On the Higgs potential in the Minimal S_3-Invariant Extension of the Standard Model

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1 Stage I: A quick overview...

- Massless massive fundamental particles
- The Higgs Mechanism
  - Spontaneous Symmetry Breaking
  - Yukawa interactions
- The flavour problem

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2 Stage II: A quick glance into the MS_3IESM

So, why to derive it again?

PHYSICAL REVIEW D **83**, 011701(R) (2011)

Exotic Higgs boson decay modes as a harbinger of S_3 flavor symmetry

Gautam Bhattacharyya,[1] Philipp Leser,[2] and Heinrich Päs[2]
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II. SCALAR POTENTIAL AND SPECTRUM

The most general S_3 invariant scalar potential involving three scalar doublet fields is given by [4,6]

$$\begin{aligned} V = {} & m^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + m_3^2\phi_3^\dagger\phi_3 + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 \\ & + \frac{\lambda_2}{2}(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2 + \lambda_3\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1 \\ & + \frac{\lambda_4}{2}(\phi_3^\dagger\phi_3)^2 + \lambda_5(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) \\ & + \lambda_6\phi_3^\dagger(\phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger)\phi_3 + [\lambda_7\phi_3^\dagger\phi_1\phi_3^\dagger\phi_2 \\ & + \lambda_8\phi_3^\dagger(\phi_1\phi_2^\dagger\phi_1 + \phi_2\phi_1^\dagger\phi_2) + \text{H.c.}]. \end{aligned} \tag{2}$$

Fritzsch neutrino mass matrix from S_3 symmetry

D Meloni[1], S Morisi[2] and E Peinado[2]

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[2] AHEP Group, Institut de Física Corpuscular—CSIC/Universitat de València, Edificio Institutos de Paterna, Apt 22085, E-46071 Valencia, Spain

3. The scalar potential

The most general Higgs potential invariant under $G \times SM$ is as follows:

$$\begin{aligned}
V = {} & \mu_1 H_S'^{\dagger} H_S' + \mu_2 \left(H_D^{\dagger} H_D\right)_1 + \mu_3 H_S^{\dagger} H_S + \mu_4 |\chi|^2 + \lambda_1 |\chi|^4 \\
& + \left(\lambda_2 H_D^{\dagger} H_D + \lambda_3 H_S^{\dagger} H_S + \lambda_4 H_S'^{\dagger} H_S'\right) |\chi|^2 + \lambda_5 \left[\left(H_D^{\dagger} H_D\right)\right]^2 + \lambda_6 \left[\left(H_D^{\dagger} H_D\right)_{1'}\right]^2 \\
& + \lambda_7 \left[\left(H_D^{\dagger} H_D\right)_2\right]^2 + \lambda_7' \left(H_D^{\dagger} H_D^{\dagger}\right)_1 (H_D H_D)_1 + \lambda_8 \left(H_S^{\dagger} H_S\right)^2 \\
& + \lambda_9' \left(H_D^{\dagger} H_D\right)_1 H_S'^{\dagger} H_S' + \lambda_9'' \left(H_D^{\dagger} H_S'\right)_2 (H_S'^{\dagger} H_D)_2 + \lambda_9''' \left(\left(H_D^{\dagger} H_S'\right)_2^2 + h.c.\right) \\
& + \lambda_{10}' \left(H_D^{\dagger} H_D\right)_1 H_S^{\dagger} H_S + \lambda_{10}'' \left(H_D^{\dagger} H_S'\right)_2 (H_S'^{\dagger} H_D)_2 + \lambda_{10}''' \left(\left(H_D^{\dagger} H_S\right)_2^2 + h.c.\right) \\
& + \lambda_{11} \left(H_D^{\dagger} H_D^{\dagger}\right)_2 (H_D H_D)_2 + \lambda_{12} (H_S'^{\dagger} H_S')^2 \\
& + \lambda_{13}' H_S'^{\dagger} H_S' H_S^{\dagger} H_S + \lambda_{13}'' \left(H_S'^{\dagger} H_S'^{\dagger} H_S H_S + h.c.\right) + \lambda_{13}''' H_S'^{\dagger} H_S H_S^{\dagger} H_S'
\end{aligned} \tag{8}$$

