UNIVERSIDAD DE GUANAJUATO
DEPARTAMENTO DE FISICA

M. Sabido

Nonconmmutativity and Λ

Work in progress in collaboration S. Pérez-Payán, E. Mena and C. Yee

XIII Mexican Workshop on Particles and Fields.

DPyC
DIVISIÓN DE PARTÍCULAS Y CAMPOS

October 24, 2011

León, Gto.

PLAN OF THE TALK

INTRODUCTION, MOTIVATION

NC-GRAVITY

NC-COSMOLOGY

NC AND LAMBDA

FINAL REMARKS

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Unfortunately on the gravity side of things, the story has been more complex.

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From these equations we can write the noncommutative lagrangian and the noncommutative theory.

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(O. Obregon, H.Compean, C. Ramirez and **M.S.**, PRD 68 (2003) .

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Noncommutative Quantum Cosmology

H. Compeán, O.Obregon and C. Ramirez PRL 88 (2002)

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(L. Mezincescu ArXiv:hep-th\0007046)

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This can be constructed from Hamiltonian Manifolds (W. Guzman,M.S., J. Socorro PLB 2011)

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$$i_{X_H}\omega_c = -dH \Rightarrow \begin{array}{c} \frac{dq^i}{dt} = \frac{\partial H}{\partial p_i}, \\ \frac{dp_j}{dt} = -\frac{\partial H}{\partial p^j}. \end{array}$$

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This is an effective noncommutativity between the minisuperspace fields, and is similar to the modification used in noncommutative quantum mechanics. H. Compean, O. Obregón and C. Ramírez PRL 88 (2002), W. Guzmán, **M.S**, J. Socorro Phys. Rev D (2007), W. Guzmán, M.S, J. Socorro, Submitted PLB(2010))

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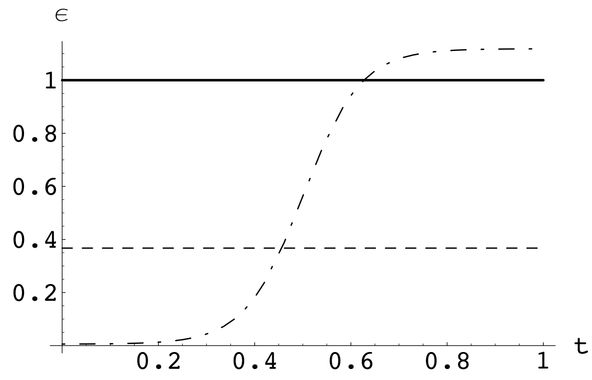

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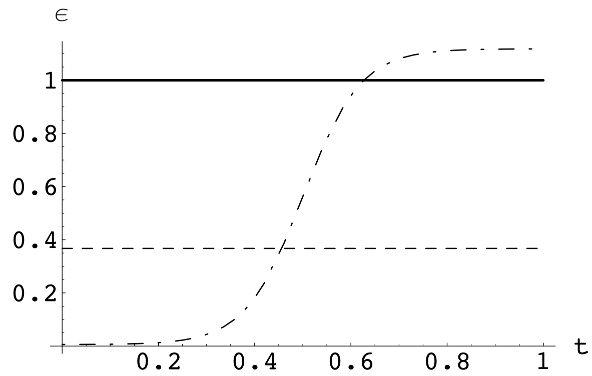

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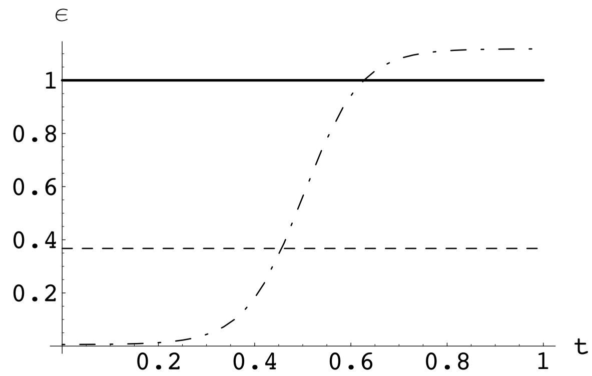

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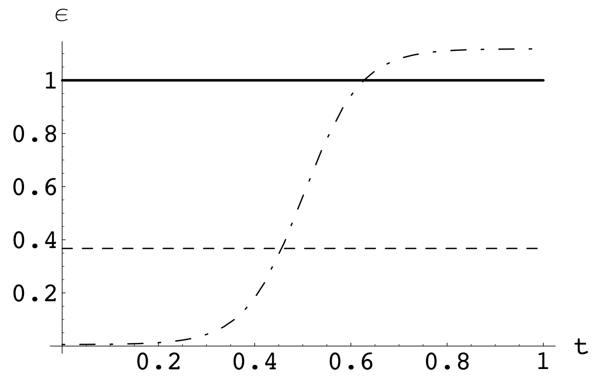

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$$\hat{x} = x + \frac{\theta}{2}p_y, \qquad \hat{y} = y - \frac{\theta}{2}p_x,$$

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$$\widehat{H} = \frac{1}{2}\left\{\hat{p}_x^2 - \hat{p}_y^2 + \omega_1^2(\hat{x}\hat{p}_y + \hat{y}\hat{p}_x) + \omega_2^2(\hat{x}^2 - \hat{y}^2)\right\}.$$

