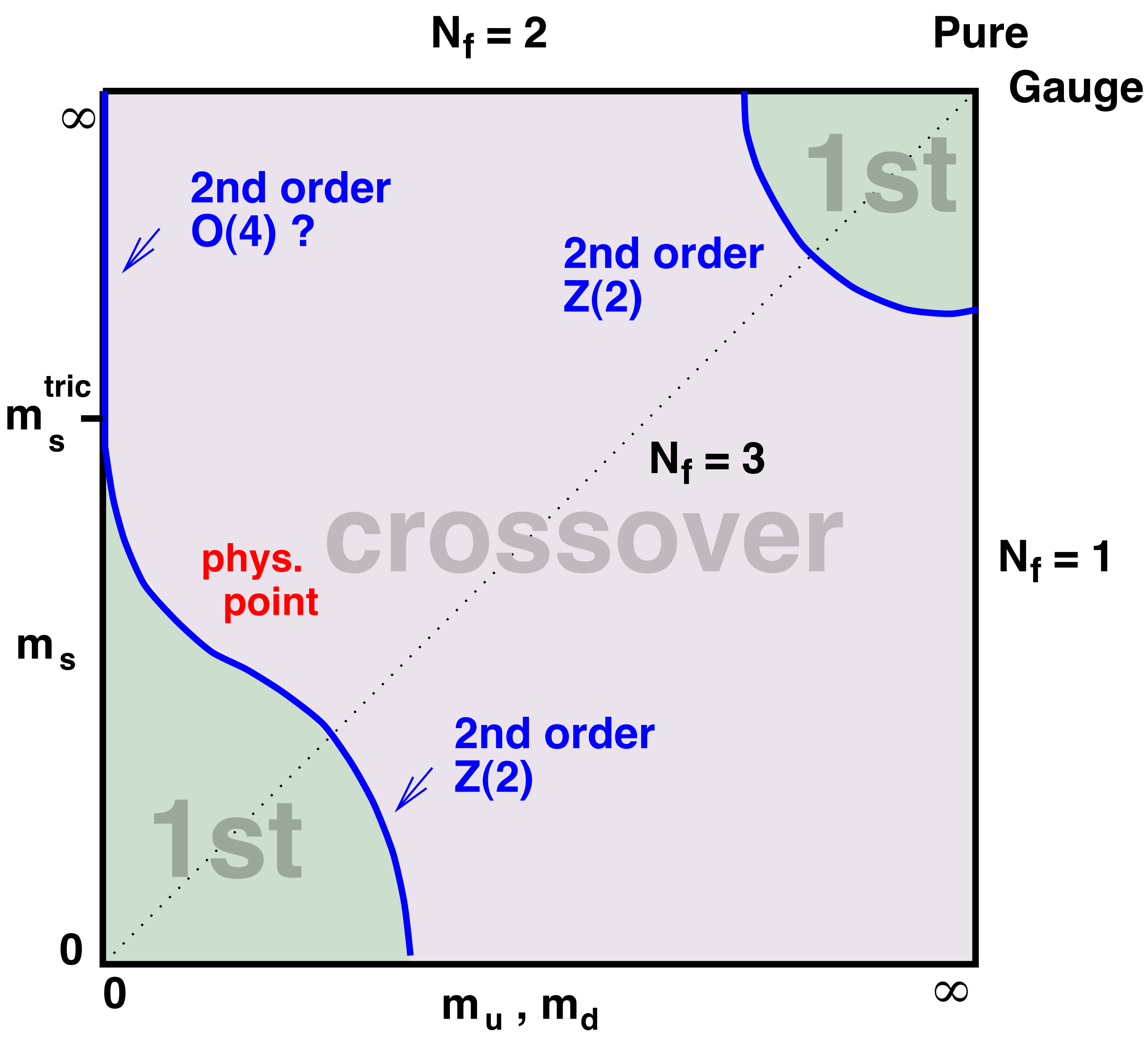
$$\langle \bullet \rangle = \frac{\mathrm{Tr}\left[\bullet\, e^{-\hat{H}/T}\right]}{\mathrm{Tr}\, e^{-\hat{H}/T}}$$

which is as “inclusive” as possible.

- Parton-hadron duality in scattering & decay seems to work once $E > 2\ \mathrm{GeV}$; at such T exchange trace over hadronic states for trace over partons.

- Does not contradict “chiral symmetry restoration” or “deconfinement” at lower temperatures: average includes a state and its chiral partner & and includes states with lots of hadrons.