PHYSICAL REVIEW D **70**, 036007 (2004)

Higgs potential in a minimal S_3 invariant extension of the standard model

Jisuke Kubo,[1,2] Hiroshi Okada,[2] and Fumiaki Sakamaki[2]
[1]*Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 Munich, Germany*
[2]*Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*
(Received 30 April 2004; published 26 August 2004)

II. S_3 INVARIANT HIGGS POTENTIAL AND SOFT S_3 BREAKING

A. S_3 invariant Higgs potential and its problem

The most general, S_3 invariant, renormalizable potential is given by [1]

$$V_H = V_{2H} + V_{4H}, \tag{9}$$

$$V_{2H} = -\mu_1^2 (H_1^\dagger H_1 + H_2^\dagger H_2) - \mu_3^2 H_S^\dagger H_S,$$

$$\begin{aligned} V_{4H} = &+ \lambda_1 (H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2 (H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\ &+ \lambda_3 [(H_1^\dagger H_2 + H_2^\dagger H_1)^2 + (H_1^\dagger H_1 - H_2^\dagger H_2)^2] \\ &+ [\lambda_4 f_{ijk} (H_S^\dagger H_i)(H_j^\dagger H_k) + \text{H.c.}] + \lambda_5 (H_S^\dagger H_S)(H_1^\dagger H_1 \\ &+ H_2^\dagger H_2) + \lambda_6 \{(H_S^\dagger H_1)(H_1^\dagger H_S) + (H_S^\dagger H_2)(H_2^\dagger H_S)\} \\ &+ \{\lambda_7 [(H_S^\dagger H_1)(H_S^\dagger H_1) + (H_S^\dagger H_2)(H_S^\dagger H_2)] + \text{H.c.}\} \\ &+ \lambda_8 (H_S^\dagger H_S)^2, \end{aligned} \tag{10}$$

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- It has the **highest** level of flavour symmetry.

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- It has the **highest** arbitrariness without breaking the flavour symmetry.

Stage I: A quick overview...

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

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Massless massive fundamental particles

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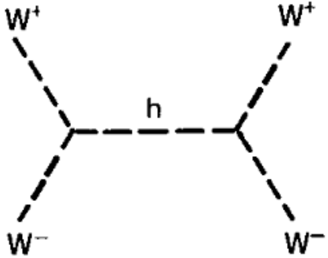
Therefore all particles within the SM (without the Higgs mechanism) appear massless.

But then,

But then, the theory itself demands the introduction of a scalar particle that help us to remove some residual divergences in order to guarantee renormalizability through diagrams of the type:



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All these issues are related.

The Englert-Brout-Higgs-Guralnik-Hagens-Kibble Mechanism

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The Higgs Mechanism

- Spontaneous Symmetry Breaking $\rightarrow$ massive gauge bosons

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- Yukawa interactions $\rightarrow$ massive fermions
- The needed scalar particle

Particle Physics For Dummies.

Spontaneous Symmetry Breaking

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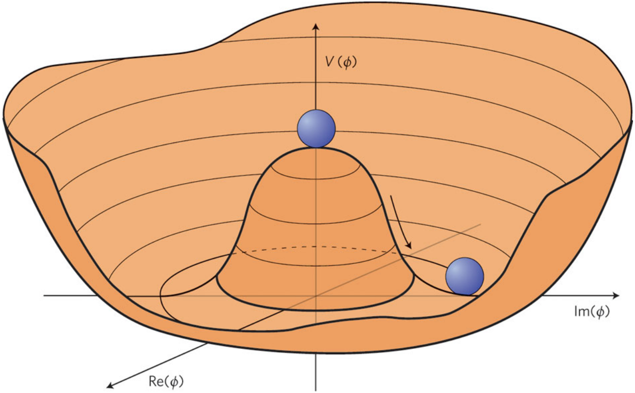
An isospin weak doublet is introduced, and with it the Higgs lagrangian is constructed as a G_{SM} invariant.

$$\mathcal{L}_H = (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$



www.nature.com/nphys/journal/v7/n1/fig_tab/nphys1874_F1.html

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$$G_{Phys.World} = SU(3)_C \otimes U(1)_{EM} \otimes U(1)_{b-l}$$

