Near-horizon geometry from flux compactification

Oscar Loaiza Brito, Liliana Vazquez Mercado

DCI-Universidad de Guanajuato
XII Mexican Workshop on Particles and Fields

October 24, 2011

Physical Review D 84, 066010(2011)

Outline

* Introduction
* Conditions that flux configuration must fulfill in order to obtain 4d effective theory with $AdS_2 \times S^2$ symmetry
* How are we going to find these conditions?
  - Einstein equation and Bianchi identities
  - Integrability conditions on the spinors
* What about the internal space?
  - Manifold with $SU(3)$ structure
  - Null curvature and torsion terms
* Near-horizon geometry of supersymmetric black hole
* Conclusions

Introduction

* The presence of extra dimensions in superstrings theories has established a relation between the physics of theories in 4d and the geometry of the 6d internal space
* An interesting topic concerns the construction of vacua with positive or null curvature
* A common assumption to study these effective 4d theories involves $10d \longrightarrow 4d \times 6d$

Introduction

- Starting point: 10d theory, namely, Type IIB Supergravity
  - Features: $N = 2$, chiral theory, n-form field strengths with n odd (bosonic content)
- Final point: effective 4d theory whose geometry is given by the product of two maximally symmetric spaces: $AdS_2 \times S^2$
- Find a solution of the 10d theory which satisfy the symmetry ($AdS_2 \times S^2$) and the Einstein-Maxwell equations
- This solution is given by the near-horizon geometry of an extremal black hole (RN), called, the Robinson-Bertotti metric

Introduction

- If we consider a fluxless compactification, it is not possible to obtain a De-Sitter vacuum (Minkowski)
- The introduction of fluxes produces important changes (no-go theorem, Maldacena & Nuñez)
  - Minkowski
  - Anti-De-Sitter
- For $N = 2$, four-dimensional supergravity admits solutions: *Minkowski* and *Robinson-Bertotti*

Introduction

- Construction of Robinson-Bertotti metric
- Compactification (with fluxes) of type IIB supergravity to a 4d spacetime conformed by the product $M_2 \times \tilde{M}_2$ (two-dimensional maximally symmetric spaces)
- $10d \longrightarrow 2d \times 2d \times 6d$

Flux supergravity compactification

Let us start by considering the most generic 10d metric,

$$ds^2 = e^{2A(y)}\left(\tilde{g}_{ij}dx^i dx^j + \tilde{g}_{ab}dx^a dx^b\right) + h_{mn}dy^m dy^n$$

By the Einstein trace-reversed equations, the Ricci scalar $R(\tilde{g}_{ij}) \equiv \tilde{R}_{(1)}$ for AdS_2 satifies

$$\tilde{R}_{(1)} + e^{2A}\left(-T^i_i + \frac{1}{4}T^L_L\right) = 2e^{-2A}\nabla^2 e^{2A}$$

where T_{MN} is the energy-momentum tensor in 10d.

Flux supergravity compactification

Let us start by considering the most generic 10d metric,

$$ds^2 = e^{2A(y)} \left(\tilde{g}_{ij} dx^i dx^j + \tilde{g}_{ab} dx^a dx^b \right) + h_{mn} dy^m dy^n$$

By the Einstein trace-reversed equations, the Ricci scalar $R(\tilde{g}_{ij}) \equiv \tilde{R}_{(1)}$ for AdS_2 satifies

$$\tilde{R}_{(1)} + e^{2A} \left(-T^i_i + \frac{1}{4} T^L_L \right) = 2e^{-2A} \nabla^2 e^{2A}$$

where T_{MN} is the energy-momentum tensor in 10d.

Flux supergravity compactification

The expression of the energy-momentum tensor for a general n-form is,

$$\mathcal{T}_1 \equiv -T_i^i + \frac{1}{4} T_L^L = -\mathcal{F}_{iM_1 \ldots M_{n-1}} \mathcal{F}^{iM_1 \ldots M_{n-1}} + \frac{n-1}{4n} \mathcal{F}^2$$

* A similar result is obtained for $\tilde{R}_{(2)}$ and $\mathcal{T}_2$ for S^2
* It is necessary to consider specific flux configurations in order to preserve a $SO(1,1) \times SO(2)$ symmetry in 4d:
    * Internal fluxes $\mathcal{F}_n^{int}$
    * Fluxes with general form $\mathcal{F}_n = \omega_2 \wedge f_{n-2}$

