Possible texture zeros for mass matrices of the quarks and leptons

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Theoretical Motivation

The idea of S_3 flavor symmetry and its explicit breaking,

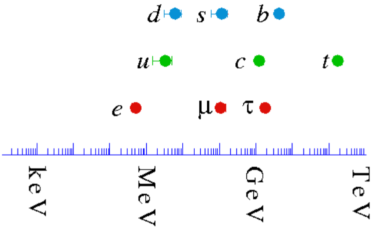
$$S_{3L} \times S_{3R} \supset S_{2L} \times S_{2R} \supset S_2^{\mathbf{diag}}$$

has been succesfully realized as a mass matrix with two texture zeroes in the quark sector to interpret the strong mass hierarchy of up and down type quarks.

$$\mathbf{M_i} = \underbrace{\left(\begin{array}{cc|c} 0 & A_i & 0 \\ A_i^* & B_i & C_i \\ \hline 0 & C_i & D_i \end{array} \right)}_{\mathbf{2 \oplus 1}} \qquad i = u, d$$

Experimental Motivation

The mass spectrum of the charged leptons exhibits a hierarchy similar to the quark's one.



A. Mondragon and E. Rodriguez Jauregui Phys. Rev. D 61 113002

Theoretical Motivation for Dirac Neutrinos

Gran Unified Theories (GUT's) with a gauge group $SO(10)$:

- **The mass matrix with two texture zeroes can describes the data on neutrino masses and mixings.**

  W. Buchmuller and D. Wyler Phys. Lett. B 521, 291 (2001)

  M. Bando and M. Obara, Prog. Theor. Phys. 109, 995 (2003) .

- **From supersymmetry, the Dirac neutrinos and quarks-u have a similar hierarchy in the mass spectrum. It would be natural to take for the Dirac neutrino a mass matrix with two texture zeros.**

First Unified Treatment of Quarks and Leptons

Universal form for the mass matrix of all Dirac fermions in the theory

$$\mathbf{M_i} = \begin{pmatrix} 0 & A_i & 0 \\ A_i^* & B_i & C_i \\ 0 & C_i & D_i \end{pmatrix} \qquad i = u, d, l, \nu_D$$

Some important features of M_i.

- **Is a Hermitian matrix, $M_i = M_i^\dagger$.**
- **The phases in M_i may be factorized out as $M_i = P^\dagger \overline{M}_i P$;**

$$\mathbf{M_i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} 0 & |a| & 0 \\ |a| & b & c \\ 0 & c & d \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix}.$$

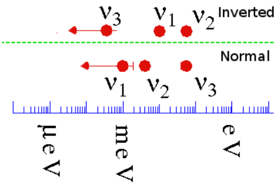
From the Experimental Data of Neutrino Oscillations

M.C. Gonzalez-Gracia Phys.Rept.460:1-129,2008.

$$\underbrace{\Delta m^2_{21}}_{m^2_{\nu_2} - m^2_{\nu_1}} = 7.67^{+0.67}_{-0.21} \times 10^{-5}\mathrm{eV}^2,$$

$$\underbrace{\Delta m^2_{31}}_{\left|m^2_{\nu_3} - m^2_{\nu_1}\right|} = \begin{cases} -2.37 \pm 0.15 \times 10^{-3}\mathrm{eV}^2, \\ (m_{\nu_2} > m_{\nu_1} > m_{\nu_3}) \\ \\ +2.46 \pm 0.15 \times 10^{-3}\mathrm{eV}^2, \\ (m_{\nu_3} > m_{\nu_2} > m_{\nu_1}) \end{cases}$$

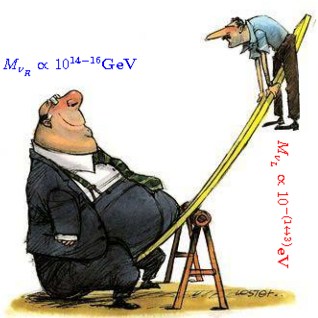
Possible Hierarchies for neutrino masses.



In minimal extensions of the standard model, representing the left handed neutrinos as Dirac particles is not favored because there is no explanation to the fact that neutrinos have a mass much lighter than their corresponding charged leptons.

Majorana Neutrinos

The neutrinos naturally acquire their small masses through type-I seesaw mechanism: $M_{\nu_L} = M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D}^T$



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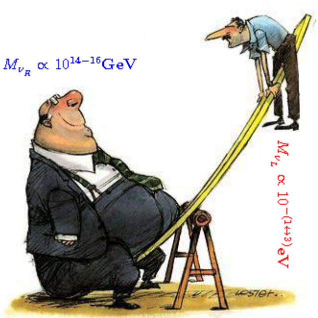

In a Unified Treatment of Dirac Fermions

$$\begin{pmatrix} 0 & A_{\nu_D} & 0 \\ A_{\nu_D}^* & B_{\nu_D} & C_{\nu_D} \\ 0 & C_{\nu_D} & D_{\nu_D} \end{pmatrix} \underbrace{M_{\nu_R}^{-1}}_{\text{shape?}} \begin{pmatrix} 0 & A_{\nu_D}^* & 0 \\ A_{\nu_D} & B_{\nu_D} & C_{\nu_D} \\ 0 & C_{\nu_D} & D_{\nu_D} \end{pmatrix},$$

Second Unified Treatment of Quarks and Leptons

Universal Form For Mass Matrices

The mass matrices of all Dirac fermions have a universal form with two texture zeros and a normal hierarchy in spectrum mass. Then, the mass matrix of the left-handed Majorana neutrinos also has form with two zeros texture.