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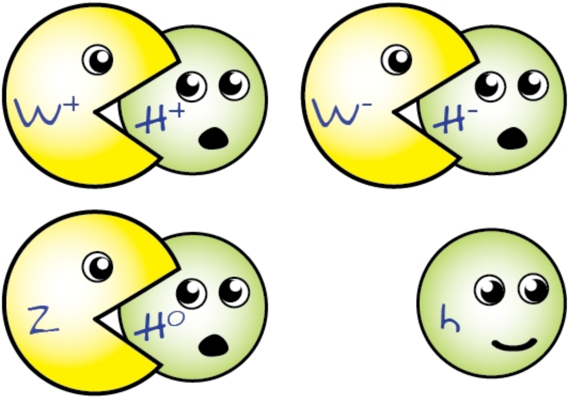
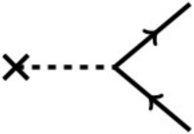


www.quantumdiaries.org/2011/10/10/who − ate − the − higgs/

Yukawa interactions

$$Y \bar{f}_L \phi f_R + h.c.$$



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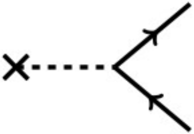
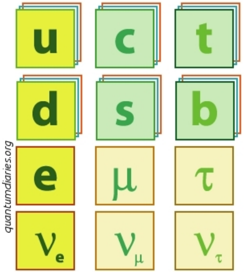


$$vY\bar{f}_L f_R + h.c.$$

Houston, we have a flavour problem...

Matter in the universe is found to be constituted by a single family of fermions (ν_e,e,u,d), nevertheless, Matter in the universe is found to be constituted by a single family of fermions (ν_e,e,u,d), nevertheless, in the "quantum world" there are additionally two more replicas.



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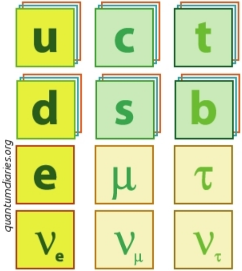

Why 3 (at least) generations of matter?

Let's take a quick deeper look into this mystery.

Some aspects of the flavour problem:

- Quark weak mixing angles (PDG 2010):
  - $\theta_{12} \approx 13.0^o$
  - $\theta_{23} \approx 2.4^o$
  - $\theta_{13} \approx 0.2^o$
- Lepton weak mixing angles (Shwetz, Tortola, & Valle 2011):
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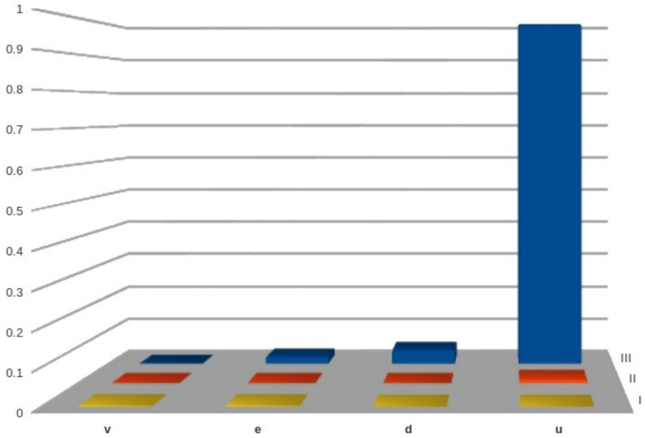
- The mass hierarchy.

Mass-Hierarchy Plot



Stage II: A quick glance into the MS_3IESM

Some references of works with an S_3 symmetry...

- S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)
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- R. Yahalom, Phys. Rev. D29, 536 (1984)
- A. Mondragón et al, Phys. Rev. D59, 093009, (1999)
- J. Kubo et al, Prog. Theor. Phys. 109, 795 (2003)
- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- A. Mondragon et al, Phys. Rev. D76, 076003, (2007)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- T. Teshima et al, arXiv:1103.6127 (2011)
- G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)

- And many more... I apologize for those references I don't include.