Ricci flat space

Contribution to the 4d Ricci scalar $\tilde{R}$ by fluxes compatible with $SO(1,1) \times SO(2)$ symmetry

CASE I

$\mathcal{F}_n = \omega_2 \wedge f_{n-2}$ fluxes, with

$$\omega_2 = \frac{1}{2}\omega_{ij} dx^i \wedge dx^j \tag{1}$$

$$\mathcal{F}_{jL_1 \ldots L_{n-1}} \mathcal{F}^{jL_1 \ldots L_{n-1}} = \frac{2}{n} F^2 \tag{2}$$

From which the corresponding contribution to $\tilde{R}_{(1)}$ by $T_{(1)}$ is

$$\mathcal{T}_1 = \frac{n-9}{4n} \mathcal{F}^2 .$$

CASE II

$G_n = \tilde{\omega}_2 \wedge g_{n-2}$ fluxes, with

$$\tilde{\omega}_2 = \frac{1}{2}\omega_{ab} dx^a \wedge dx^b \tag{3}$$

Where the contribution to $\tilde{R}_{(2)}$ by $\mathcal{T}_{(2)}$ is given by

$$\mathcal{T}_2 = \frac{n-9}{4n}\mathcal{G}_n^2.$$

Ricci flat space

Contribution of internal fluxes, $\mathcal{F}_n$,

$$\mathcal{T} = \frac{n-1}{2n}\mathcal{F}_n^2$$

Contribution of fluxes, $\mathcal{G}_n = Vol_4 \wedge h_{n-4}$,

$$\mathcal{T} = -\frac{9-n}{2n}\mathcal{G}_n^2$$

Ricci-flat 4d space-time is an allowed solution from 10d supergravity flux compactification into a 4d space-time given by $AdS_2 \times S^2$, since

$$\tilde{R} = \tilde{R}_{(1)} + \tilde{R}_{(2)} = -e^{2A}(\mathcal{T}_1 + \mathcal{T}_2)$$

Ricci flat space

Contribution of internal fluxes, $\mathcal{F}_n$,

$$\mathcal{T} = \frac{n-1}{2n}\mathcal{F}_n^2$$

Contribution of fluxes, $\mathcal{G}_n = Vol_4 \wedge h_{n-4}$,

$$\mathcal{T} = -\frac{9-n}{2n}\mathcal{G}_n^2$$

Ricci-flat 4d space-time is an allowed solution from 10d supergravity flux compactification into a 4d space-time given by $AdS_2 \times S^2$, since

$$\tilde{R} = \tilde{R}_{(1)} + \tilde{R}_{(2)} = -e^{2A}(\mathcal{T}_1 + \mathcal{T}_2)$$

Ricci flat space

Contribution of internal fluxes, $\mathcal{F}_n$,

$$\mathcal{T} = \frac{n-1}{2n}\mathcal{F}_n^2$$

Contribution of fluxes, $\mathcal{G}_n = Vol_4 \wedge h_{n-4}$,

$$\mathcal{T} = -\frac{9-n}{2n}\mathcal{G}_n^2$$

Ricci-flat 4d space-time is an allowed solution from 10d supergravity flux compactification into a 4d space-time given by $AdS_2 \times S^2$, since

$$\tilde{R} = \tilde{R}_{(1)} + \tilde{R}_{(2)} = -e^{2A}(\mathcal{T}_1 + \mathcal{T}_2)$$

Examples

- We focus on type *IIB* supergravity compactifications
- Consider a 5-form F_5 of the form $f_2 \wedge F_3$, with coefficients F_{ijmnp} and F_{abmnp}
- The corresponding 2d scalar curvatures are

$$\tilde{R}_{(1)} = -\frac{e^{2A(y)}}{5}|F_5|^2,$$

$$\tilde{R}_{(2)} = \frac{e^{2A(y)}}{5}|F_5|^2.$$

Let us consider the flux configuration consisting on a NS-NS flux H_3 and a RR flux F_3 given by

$$\begin{aligned} H_3 &= (Ndx^0 \wedge dx^1 + Mdx^2 \wedge dx^3) \wedge d\alpha, \\ F_3 &= (Pdx^0 \wedge dx^1 + Qdx^2 \wedge dx^3) \wedge d\alpha, \end{aligned}$$

with α a function of internal coordinates. The curvatures are

$$\begin{aligned} \tilde{R}_{(1)} &= -2e^{2A(y)}(N^2 + P^2)(\nabla\alpha)^2, \\ \tilde{R}_{(2)} &= 2e^{2A(y)}(M^2 + Q^2)(\nabla\alpha)^2 \end{aligned}$$

Taking $M^2 + Q^2 = N^2 + P^2$, the total 4d curvature vanishes.