$$\begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix} = \begin{pmatrix} 0 & A_{\nu_D} & 0 \\ A^*_{\nu_D} & B_{\nu_D} & C_{\nu_D} \\ 0 & C_{\nu_D} & D_{\nu_D} \end{pmatrix} \underbrace{M^{-1}_{\nu_R}}_{\text{shape?}} \begin{pmatrix} 0 & A^*_{\nu_D} & 0 \\ A_{\nu_D} & B_{\nu_D} & C_{\nu_D} \\ 0 & C_{\nu_D} & D_{\nu_D} \end{pmatrix},$$

$$\overbrace{\begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & b_{\nu_R} & c_{\nu_R} \\ 0 & c_{\nu_R} & d_{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & 0 & c_{\nu_R} \\ 0 & c_{\nu_R} & d_{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & b_{\nu_R} & 0 \\ 0 & 0 & d_{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & 0 & 0 \\ 0 & 0 & d_{\nu_R} \end{pmatrix}}^{M_{\nu_R}},$$

$$M_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix},$$

$$a_{\nu_L} = \frac{|A_{\nu_D}|^2}{a_{\nu_R}}; \qquad d_{\nu_L} = \frac{D_{\nu_D}^2}{d_{\nu_R}},$$

$$b_{\nu_L} = \frac{C_{\nu_D}^2}{d_{\nu_R}} + \frac{c_{\nu_R}^2 - b_{\nu_R} d_{\nu_R}}{d_{\nu_R}} \frac{A_{\nu_D}^{*2}}{a_{\nu_R}^2}$$

$$+2\left(B_{\nu_D} - \frac{C_{\nu_D} c_{\nu_R}}{d_{\nu_R}}\right) \frac{A_{\nu_D}^*}{a_{\nu_R}},$$

$$c_{\nu_L} = \frac{C_{\nu_D} D_{\nu_D}}{d_{\nu_R}} + \left(C_{\nu_D} - \frac{c_{\nu_R} D_{\nu_D}}{d_{\nu_R}}\right) \frac{A_{\nu_D}^*}{a_{\nu_R}}$$

Texture zeros

The number of texture zeros in the mass matrix M_{ν_R}, is present in the elements b_{ν_L} and c_{ν_L}.

$$M_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix},$$

$$a_{\nu_L} = \frac{|A_{\nu_D}|^2}{a_{\nu_R}}; \qquad d_{\nu_L} = \frac{D^2_{\nu_D}}{d_{\nu_R}},$$

$$b_{\nu_L} = \frac{C^2_{\nu_D}}{d_{\nu_R}} + \frac{c^2_{\nu_R} - b_{\nu_R} d_{\nu_R}}{d_{\nu_R}} \frac{A^{*2}_{\nu_D}}{a^2_{\nu_R}}$$

$$+2\left(B_{\nu_D} - \frac{C_{\nu_D} c_{\nu_R}}{d_{\nu_R}}\right) \frac{A^*_{\nu_D}}{a_{\nu_R}},$$

$$c_{\nu_L} = \frac{C_{\nu_D} D_{\nu_D}}{d_{\nu_R}} + \left(C_{\nu_D} - \frac{c_{\nu_R} D_{\nu_D}}{d_{\nu_R}}\right) \frac{A^*_{\nu_D}}{a_{\nu_R}}$$

Texture zeros

The number of texture zeros in the mass matrix M_{ν_R}, is present in the elements b_{ν_L} and c_{ν_L}.

Matrices para los neutrinos: $M_{\nu_L} = Q\bar{M}_{\nu_L}Q,\ M_{\nu_R} = R\bar{M}_{\nu_R}R$

$$\bar{M}_{\nu_R} = \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & |b_{\nu_R}| & |c_{\nu_R}| \\ 0 & |c_{\nu_R}| & d_{\nu_R} \end{pmatrix}$$

$$R = \mathrm{diag}\left\{e^{-i\phi_c}, e^{i\phi_c}, 1\right\}$$

$$M_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix},$$

$$a_{\nu_L} = \frac{|A_{\nu_D}|^2}{a_{\nu_R}}; \qquad d_{\nu_L} = \frac{D^2_{\nu_D}}{d_{\nu_R}},$$

$$b_{\nu_L} = \frac{C^2_{\nu_D}}{d_{\nu_R}} + \frac{c^2_{\nu_R} - b_{\nu_R} d_{\nu_R}}{d_{\nu_R}} \frac{A^{*2}_{\nu_D}}{a^2_{\nu_R}}$$
$$+2\left(B_{\nu_D} - \frac{C_{\nu_D} c_{\nu_R}}{d_{\nu_R}}\right)\frac{A^*_{\nu_D}}{a_{\nu_R}},$$
$$c_{\nu_L} = \frac{C_{\nu_D} D_{\nu_D}}{d_{\nu_R}} + \left(C_{\nu_D} - \frac{c_{\nu_R} D_{\nu_D}}{d_{\nu_R}}\right)\frac{A^*_{\nu_D}}{a_{\nu_R}}$$

Texture zeros

The number of texture zeros in the mass matrix M_{ν_R}, is present in the elements b_{ν_L} and c_{ν_L}.

