Lorentz violation in 5D Susy model.

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Motivation

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- Supersymmetry.

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- Supersymmetry.
- Extra Dimensions.
- Other extensions to Standard Model.

Outline.

Lorentz-Violating Extension of the Standard Model

Colladay and Kostelecky found an extension of the Standard Model without Lorentz invariance but it has gauge invariance, energy momentum conservation, and covariance under observer rotations and boosts, while covariance under particle rotations and boosts is broken.

hep-ph/9809521

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Their model uses a method intrinsically perturbative, that is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'.$$

An example for $\mathcal{L}'$ is

$$\mathcal{L}' \supset \frac{\lambda}{M^k}\langle T\rangle \cdot \bar{\psi}\Gamma\,(i\partial)^k\,\psi \supset \{a_\mu\bar{\psi}\gamma^\mu\psi,\ b_\mu\bar{\psi}\gamma_5\gamma^\mu\psi\},$$

where

- λ is a dimensionless coupling constant
- $(i\partial)^k$ represents a four derivative acting in some combination on the fermion fields.
- Γ represents some gamma-matrix structure.
- $\langle T\rangle$ is a nonzero expectation value for one Lorentz tensor.

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Examples of $\mathcal{L}'$:

$$-b\,(\partial_5\phi)\,(\partial_5\phi)\,,$$
$$ia\bar{\Psi}\gamma^5\partial_5\Psi,$$
$$c\left(F_{\mu 5}F^{\mu 5}+F_{5\mu}F^{\mu 5}\right).$$

Scalar field

Taking into account the action

$$\begin{aligned} S &= \int dx^{\mu} dy \left[(\partial_M \Phi)^{\dagger} \left(\partial^M \Phi \right) - a (\partial_5 \Phi)^{\dagger} (\partial_5 \Phi) \right] \\ &= \int dx^{\mu} dy \left[(\partial_{\mu} \Phi)^{\dagger} (\partial^{\mu} \Phi) + (1 + a) \left(\partial^5 \Phi \right)^{\dagger} (\partial_5 \Phi) \right]. \end{aligned}$$

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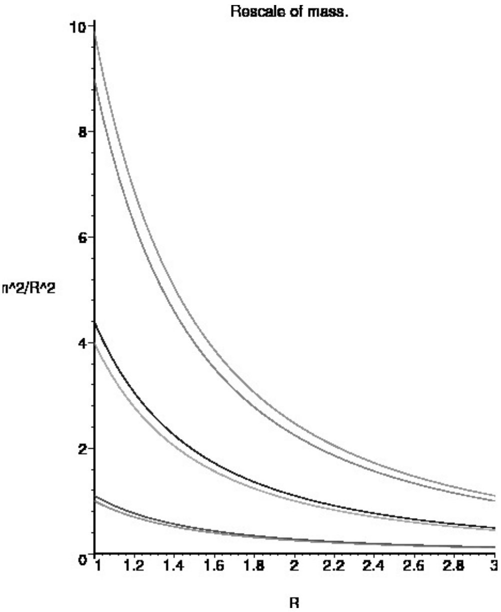
$$\begin{aligned} S &= \int dx^{\mu} dy \left[(\partial_M \Phi)^{\dagger} \left(\partial^M \Phi \right) - a \left(\partial_5 \Phi \right)^{\dagger} \left(\partial_5 \Phi \right) \right] \\ &= \int dx^{\mu} dy \left[(\partial_\mu \Phi)^{\dagger} \left(\partial^\mu \Phi \right) + (1+a) \left(\partial^5 \Phi \right)^{\dagger} \left(\partial_5 \Phi \right) \right]. \end{aligned}$$

The Kaluza-Klein decomposition

$$\Phi = \sum_n \phi_n(x) \chi_n(y),$$

and the use to orbifolds leads to

$$m_n^2 = (1+a) \frac{n^2}{R^2}.$$



Considering $a = 0.1$.

Compactification on a circle.

Now, taking the restriction

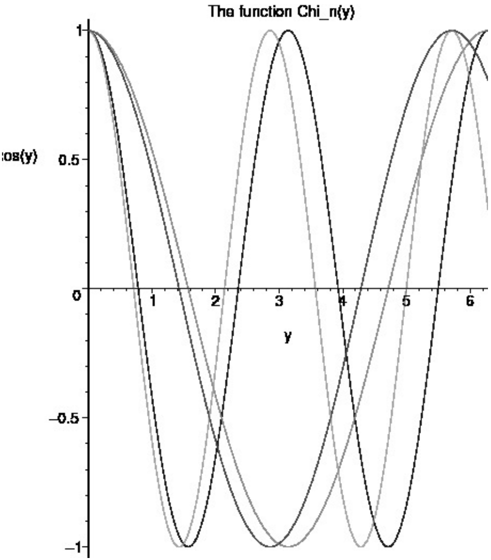
$$y \sim y + \frac{2\pi R}{\omega},$$

and considering

$$S = \int dx^{\mu} dy \left[(\partial_M \Phi)^{\dagger} \left(\partial^M \Phi \right) \right], \quad \Phi = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{2\pi R}} \phi_n(x) \exp(i\omega y \frac{n}{R}),$$

then

$$m_n^2 = \frac{n^2}{R^2} \omega^2.$$



Shifting in the periodic conditions.

Convention to Gamma matrices.