Einstein equations

We start computing the corresponding 2d Ricci tensors.
The 10-dimensional component of the Ricci tensor is,

$$R_{MN} = -\frac{1}{Im\tau}\left(\frac{G_3^2}{48}G_{MN} - \frac{1}{4}G_{MQR}\bar{G}_N^{QR}\right), \quad (4)$$

where $G_3 = F_3 - \tau H_3$.
To preserve the symmetries of the compactification setup, the most general metric to consider is

$$ds^2 = e^{2A(y)}\tilde{g}_{ij}dx^i dx^j + e^{2B(y)}\tilde{g}_{ab}dx^a dx^b + e^{-2A(y)}\tilde{h}_{mn}dy^m dy^n \quad (5)$$

Einstein equations

Expressions of Ricci Tensor for M_2 and $\tilde{M}_2$, respectively

$$R_{ij}(g) = e^{4A}\left(\tilde{\nabla}^2 A + 2\tilde{\nabla}A\cdot\tilde{\nabla}B - 2(\tilde{\nabla}A)^2\right)\tilde{g}_{ij} - \frac{1}{Im\tau}\left(\frac{G_3^2}{48}G_{ij} - \frac{1}{4}G_{iQR}\bar{G}_j^{\;QR}\right)$$

$$R_{ab}(g) = e^{2(A+B)}\left(\tilde{\nabla}^2 B - 2\tilde{\nabla}A\cdot\tilde{\nabla}B + 2(\tilde{\nabla}B)^2\right)\tilde{g}_{ab} - \frac{1}{Im\tau}\left(\frac{G_3^2}{48}G_{ab} - \frac{1}{4}G_{aQR}\bar{G}_b^{\;QR}\right)$$

with $G_3 = F_3 - \tau H_3$ complex 3-form.

Bianchi Identities

- From dual Bianchi Identities, $d * F_3 = d * H_3 = 0$, we get

$$(Q + \tau M)\left(-2\partial_m(2A + B)\tilde{\partial}^m\alpha + \tilde{\partial}^2\alpha\right) = 0$$
$$(P + \tau N)\left(2\partial_m(-4A + B)\tilde{\partial}^m\alpha + \tilde{\partial}^2\alpha\right) = 0$$

- It is worth mentioning that even though M_2 and $\tilde{M}_2$ are independent, it seems that they share the same warping factor
- Both equations reduce to

$$\tilde{\partial}^2\alpha = 6e^{-2A}\partial_m A\,\partial^m\alpha = \frac{3}{2}e^{-6A}(\partial_m e^{4A})(\partial^m\alpha)$$

Bianchi Identities

- If we compare Ricci tensor R_{ij} with Bianchi Identities, we have, for M_2

$$\tilde{\nabla}^2(e^{4A}-\alpha) = 2\tilde{R}_1 + \frac{1}{2}e^{-6A}\left(\partial_m e^{4A}\,\partial^m e^{4A}\right) - \frac{3}{2}\partial_m e^{4A}\partial^m\alpha$$
$$+\frac{1}{4Im\,\tau}\left[-P^2-\tau\tilde{\tau}N^2+2(Im\,\tau)PN\right]e^{-2A}\partial_m\alpha\partial^m\alpha$$

A similar expression is obtained for $\tilde{M}_2$

- If we add both contributions the result is

$$\tilde{\nabla}^2(e^{4A}-\alpha) = e^{-6A}\left(\partial_m e^{4A}\,\partial^m e^{4A}\right) - 3\partial_m e^{4A}\partial^m\alpha$$
$$+\frac{1}{4Im\,\tau}[(Q^2-P^2)-\tau\tilde{\tau}(M^2-N^2)+2(Im\,\tau)(QM+PN)]e^{-2A}\partial_m\alpha\partial^m\alpha$$