Matrices para los neutrinos: $M_{\nu_L} = Q\bar{M}_{\nu_L}Q$, $M_{\nu_R} = R\bar{M}_{\nu_R}R$

$$\bar{M}_{\nu_R} = \begin{pmatrix} 0 & a_{\nu_R} & 0 \\ a_{\nu_R} & |b_{\nu_R}| & |c_{\nu_R}| \\ 0 & |c_{\nu_R}| & d_{\nu_R} \end{pmatrix} \qquad \bar{M}_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix}$$

$$R = \mathrm{diag}\left\{e^{-i\phi_c}, e^{i\phi_c}, 1\right\} \qquad Q = \mathrm{diag}\left\{e^{-i\varphi}, e^{i\varphi}, 1\right\}$$

Second Unified Treatment of Quarks and Leptons

For all Dirac fermions

$M_j = P_j^\dagger \bar{M}_j P_j \ (j = u, d, l)$

$P_j = \mathrm{diag}\left\{1, e^{-i\phi_j}, e^{-i\phi_j}\right\}$

$$\begin{pmatrix} 0 & |a_j| & 0 \\ |a_j| & b_j & c_j \\ 0 & c_j & d_j \end{pmatrix}$$

For left-handed neutrinos

$M_{\nu_L} = Q \bar{M}_{\nu_L} Q,$

$Q = \mathrm{diag}\left\{1, e^{-i\varphi}, e^{i\varphi}\right\}$

$$\begin{pmatrix} 0 & a_{\nu_L} & 0 \\ a_{\nu_L} & |b_{\nu_L}| & |c_{\nu_L}| \\ 0 & |c_{\nu_L}| & d_{\nu_L} \end{pmatrix}.$$

The mass matrices can be diagonalized by an unitary matrix , so that:

$$M_j = U_j^\dagger \ \mathrm{diag} \ (m_{j1}, m_{j2}, m_{j3}) \, U_j$$

$$M_{\nu_L} = U_\nu \ \mathrm{diag} \ (m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \, U_\nu^T$$

with the unitary matrix:

$$U_j \equiv \mathbb{O}_j^T P_j, \qquad U_\nu \equiv Q \mathbb{O}_\nu$$

Equivalent Mass Matrices

The real representation of the group S_3

$$D(E) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D(A_1) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D(A_2) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$D(A_3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D(A_4) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad D(A_5) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Similarity transformation

$$DU_j \underbrace{\mathbb{I}}_{DD^T} M_j \underbrace{\mathbb{I}}_{DD^T} U_j^{\dagger} D^T = D \, \mathrm{diag} \, (m_{j1}, m_{j2}, m_{j3}) \, D^T$$

$$DU_{\nu}^{\dagger} \underbrace{\mathbb{I}}_{DD^T} M_{\nu_{\nu_L}} \underbrace{\mathbb{I}}_{DD^T} U_{\nu}^{*} D^T = D \, \mathrm{diag} \, (m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \, D^T$$

Equivalent Class of Mass Matrices

$$\begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & d \end{pmatrix} \quad \begin{pmatrix} b & 0 & c \\ 0 & 0 & a \\ c & a & d \end{pmatrix}$$

$$\begin{pmatrix} d & c & a \\ c & b & 0 \\ a & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} b & c & 0 \\ c & d & a \\ 0 & a & 0 \end{pmatrix} \quad \begin{pmatrix} d & a & c \\ a & 0 & 0 \\ c & 0 & b \end{pmatrix}$$

- All these matrices are equivalent, since they have the same invariant

$$\mathrm{tr}\left\{M\right\} = b + d, \quad \det\left\{M\right\} = -a^2 d$$

$$\chi \equiv \frac{1}{2}\left(\mathrm{tr}\left\{M^2\right\} - \mathrm{tr}\left\{M\right\}^2\right) = a^2 + c^2 - bd$$

Equivalent Class of Mass Matrices

$$\begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & d \end{pmatrix} \quad \begin{pmatrix} b & 0 & c \\ 0 & 0 & a \\ c & a & d \end{pmatrix}$$

$$\begin{pmatrix} d & c & a \\ c & b & 0 \\ a & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} b & c & 0 \\ c & d & a \\ 0 & a & 0 \end{pmatrix} \quad \begin{pmatrix} d & a & c \\ a & 0 & 0 \\ c & 0 & b \end{pmatrix}$$

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- Eigenvalues

$$\lambda_i^3 - \mathrm{Tr}\left\{M\right\}\lambda_i^2 - \chi\lambda_i - \det\left\{M\right\} = 0.$$

Equivalence Classes for matrices with two texture zeros. The "$\star$" and "$\times$" denote an arbitrary non-vanishing matrix element on the diagonal and off-diagonal entries, respectively.