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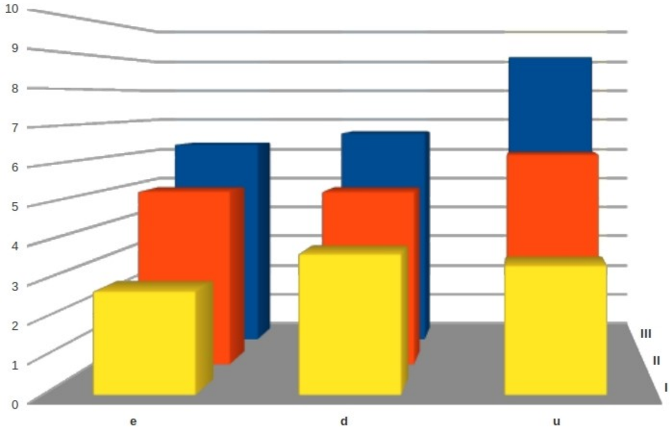
- After fermions gain mass, families become **distinguishable**.
- Was the mass distributed following the irreps of S_3 in the 3d-real representation?

The Log-Mass Plot



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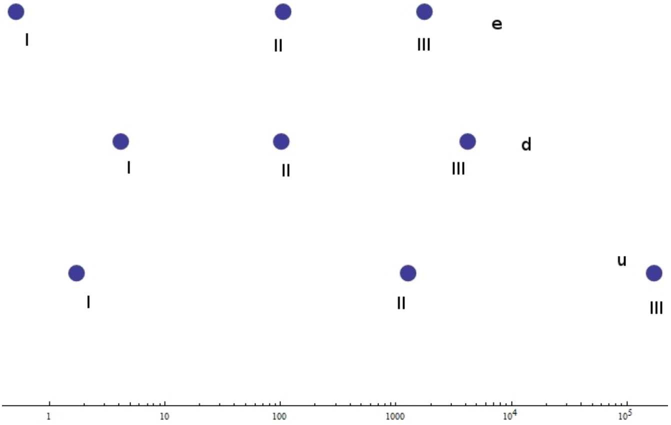

Smile!

Three generations (flavours):

- Grandparents
- Parents
- Children

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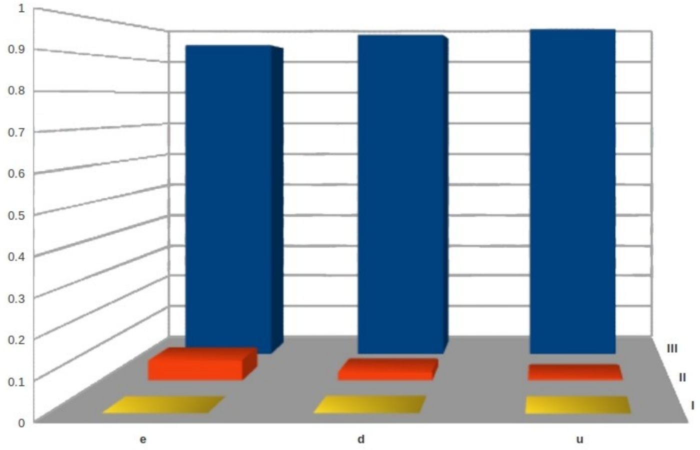
Any conclusion about the mass hierarchy (flavour structure) **can not** be done from this **global picture**.

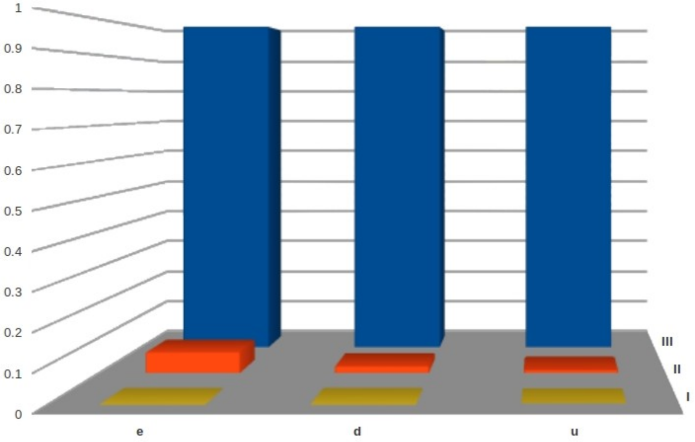
To see the **flavour structure** we **need to interrelate** families in each **independent sector**.