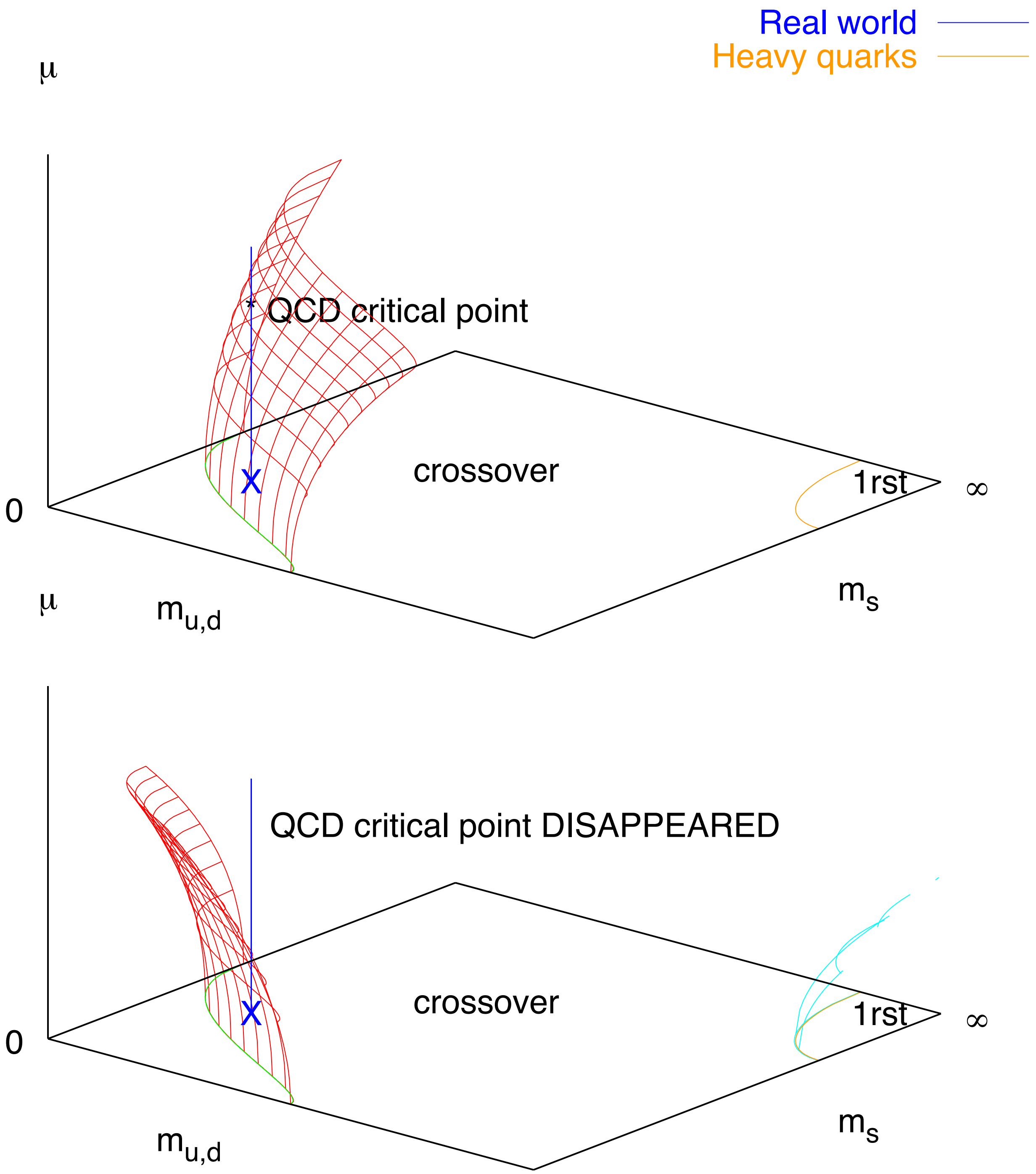
Quark Masses are Key



from de Forcrand & Philipsen
arXiv:0808.1096

- Explicit χSB softens the transition.
- Quark masses are small, but ...
- ... if even smaller, the $\mu = 0$ transition would be second order, or even first order.
- Implications for the early universe.

Equation of State ($\mu \neq 0$)



- Studies limited to $\mu \approx 0$.
- Curvature a matter of controversy.
- Models and qualitative arguments suggest top picture, whence a critical point for some $\mu \neq 0$.
- Several groups find the opposite:
  - a cutoff effect?