Class	Textures	Invariants: Symmetric	Invariants: Hermitian
I	$\begin{pmatrix} 0 & \times & 0 \\ \times & \star & \times \\ 0 & \times & \star \end{pmatrix} \begin{pmatrix} 0 & 0 & \times \\ 0 & \star & \times \\ \times & \times & \star \end{pmatrix} \begin{pmatrix} \star & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \star \end{pmatrix}$ $\begin{pmatrix} \star & \times & \times \\ \times & 0 & 0 \\ \times & 0 & \star \end{pmatrix} \begin{pmatrix} \star & \times & 0 \\ \times & \star & \times \\ 0 & \times & 0 \end{pmatrix} \begin{pmatrix} \star & \times & \times \\ \times & \star & 0 \\ \times & 0 & 0 \end{pmatrix}$	$\mathrm{Tr} = b + d$ $\det = -a^2 d$ $\chi = a^2 + c^2 - bd$	$\mathrm{Tr} = b + d$ $\det = -\|a\|^2 d$ $\chi = \|a\|^2 + \|c\|^2 - bd$
II	$\begin{pmatrix} 0 & \times & \times \\ \times & \star & 0 \\ \times & 0 & \star \end{pmatrix} \begin{pmatrix} \star & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \star \end{pmatrix} \begin{pmatrix} \star & 0 & \times \\ 0 & \star & \times \\ \times & \times & 0 \end{pmatrix}$	$\mathrm{Tr} = b + d$ $\det = -a^2 d - e^2 b$ $\chi = a^2 + e^2 - bd$	$\mathrm{Tr} = b + d$ $\det = -\|a\|^2 d - \|e\|^2 b$ $\chi = \|a\|^2 + \|e\|^2 - bd$
III	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \star \end{pmatrix} \begin{pmatrix} 0 & \times & \times \\ \times & \star & \times \\ \times & \times & 0 \end{pmatrix} \begin{pmatrix} \star & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$\mathrm{Tr} = d$ $\det = 2ace - a^2 d$ $\chi = a^2 + c^2 + e^2$	$\mathrm{Tr} = d$ $\det = a^* c^* e + ace^* - \|a\|^2 d$ $\chi = \|a\|^2 + \|c\|^2 + \|e\|^2$
IV	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & \times \\ 0 & \times & \star \end{pmatrix} \begin{pmatrix} \star & 0 & \times \\ 0 & \star & 0 \\ \times & 0 & \star \end{pmatrix} \begin{pmatrix} \star & \times & 0 \\ \times & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$	$\mathrm{Tr} = g + b + d$ $\det = -gc^2 + gbd$ $\chi = c^2 - gb - gd - bd$	$\mathrm{Tr} = g + b + d$ $\det = -g\|c\|^2 - gbd$ $\chi = \|c\|^2 - gb - gd - bd$

This classification reduces the number of non-singular matrices, from thirty-three to only eleven sets of matrices, or equivalence classes.

In our case, with a normal hierachy in the eigenvalues,

$$m_{i1} < m_{i2} < m_{i3}, \quad \textbf{with} \quad i = u, d, l, \nu.$$

Also $m_{i2} = -|m_{i2}|$ **and** $d_i \equiv 1 - \delta_i$**.** $0 < \delta < 1 - \widetilde{m}_1$**.**

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Also $m_{i2} = -|m_{i2}|$ **and** $d_i \equiv 1 - \delta_i$. $0 < \delta < 1 - \widetilde{m}_1$.
The orthogonal real matrix is;

$$\mathbb{O} = \begin{pmatrix} \left[\frac{\widetilde{m}_{i2} f_{i1}}{\mathcal{D}_{i1}}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_{i1} f_{i2}}{\mathcal{D}_{i2}}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_{i1}\widetilde{m}_{i2}\delta_i}{\mathcal{D}_{i3}}\right]^{\frac{1}{2}} \\ \left[\frac{\widetilde{m}_{i1}(1-\delta_i) f_{i1}}{\mathcal{D}_{i1}}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_{i2}(1-\delta_i) f_{i2}}{\mathcal{D}_{i2}}\right]^{\frac{1}{2}} & \left[\frac{(1-\delta_i)\delta_i}{\mathcal{D}_{i3}}\right]^{\frac{1}{2}} \\ -\left[\frac{\widetilde{m}_{i1} f_{i2}\delta_i}{\mathcal{D}_{i1}}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_{i2} f_{i1}\delta_i}{\mathcal{D}_{i2}}\right]^{\frac{1}{2}} & \left[\frac{f_{i1} f_{i2}}{\mathcal{D}_{i3}}\right]^{\frac{1}{2}} \end{pmatrix}$$

In our case, with a normal hierachy in the eigenvalues,

$$m_{i1} < m_{i2} < m_{i3}, \quad \textbf{with} \quad i = u, d, l, \nu.$$

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$$\begin{aligned} D_{i1} &= (1-\delta_i)(\widetilde{m}_{i1} + \widetilde{m}_{i2})(1 - \widetilde{m}_{i1}), \\ D_{i2} &= (1-\delta_i)(\widetilde{m}_{i1} + \widetilde{m}_{i2})(1 + \widetilde{m}_{i2}), \\ D_{i3} &= (1-\delta_i)(1 - \widetilde{m}_{i1})(1 + \widetilde{m}_{i2}) \end{aligned}$$

Mixing Matrices

Mixing Matrix for Quarks

- $$V_{CKM} = U_u U_d^{\dagger} = \mathbf{O_u}^T P^{(u-d)} \mathbf{O}_d,$$
  where $P^{(u-d)} = \mathrm{diag}\left[1, e^{i\phi}, e^{i\phi}\right]$ with $\phi = \phi_u - \phi_d$
- The Jarlskog invariant is
  $$J_q = \Im m \left[V_{us} V_{cs}^* V_{ub}^* V_{cb}\right],$$
- The inner angles of the unitarity triangle are
  $$\alpha \equiv \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right).$$

Mixing Matrix V_{CKM}

The allowed ranges of the magnitudes of all nine *CKM* elements are[1]:

$$\begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}.$$

[1] A. Ceccucci, et. al. (PDG) Phys.Lett.B667:1,2008.

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The Jarlskog invariant :

$$J_q = \left(3.05^{+0.19}_{-0.20}\right) \times 10^{-5}.$$

The three angles of the unitary triangle values are:

$$\alpha = (88^{+6}_{-5})^o, \quad \beta = (21.46 \pm 0.71)^o, \quad \gamma = (77^{+30}_{-32})^o.$$

[1] A. Ceccucci, et. al. (PDG) Phys.Lett.B667:1,2008.