Bianchi Identities

- Solutions:
  - Relation between A, α and flux numbers
  - Constant A (R-B metric in 4d)
- Some cases are ($\mathcal{T} = 0, M^2 + Q^2 = N^2 + P^2$):
  - $(M, N, P, Q) \neq 0$ y $M = -Q$, $Q = P$
  - $P = 0$ and $\frac{M}{Q} = \frac{\tau\bar{\tau}-1}{2Im\tau}$
  - $Q = 0$ and $\frac{N}{P} = \frac{\tau\bar{\tau}+1}{2Im\tau}$
  - $N = 0$ and $\frac{Q}{M} = \frac{\tau\bar{\tau}-1}{2Im\tau}$
  - $M = 0$ and $\frac{P}{N} = \frac{\tau\bar{\tau}+1}{2Im\tau}$
- For the above expressions, $H_3 \wedge F_3 = 0$

Up to now, what have we done?

* RB metric (R=0) in 4d spacetime
* We turned on fluxes which satify the condition of null curvature, H_3 and F_3 with $M^2 + Q^2 = N^2 + P^2$
* We computed the 2d Ricci tensors
* We verified that the flux configuration is compatible with Einstein equations and Bianchi identities
* Finally, we concentrated on the simplest solution involvin a constant warping factor A, because our goal is to find the **minimal conditions** under which we can construct the RB solution in 4d

Curvature from integrability conditions

- By choosing one of the above constraints, let us proceed to compute the scalar curvature of the 2d spaces from the integrability conditions on the 2d components of 10d spinors
- When supersymmetry is preserved, variation of the gravitino is

$$\delta\Psi_M = \nabla_M \epsilon - \frac{1}{4} \not{H}_M \sigma^3 \epsilon + \frac{1}{16} e^{\phi} \not{F}_3 \Gamma_M \sigma^1 \epsilon = 0$$

where σ are Pauli matrices and $\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$

Curvature from integrability conditions

It is desirable to find an independent expression for 10d spinors ϵ^1 and ϵ^2.

We start by taking the 4d component of $\delta\Psi$

- $[\Gamma_i, \Gamma_{jkm}] = \{\Gamma_i, \Gamma_{abm}\} = 0$
- M_2 component of gravitino variation

$$\nabla_i \epsilon - \frac{1}{4} \not{H}_i \sigma^3 \epsilon - \frac{1}{16} e^{\phi} \Gamma_i \left(\not{F}_3^{(1)} - \not{F}_3^{(2)} \right) \sigma^1 \epsilon = 0,$$

$$\not{F}_3^{(1)} = F^{01m} \Gamma_{01m}$$

$$\not{F}_3^{(2)} = F^{23m} \Gamma_{23m}$$

- Dilatino variation

$$\delta\lambda = -\frac{1}{2} \not{H}_3 \sigma^3 \epsilon - \frac{1}{4} e^{\phi} \not{F}_3 \sigma^1 \epsilon = 0$$

Curvature from integrability conditions

It is desirable to find an independent expression for 10d spinors ϵ^1 and ϵ^2.

We start by taking the 4d component of $\delta\Psi$

* $[\Gamma_i, \Gamma_{jkm}] = \{\Gamma_i, \Gamma_{abm}\} = 0$
* M_2 component of gravitino variation

$$\nabla_i \epsilon - \frac{1}{4} \not{H}_i \sigma^3 \epsilon - \frac{1}{16} e^{\phi} \Gamma_i \left(\not{F}_3^{(1)} - \not{F}_3^{(2)} \right) \sigma^1 \epsilon = 0,$$

$$\not{F}_3^{(1)} = F^{01m} \Gamma_{01m}$$

$$\not{F}_3^{(2)} = F^{23m} \Gamma_{23m}$$

* Dilatino variation

$$\delta\lambda = -\frac{1}{2} \not{H}_3 \sigma^3 \epsilon - \frac{1}{4} e^{\phi} \not{F}_3 \sigma^1 \epsilon = 0$$

Curvature from integrability conditions

We choose $P = 0$. Components **a**, **i** of gravitino variation are

$$\left(\nabla_a - \frac{1}{4}\not{H}_a + \frac{1}{8}\Gamma_a \not{H}_3\right)\epsilon^1 = 0,$$
$$\left(\nabla_i - \frac{1}{4}\not{H}_i + \frac{1}{8}\Gamma_i \not{H}_3\right)\epsilon^1 = 0.$$

It is important to notice that both spinors are decoupled, just in the presence of non-trivial fluxes H_3 and F_3.