Summary and Challenges

Summary: from "QCD *should* work this way" to "QCD *does* work this way"

- Quantitative
  - Precise α_{s}, m_{c}, m_{b}:
  - Precise m_{s}, m_{d}, m_{u}:
  - Hadron masses:
  - CKM V_{cb}, V_{ub}, $V_{td}/V_{ts} \oplus B_K, f_B$:
  - Chiral condensate $\langle \bar{q}q \rangle$:
  - Smooth crossover:

- Qualitative
  - $\mathrm{QCD}_{\mathrm{hadrons}} = \mathrm{QCD}_{\mathrm{partons}}$
  - strong CP is a problem
  - Your mass = E_0/c^2;
  - $\mathrm{Nobel}_{\mathrm{KM}} \oplus$ BSM hints;
  - QCD breaks chiral symmetry;
  - Cooling universe.

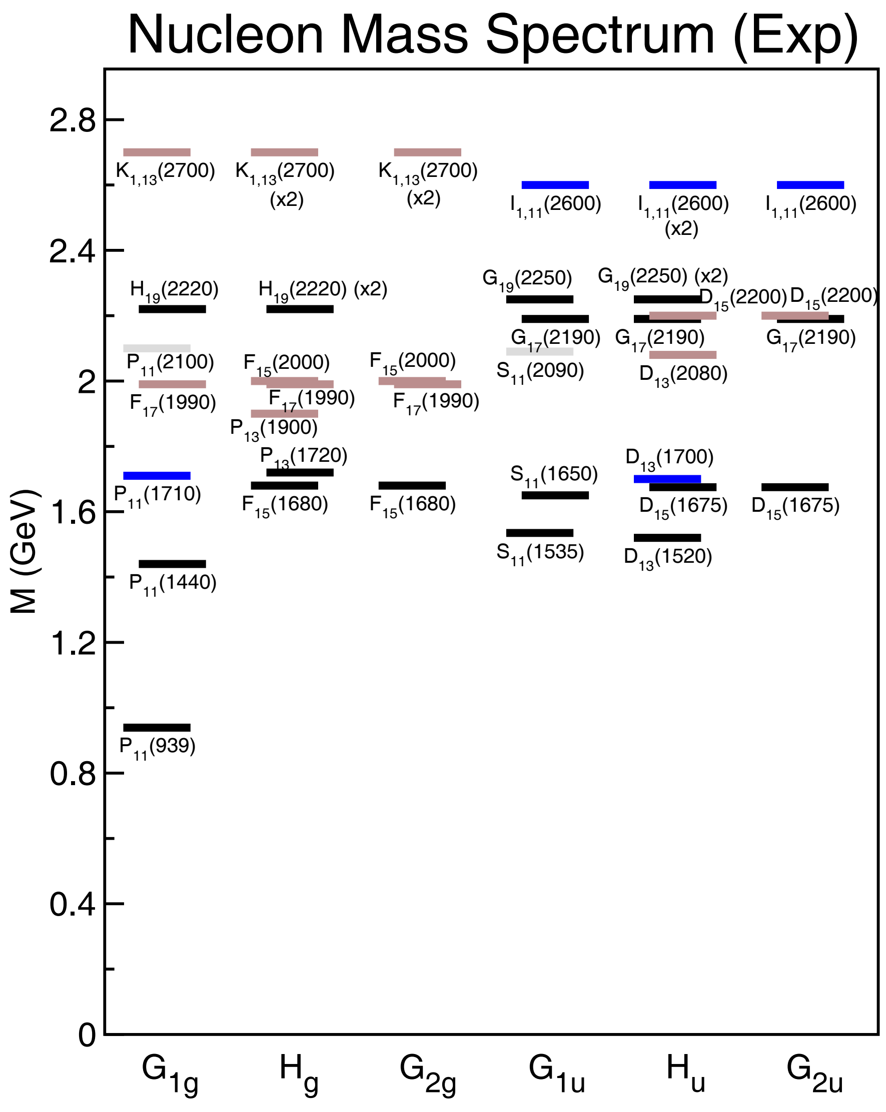
Challenges

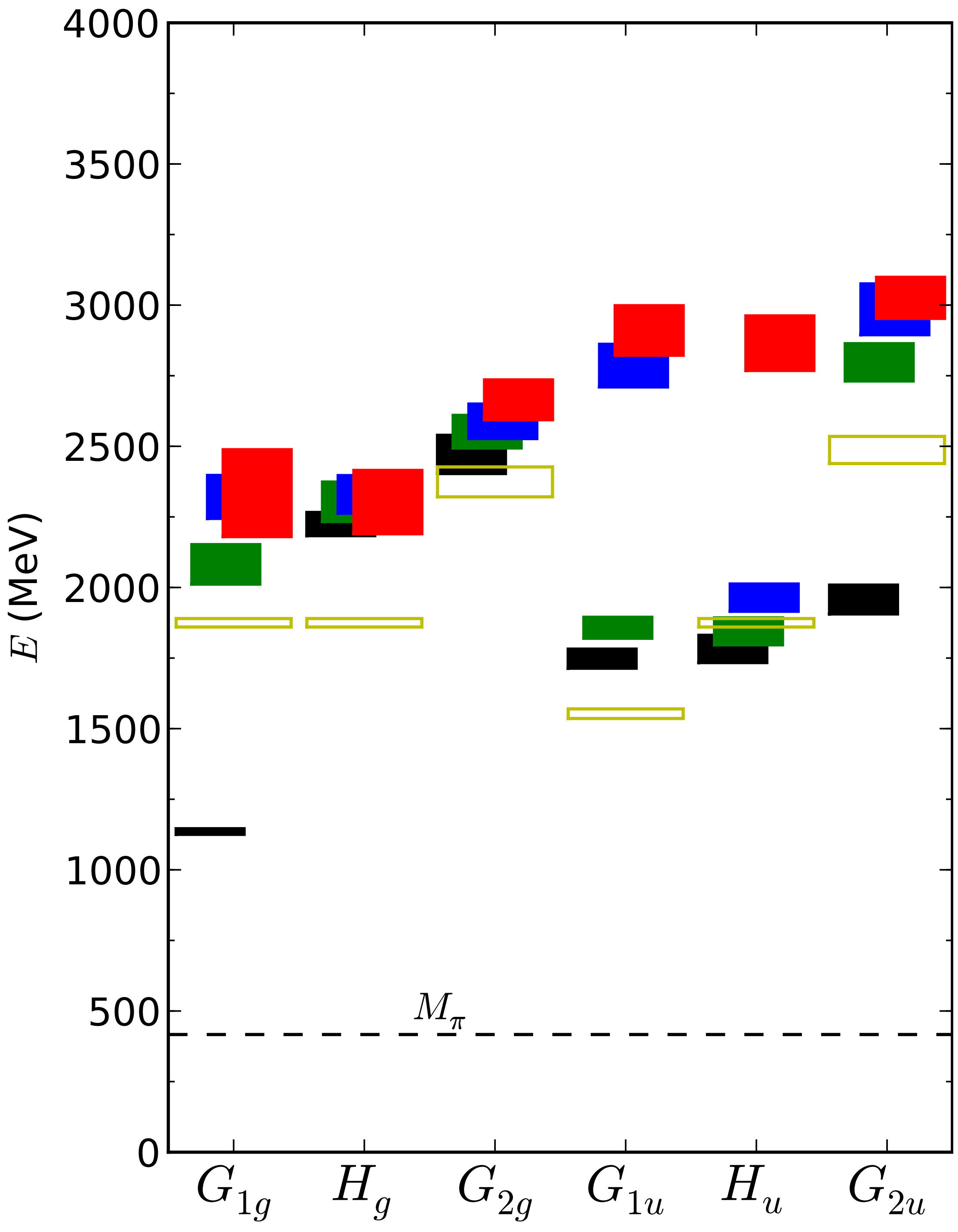
- Particle physics:
    - 1% precision for flavor physics;
    - reliable moments for the nucleon’s gluon distribution;
    - non-QCD gauge theories of electroweak symmetry breaking.
- Nuclear physics:
    - larger chemical potential;
    - excited states;
    - multi-hadron states, mixing, and, soon enough, nuclei—

Excited Baryons

J. Bulava et al., arXiv:0901.0027, arXiv:0907.4516

- Future applications to glueball spectra and mixing.





Atomic Nuclei from QCD

HALQCD, *PRL* **106** (2011) 162002;
NPLQCD, *PRL* **106** (2011) 162001, arXiv:1103.2821

- The simplest nucleus is the deuteron, pn.
- Barely bound: fine tuning of QCD parameters.
- Do similar dibaryons exist? Conjectured $H = \Lambda\Lambda$.
- Recent lattice QCD calculations, with slightly unphysical quark masses, suggest that the H is indeed bound.