The χ^2 fit

With the following numerical values of quark mass ratio,

$$\widetilde{m}_u = 2.5469 \times 10^{-5}, \quad \widetilde{m}_c = 3.9918 \times 10^{-3},$$
$$\widetilde{m}_d = 1.5261 \times 10^{-3}, \quad \widetilde{m}_s = 3.2319 \times 10^{-2},$$

The resulting best values of the parameters δ_u and δ_d, at 90% are:

$$\delta_u = \left(5.255^{+8.74}_{-3.53}\right) \times 10^{-3}, \quad \delta_d = \left(9.904^{+50.1}_{-9.89}\right) \times 10^{-4}.$$

the Dirac CP violating phase is $\left(\phi = 89.86^{+0.28}_{-0.30}\right)^o$.

The best values for the moduli of the entries of the *CKM* mixing matrix are given in the following expresion

$$\left| V^{th}_{CKM} \right|_{90\%} = \begin{pmatrix} 0.97425^{+0.00012}_{-0.00006} & 0.2253 \pm 0.0004 & 0.00334^{+0.00009}_{-0.00006} \\ 0.2252^{+0.0005}_{-0.0004} & 0.97344^{+0.00010}_{-0.00009} & 0.0413^{+0.0013}_{-0.0014} \\ 0.00858^{+0.00025}_{-0.00029} & 0.0405^{+0.00066}_{-0.00067} & 0.999143^{+0.000027}_{-0.000026} \end{pmatrix}$$

The χ^2 fit

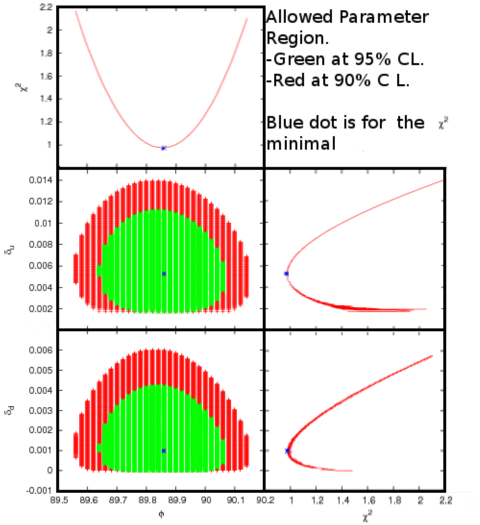
Inner angles of the unitary triangle

$$\alpha^{th} = (92.46^{+5.29}_{-3.52})^o, \quad \beta^{th} = (20.41^{+0.16}_{-0.13})^o. \quad \gamma^{th} = (68.33^{+3.74}_{-5.57})^o.$$

The Jarlskog invariant takes the value

$$J_q^{th} = (2.77 \pm 0.10) \times 10^{-5}.$$

All these results are in good agreement with the experimental values. The minimun value of χ^2 obtained in this fit is $\chi^2_{min} = 0.974$ and the resulting value of χ^2 for degree of freedom is $\frac{\chi^2_{min}}{d.o.f.} = 0.162$.


Allowed Parameter Region.
-Green at 95% CL.
-Red at 90% C L.
Blue dot is for the χ^2 minimal
χ^2
δ_u
δ_d
ϕ
χ^2
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0.002
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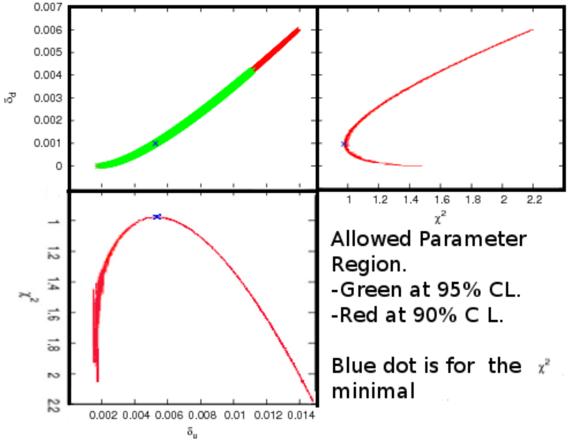
0.007
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χ²
0.002 0.004 0.006 0.008 0.01 0.012 0.014
δu
Allowed Parameter Region.
-Green at 95% CL.
-Red at 90% C L.
Blue dot is for the χ² minimal

- The lepton mixing matrix U_{PMNS}

$$U_{PMNS}^{th} = \mathbf{O}_l^T P^{(\nu-l)} \mathbf{O}_\nu K,$$

The matrix $P^{(\nu-l)} = \mathrm{diag}\left[1, e^{i\Phi_1}, e^{i\Phi_2}\right]$ is the diagonal matrix of the Dirac phases, with $\Phi_1 = 2\varphi - \phi_l$ and $\Phi_2 = \varphi - \phi_l$.