* Then, we can express the gravitino variation as

$$(\nabla_i + \kappa_i)\epsilon^1 = \nabla_i^{(T)}\epsilon^1 = 0,$$

$\kappa_i = -\frac{1}{4}\not{H}_i + \frac{1}{8}\Gamma_i \not{H}_3$

Curvature from integrability conditions

- Now, we can compute the metric connection components
  $ds^2 = e^{-2A} g_{\mu\nu} dx^\mu dx^\nu + h_{mn} dy^m dy^n$
- The i component of covariant derivative of ϵ^1 is

$$\nabla_i^T \epsilon^1 = \left(\tilde{\nabla}_i - \frac{1}{2} \gamma_i \tilde{\gamma} \otimes \tilde{\sigma} \otimes \not{\partial} A + \kappa_i \right) \epsilon^1 = 0,$$

where $\nabla_i = \tilde{\nabla}_i - \frac{1}{2} \gamma_i \tilde{\gamma} \otimes \tilde{\sigma} \otimes \not{\partial} A$

Curvature from integrability conditions

* Riemann Tensor

$$\frac{1}{4}\tilde{R}_{ij}{}^{kl}\gamma_{kl} - [\kappa_i, \kappa_j] = 0$$

In the fluxless case, the contorsion term and Riemann tensor vanish for a constant warping factor.

* In the presence of fluxes, the contorsion contributes with an extra term, though we consider a constant warping factor

$$[\kappa_0, \kappa_1] = -\frac{1}{32}\left(N^2 + M^2\right)(\nabla\alpha^2)\gamma_{01}$$

Curvature from integrability conditions

* Then, the corresponding 2d Ricci scalar is given by

$$R_{(1)} = -\frac{1}{8}(N^2 + M^2)(\nabla\alpha)^2.$$

* Similarly, for S^2, the scalar curvature is $\tilde{R}_{(2)} = -\tilde{R}_{(1)}$.
  $R = R_{(1)} + R_{(2)} = 0$

Some remarks:

- There is a unique 4d solution of this system, namely, the near-horizon geometry $AdS_2 \times S^2$ with $R_4 = 0$
- The relation among fluxes is established by requiring N=2 supergravity in 4d (which implies the decoupling of the spinors)
- For $M = N = 0$, i.e., in the fluxless case, both curvatures vanish and we recover Minkowski space-time
- Although it seems that RR fluxes do not play a role in the curvature, they are necessary, otherwise the contribution to R_4 by T would not be zero

Integrability conditions

* From R_1 and R_2, it is possible to induce the 4d metric $\tilde{g}_{\mu\nu}$,

$$ds_4^2 = -\mathcal{A}(A)\frac{x_1^2}{h}dx_0^2 - \Theta(A)\frac{h}{x_1^2}dx_1^2 + \Theta(A)h dx_2^2 + \mathcal{B}(A)h\sin^2 x_2 dx_3^2$$

$$h = 2/|R_{(1)}|$$

$\mathcal{A}$, $\mathcal{B}$:A arbitraty functions

* It is not enough to reproduce the curvatures. In addition, this metric must be solution of an effective theory in 4d

Near-horizon metric

* Therefore, in the presence of an homogenous electromagnetic field of the form,

$$F_{t\rho} = |2\tilde{R}_{(1)}| = \left(\frac{1}{4}(N^2 + M^2)(\nabla\alpha)^2\right)$$

there is a unique solution for Einstein-Maxwell effective equations

* This solution is the near-horizon metric of an extremal black hole, known as Robinson-Bertotti solution

Near-horizon metric

* The curvature of each subspace is proportional to the flux number $(N^2 + M^2)$
* From the integrability conditions for ϵ^2, it is possible to get the same result,

$$\kappa_i = \frac{1}{4} \not{H}_i - \frac{1}{8} \Gamma_i \not{H}$$

* The 2d curvatures remain the same with respect to those obtained from ϵ^1 equations
* A different case would consider a RR flux configuration in which $Q = 0$. A solution of the type $AdS_2 \times S^2$ is also obtained.

Conclusions

* We studied the required conditions that a flux configuration must satisfy in order to obtain 4d space-time of the type $AdS_2 \times S^2$
* A way to construct a 4d space-time with $AdS_2 \times S^2$ symmetry by turning on 3-form fluxes was developed
* Solutions of this type as near-horizon geometry are not uniquely constructed as the limit of extremal black-holes, but also by compactifications on internal manifolds with torsion derived by the presence of flux compactification

Thank you!