4D Dirac Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix},$$

which satisfy the property

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}.$$

5D Dirac Matrices

$$\Gamma^\mu = \gamma^\mu, \qquad \Gamma^5 = i\gamma^5 = i\begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \qquad \{\Gamma^M, \Gamma^N\} = 2\eta^{MN}.$$

Modified Dirac action.

$$
\begin{aligned}
S &= \int dx^{\mu} dy \left[\frac{i}{2} \bar{\Psi} \Gamma^{M} \partial_{M} \Psi - \frac{1}{2} b \bar{\Psi} \gamma^{5} \partial_{5} \Psi \right] \\
&= \int dx^{\mu} dy \left[\frac{i}{2} \bar{\Psi} \left(\Gamma^{M} \partial_{M} + b \Gamma^{5} \partial_{5} \right) \Psi \right].
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S = \int dx^\mu dy \left(\frac{i}{2} \left(\psi \sigma^\mu \partial_\mu \bar{\psi} + \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi \right) - \frac{1+b}{2} \left(\psi \partial_5 \chi - \bar{\chi} \partial_5 \bar{\psi} \right) \right)
$$

Building the model.

- Consider the sum of Dirac action (Ψ) and two scalar field actions ($\phi_1,\ \phi_2$) in 5D which include terms on Lorentz Violation (constant b for Dirac action and a for scalar field actions).

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- Impose a $SU(2)$ symmetry to the scalar fields.
- Rewrite Dirac spinors as symplectic-Majorana spinors, which have the properties

$$\psi^i = c^{ij} C \bar{\psi}^{jT}, \qquad c = -i\sigma^2, \qquad C = \begin{pmatrix} c & 0 \\ 0 & c. \end{pmatrix},$$

$$\bar{\psi}^i \Gamma^M \ldots \Gamma^P \chi^j = -c^{ik} c^{jl} \bar{\chi}^l \Gamma^P \ldots \Gamma^M \psi^k.$$

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- Search the necessary conditions to found the supersymmetry transformations and relations between a and b, starting with transformation:

$$\delta_\zeta \phi^i = \epsilon^{ij} \bar{\zeta}_j \Psi.$$

Check this

$$\left(\Gamma^M\partial_M + b\Gamma^5\partial_5\right)\left(\Gamma^N\partial_N + b\Gamma^5\partial_5\right) = \partial^M\partial_M + \left(b^2+2b\right)\partial^5\partial_5$$
$$= \partial^\mu\partial_\mu + (1+b)^2\,\partial^5\partial_5.$$

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$$= \partial^\mu \partial_\mu + (1+b)^2 \partial^5 \partial_5.$$

"A modified Klein-Gordon operator."

$$S_{model} = \int dx^{\mu} dy \frac{1}{2} \left((\partial_M \Phi_i)^\dagger \left(\partial^M \Phi^i \right) - a \left(\partial_5 \Phi_i \right)^\dagger \left(\partial_5 \Phi^i \right) \right)$$
$$+ \int dx^{\mu} dy \left[\frac{i}{2} \bar{\Psi} \left(\Gamma^M \partial_M + b \Gamma^5 \partial_5 \right) \Psi + F_i^\dagger F^i \right].$$

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\end{aligned}
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The transformation of Dirac field is

$$
\delta_{\zeta} \Psi = i \epsilon_{ij} \left(\Gamma^N \partial_N + b \Gamma^5 \partial_5 \right) \bar{\zeta}^j \phi^i + \epsilon_{ij} \zeta^i F^j,
$$

iff

$$
a^2 = b^2 + 2b.
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iff

$$a^2 = b^2 + 2b.$$

Also

$$\delta_\zeta F^i = i\epsilon^{ij}\bar{\zeta}_j\left(\Gamma^N \partial_N + b\Gamma^5 \partial_5\right)\Psi.$$

Superfield formalism.

Taking into account two families of chiral superfields

$$\begin{aligned} \Phi(y,\theta) &= \phi_1(y) + \sqrt{2}\theta\psi_L(y) + \theta^2 F(y), \\ \Phi^c(y,\theta) &= \phi_2(y) + \sqrt{2}\theta\psi_R(y) + \theta^2 F^c(y). \end{aligned}$$

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Compute the terms

$$\begin{aligned}\Phi^c\partial_5\Phi &\supset \theta^2\left(F^c\partial_5\phi_1 + \phi_2\partial_5 F - \psi_R\partial_5\psi_L\right),\\ \bar{\Phi}^c\partial_5\bar{\Phi} &\supset \bar{\theta}^2\left(F^{c\dagger}\partial_5\phi_1^\dagger + \phi_2^\dagger\partial_5 F^\dagger - \bar{\psi}_R\partial_5\bar{\psi}_L\right).\end{aligned}$$

The equations of motion for F fields leads to

$$\mathcal{L}_F = -\partial_5 \phi_1^\dagger \partial_5 \phi_1 + \partial_5 \left(\phi_2 \partial_5 \phi_2^\dagger + \phi_2^\dagger \partial_5 \phi_2 \right) - \partial_5 \phi_2^\dagger \partial_5 \phi_2.$$

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So, the supersymmetry action in 5D can be written as

$$S_5^{Hyp.} = \int d^5x \left\{ \int d^4\theta \left(\bar{\Phi}\Phi + \bar{\Phi}^c \Phi^c \right) + \int d^2\theta \Phi^c \partial_5 \Phi + \int d^2\bar{\theta} \bar{\Phi}^c \partial_5 \bar{\Phi} \right\} .$$

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So, next terms arise

$$-b^2\left(\partial_5\phi_2^\dagger\partial_5\phi_2 + \partial_5\phi_1^\dagger\partial_5\phi_1\right),$$
$$-b\left(\psi_R\partial_5\psi_L + \bar{\psi}_R\partial_5\bar{\psi}_L\right).$$

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The same relationship for the coupling constant.

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- These models require a deformation in the supersymmetric generators and the use of the symplectic-Majoran spinors.
- The superfield formalism for the free theory requires another term, the Kähler potential is not enough.