- The rephasing invariant related to the Dirac phase, analogous to the Jarlskog invariant in the quark sector, is given as:

$$J_l \equiv \Im m \left[U_{e1}^* U_{\mu 3}^* U_{e3} U_{\mu 1}\right],$$

- For mixing angles

$$\theta^{l}_{12} = 34.5^{\circ} \pm 1.4^{\circ} \left({}^{+4.8}_{-4.0} \right)^{\circ} ,$$

$$\theta^{l}_{23} = 42.3^{\circ}\,{}^{+5.1}_{-3.3} \left({}^{+11.3}_{-7.7} \right)^{\circ} ,$$

$$\theta^{l}_{13} = 0.0^{\circ}\,{}^{+7.9}_{-0.0} \left({}^{+12.9}_{-0.0} \right)^{\circ} ,$$

$$\delta_{CP} \in [0^{\circ},\ 360^{\circ}] .$$

- The magnitude of the elements of the complete matrix U_{PMNS}, at 90% CL:

$$\begin{pmatrix} 0.80 \to 0.84 & 0.53 \to 0.60 & 0.0 \to 0.17 \\ 0.29 \to 0.52 & 0.51 \to 0.69 & 0.61 \to 0.76 \\ 0.26 \to 0.50 & 0.46 \to 0.66 & 0.64 \to 0.79 \end{pmatrix}$$

The χ^2 fit

The charged lepton masses;

$$m_e = 0.510998910\,\mathrm{MeV}, \qquad m_\mu = 105.658367\,\mathrm{MeV}, \qquad m_\tau = 1776.82\,\mathrm{MeV}.$$

and

$$\widetilde{m}_{\nu_1} = \sqrt{1 - \frac{\left(\Delta m_{32}^2 + \Delta m_{21}^2\right)}{m_{\nu_3}^2}}, \ \widetilde{m}_{\nu_2} = \sqrt{1 - \frac{\Delta m_{32}^2}{m_{\nu_3}^2}}.$$

The best values for the neutrino masses, at 90% CL:

$$m_{\nu_3} = \left(4.92^{+0.21}_{-0.22}\right) \times 10^{-2}\mathrm{eV}.$$

$$m_{\nu_1} = \left(3.22^{+0.67}_{-0.39}\right) \times 10^{-3}\mathrm{eV}, \quad m_{\nu_2} = \left(9.10^{+0.25}_{-0.13}\right) \times 10^{-3}\mathrm{eV},$$

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The resulting best values of the parameters δ_l and δ_ν are

$$\delta_l = (3 \pm 2.98) \times 10^{-2}, \qquad \delta_\nu = 0.510^{+0.09}_{-0.12}$$

The best values of the Dirac CP violating phases are

$$\Phi_1 = 270^\circ \pm 20^\circ. \qquad \text{y} \qquad \Phi_2 = 180^\circ \pm 20^\circ$$

The best values for the moduli of the entries of the *PMNS* mixing matrix are given in the following expression

$$\left| U_{PMNS}^{th} \right|_{90\%} = \begin{pmatrix} 0.8204^{+0.008}_{-0.010} & 0.5616^{+0.012}_{-0.014} & 0.1181^{+0.017}_{-0.011} \\ 0.3748^{+0.018}_{-0.031} & 0.6280^{+0.019}_{-0.010} & 0.6819 \pm 0.025 \\ 0.4345^{+0.024}_{-0.020} & 0.5388^{+0.022}_{-0.024} & 0.7216^{+0.024}_{-0.027} \end{pmatrix}.$$

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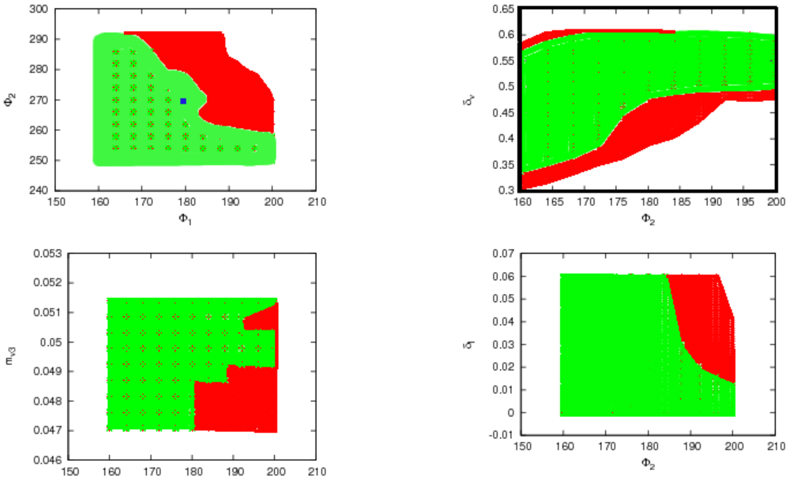
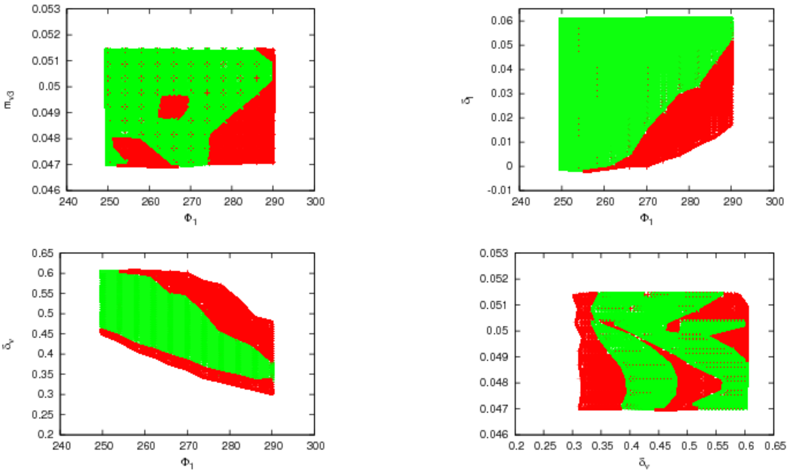
The value of the rephasing invariant related to the Dirac phase is

$$J^{th}_{l} = (1.2 - 2.4) \times 10^{-2}.$$



Allowed parameter region. Red at 90% CL. Green at 95% CL.