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$$\dot{x} = \{x, \widehat{H}\} = \frac{1}{2}[2\hat{p}_x + \omega_1^2 y], \quad \dot{y} = \{y, \widehat{H}\} = \frac{1}{2}[-2\hat{p}_y + \omega_1^2 x],$$

$$\dot{\hat{p}}_x = \{\hat{p}_x, \widehat{H}\} = \frac{1}{2}[-\omega_1^2\hat{p}_y - 2\omega_2^2 x], \quad \dot{\hat{p}}_y = \{\hat{p}_y, \widehat{H}\} = \frac{1}{2}[-\omega_1^2\hat{p}_x + 2\omega_2^2 y],$$

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$$\ddot{\eta} - \omega_1^2 \dot{\eta} + \frac{1}{4}(4\omega_2^2 + \omega_1^4)\eta = 0,$$
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This equations are very easy to solve for any value of Λ, θ and β.

Let analyze the case when the cosmological constant is zero, as well as the parameter θ. In this case the solution is very simple, in particular the scale factor is

$$a^3(t) = V_0 \cosh^2\left(\frac{1}{4}t\beta\right),$$

Comparing the to models we arrive to the relationship

$$\Lambda \sim \beta^2.$$

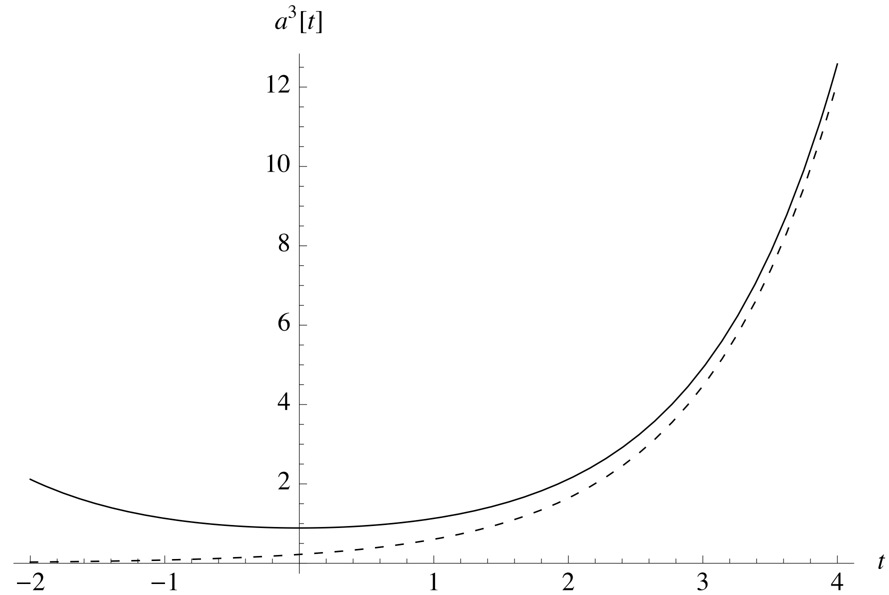

Figure 1: Dynamics of the phase space deformed model for the values $X_0 = Y_0 = 1, \delta_2 = \delta 1 = 0, \omega = 0and\beta = 1$. Thesolidlinecorrespondstothe volume of the universe, calculated with the noncommutative model. The dotted line corresponds to the volume of the de Sitter spacetime. For large values of t the behavior is the same.

NONCOMMUTATIVITY AND LAMBDA

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$$H = \left\{ \left(\hat{p}_u - \frac{2(\beta - \theta\omega^2)}{4 - \omega^2\theta^2} \hat{v} \right)^2 - \left(\hat{p}_v + \frac{2(\beta - \theta\omega^2)}{4 - \omega^2\theta^2} \hat{u} \right)^2 + \left(\frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2} \right) \hat{u}^2 - \left(\frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2} \right) \hat{v}^2 \right\}$$

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$$\left.+\left(\frac{4(\beta-\theta\omega^2)^2}{(4-\omega^2\theta^2)^2}+\frac{4(\omega^2-\beta^2/4)}{4-\omega^2\theta^2}\right)\hat{u}^2-\left(\frac{4(\beta-\theta\omega^2)^2}{(4-\omega^2\theta^2)^2}+\frac{4(\omega^2-\beta^2/4)}{4-\omega^2\theta^2}\right)\hat{v}^2\right\}$$

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This can be written in simpler form if we define a 2 dimensional vector potential

$$H = \left\{ \left[(\hat{p}_u - A_{\hat{u}})^2 + \omega'^2 \hat{u}^2 \right] - \left[(\hat{p}_v - A_{\hat{v}})^2 + \omega'^2 \hat{v}^2 \right] \right\}$$

NONCOMMUTATIVITY AND LAMBDA

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$$\omega'^2 \equiv \frac{4(\beta - \theta\omega^2)^2}{(4 - \omega^2\theta^2)^2} + \frac{4(\omega^2 - \beta^2/4)}{4 - \omega^2\theta^2}$$

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We can calculate the effective magnetic field

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$$B = \frac{4(\beta - \omega^2\theta)}{4-\omega^2\theta^2}$$

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- A simple 5 dimensional model was proposed, wehere the origin of Λ is related to the noncommutativity between the compact dimension and the radius of the noncompact dimension.