Allowed parameter region. Red at 90% CL. Green at 95% CL.

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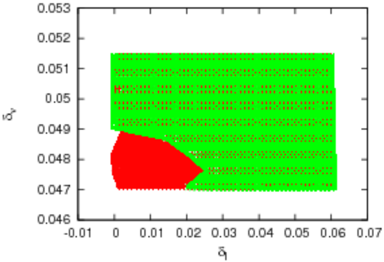
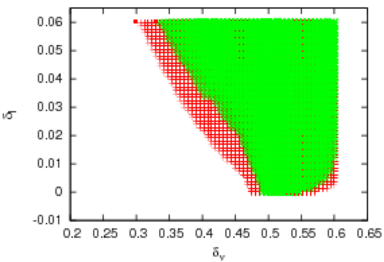
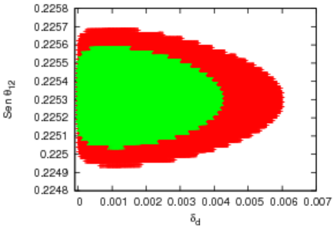
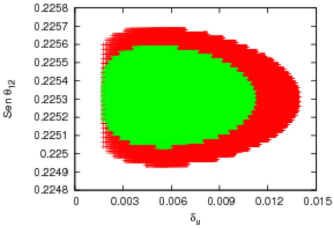
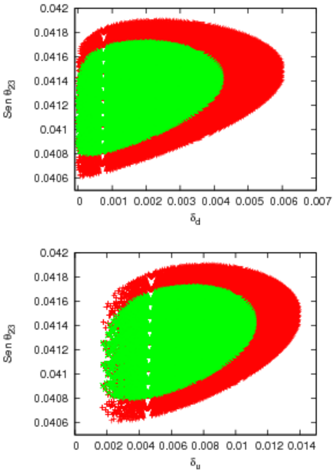




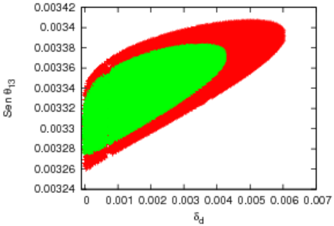


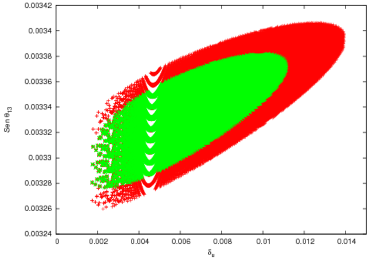


- Mixing Angles θ^q_{12}. Red at 90% CL. Green at 95% CL.
- $$\sin^2\theta^q_{12} \approx \frac{\frac{\widetilde{m}_d}{\widetilde{m}_s} + \frac{\widetilde{m}_u}{\widetilde{m}_c} - 2\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}\frac{\widetilde{m}_d}{\widetilde{m}_s}}\cos\phi}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\right)},$$
- $$\theta^q_{12} = 13.02^\circ \pm 0.03^\circ,$$



- Mixing Angles θ^q_{23}. Red at 90% CL. Green at 95% CL.
-
$$\sin^2\theta^q_{23} \approx \frac{\left(\sqrt{\delta_u} - \sqrt{\delta_d}\right)^2}{\left(1 + \frac{\widetilde{m}_u}{\widetilde{m}_c}\right)},$$
-
$$\theta^q_{23} = \left(2.36^{+0.04}_{-0.03}\right)^\circ,$$





- Mixing Angles θ_{23}^{q}. Red at 90% CL. Green at 95% CL.
- $$\sin^2\theta_{13}^{q} \approx \frac{\frac{\widetilde{m}_u}{\widetilde{m}_c}\left(\sqrt{\delta_u}-\sqrt{\delta_d}\right)^2}{\left(1+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)}.$$
- $$\theta_{13}^{q} = 0.19^\circ \pm 0.03^\circ,$$

Mixing angles for lepton sector

$$\sin^2\theta^l_{12} = \frac{f_{\nu 2}}{(1+\widetilde{m}_{\nu_2})(1-\delta_\nu)}\left\{\frac{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}} + \frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu) + 2\sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\cos\phi_1}{\left(1+\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)}\right\},$$

$$\sin^2\theta^l_{23} \approx \frac{\delta_\nu + \delta_e f_{\nu 2} - \sqrt{\delta_\nu\delta_e f_{\nu 2}}\cos(\phi_1-\phi_2)}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})},$$

$$\sin^2\theta^l_{13} \approx \frac{\delta_\nu}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})}\left\{\frac{\widetilde{m}_e}{\widetilde{m}_\mu} + \frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)} - 2\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}}\cos\phi_1\right\},$$

Mixing angles for lepton sector

$$\sin^2\theta_{12}^l = \frac{f_{\nu 2}}{(1+\widetilde{m}_{\nu_2})(1-\delta_\nu)} \left\{ \frac{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}} + \frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu) + 2\sqrt{\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}(1-\delta_\nu)}\cos\phi_1}{\left(1+\frac{\widetilde{m}_{\nu_1}}{\widetilde{m}_{\nu_2}}\right)\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)} \right\},$$

$$\sin^2\theta_{23}^l \approx \frac{\delta_\nu + \delta_e f_{\nu 2} - \sqrt{\delta_\nu \delta_e f_{\nu 2}}\cos(\phi_1-\phi_2)}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})},$$

$$\sin^2\theta_{13}^l \approx \frac{\delta_\nu}{\left(1+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)(1+\widetilde{m}_{\nu_2})} \left\{ \frac{\widetilde{m}_e}{\widetilde{m}_\mu} + \frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)} - 2\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\frac{\widetilde{m}_{\nu_1}\widetilde{m}_{\nu_2}}{(1-\delta_\nu)}}\cos\phi_1 \right\},$$

We obtain the following numerical values

$$\theta_{12}^l = 34.3^o, \quad \theta_{23}^l = 43.6^o, \quad \theta_{13}^l = 3.4^o$$

in very good agreement with experimental data

1. The strong mass hierarchy of the Dirac fermions produces small and very small mass ratios of u and d-type quarks and charged leptons. The quark mass hierarchy is then reflected in a similar hierarchy of small and very small quark mixing angles.

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2. The normal seesaw mechanism gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio m_{ν_1}/m_{ν_2} and allows for large θ^l_{12} and θ^l_{23} mixing angles.

The effective Majorana masses

The square of the magnitudes of the effective Majorana neutrino masses are

$$|\langle m_{ll}\rangle|^2 = \sum_{j=1}^{3} m_{\nu_j}^2 \, |U_{lj}|^4 + 2\sum_{j<k}^{3} m_{\nu_j} m_{\nu_k} \, |U_{lj}|^2 \, |U_{lk}|^2 \cos 2\left(w_{lj} - w_{lk}\right),$$

where $w_{lj} = \arg\{U_{lj}\}$**, with** $l = e, \mu, \tau$**; this term includes phases of both types, Dirac and Majorana.**

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where $w_{lj} = \arg\left\{U_{lj}\right\}$, with $l = e, \mu, \tau$; this term includes phases of both types, Dirac and Majorana.

$$|\langle m_{ee}\rangle|^2 \approx \{9.41 + 8.29\cos(1^o - 2\beta_1) + 4.3\cos(1^o - 2w_{e3}) + 4.31\cos 2(\beta_1 - w_{e3})\} \times 10^{-6}\ \mathrm{eV}^2$$

where $w_{e3} = \arctan\left\{\frac{0.15\tan\beta_2 - 0.013}{0.15 + 0.013\tan\beta_2}\right\}.$

Similarly,

$$|\langle m_{\mu\mu}\rangle|^2 \approx \{4.8 + 0.17\cos 2(44^o - w_{\mu 2}) + 1.8\cos 2(w_{\mu 2} - w_{\mu 3})\} \times 10^{-4}\ \mathrm{eV}^2$$

where

$$w_{\mu 2} \approx \arctan\left\{\frac{0.65\tan\beta_1 + 0.13}{0.65 - 0.13\tan\beta_1}\right\}, \quad w_{\mu 3} \approx \arctan\left\{\frac{\tan\beta_2 - 0.13}{1 + 0.13\tan\beta_2}\right\}.$$

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Therefore in order to make an estimate we maximize the effective Majorana neutrino masses, we obtained

$$|\langle m_{ee}\rangle|^{\max} \approx 4.6 \times 10^{-3}\mathrm{eV}, \qquad |\langle m_{\mu\mu}\rangle|^{\max} \approx 2.1 \times 10^{-2}\mathrm{eV}.$$

These numerical values are consistent with the vary small experimentally determined upper bounds for the reactor neutrino mixing angle θ^l_{13}.

Equivalence Classes for matrices with two texture zeros.

Class	Textures
II	$\begin{pmatrix} 0 & \times & \times \\ \times & \star & 0 \\ \times & 0 & \star \end{pmatrix} \begin{pmatrix} \star & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \star \end{pmatrix} \begin{pmatrix} \star & 0 & \times \\ 0 & \star & \times \\ \times & \times & 0 \end{pmatrix}$
III	$\begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \star \end{pmatrix} \begin{pmatrix} 0 & \times & \times \\ \times & \star & \times \\ \times & \times & 0 \end{pmatrix} \begin{pmatrix} \star & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$
IV	$\begin{pmatrix} \star & 0 & 0 \\ 0 & \star & \times \\ 0 & \times & \star \end{pmatrix} \begin{pmatrix} \star & 0 & \times \\ 0 & \star & 0 \\ \times & 0 & \star \end{pmatrix} \begin{pmatrix} \star & \times & 0 \\ \times & \star & 0 \\ 0 & 0 & \star \end{pmatrix}$

- **Class II**
  $m_\nu \geq 0.01$ **eV,**
  $|\langle m_{ee} \rangle| \geq 0.02$ **eV**
- **Class III**
  $0.01 \geq m_\nu \geq$ **eV,**
  $|\langle m_{ee} \rangle| \geq 0.02$ **eV**
- **Class IV is not viable phenomenologically.**

Conclusions

- The strong hierarchy in the mass spectra of the quarks and charged leptons explains the small or very small quark mixing angles, the very small charged lepton mass ratio explain the very small θ^{l}_{13} which, in our scheme, is independent of the neutrino masses.
- The see-saw mechanism type I gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio m_{ν_1}/m_{ν_2} and allows for large θ^{l}_{12} and θ^{l}_{23} mixing angles.

- We obtain the analytical expresion for mixing angles, mixing matrices, invariant associated at CP violation phases and for the effective Majorana masses.
- We can reproduce the numerical values of both mixing matrices, V_{CKM} and U_{PMNS}, in very good agreement with experimental data.
- We have then classified the matrix with two texture zero. Also we analysed for lepton sector was obtained that only the matrices with two zeros class I,II, III are viable phenomenologically.