Say Tequila!

Mass-Percent Plot



The Flavour Spectrum



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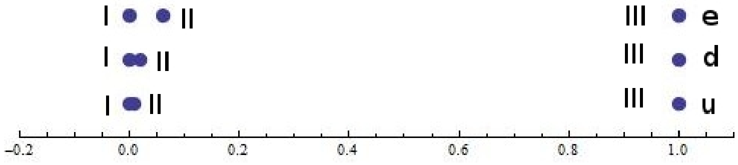

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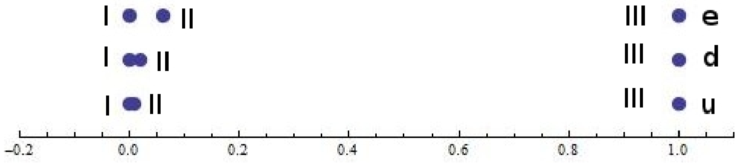
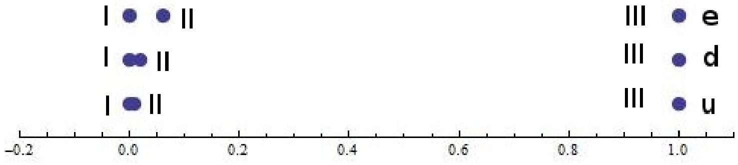


- S_3 it's conserved.

The Flavour Spectrum

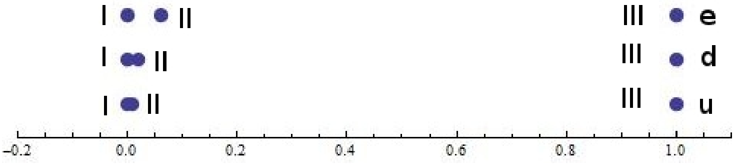


- S_3 it's conserved.

  A single Higgs weak doublet:

$$\mathcal{M}_f = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

The Flavour Spectrum



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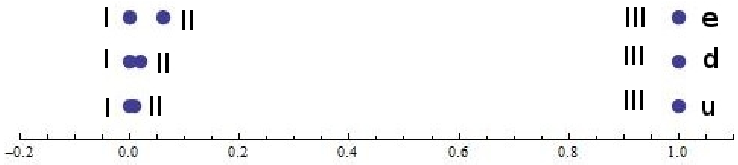
- We need at least to introduce additionally two Higgs weak-doublets more to the SM.

The Flavour Spectrum



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$$\mathcal{M}_f = \begin{pmatrix} \mu_1 + \mu_2 & \mu_4 & \mu_5 \\ \mu_4 & \mu_1 - \mu_2 & \mu_6 \\ \mu_7 & \mu_8 & \mu_3 \end{pmatrix}$$

A **special feature** arises from the theory:

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The concepts of flavours and generations are taken to a more fundamental level.

Stage III: Flavouring the Higgs potential

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- E. Derman, Phys. Rev. D19, 317 (1979)
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- O. Félix-Beltrán et al, J. Phys.: Conf. Ser. 171 012028 (2009)
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- G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)
- Again, there are many more, I apologize for those not included.

Cooking recipe for a Higgs Potential with S_3 flavour

The essential ingredients to work it out were:

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The tensorial products between irreps:

- $\mathbf{1}_S \otimes \mathbf{1}_S = \mathbf{1}_S$
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- $\mathbf{1}_A \otimes \mathbf{1}_S = \mathbf{1}_A$
- $\mathbf{1}_S \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_S + \mathbf{1}_A + \mathbf{2}$

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To carefully carry the weak $(SU(2)_L)$ index.

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$n = 2$:

- $\mathbf{1_S} \otimes \mathbf{1_S}$
- $[\mathbf{2} \otimes \mathbf{2}]_\mathbf{S}$

1. Find out all the **l.i.** S_3-invariant terms for **2** and **4** scalar fields.

$n = 2$:

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- $[\mathbf{2} \otimes \mathbf{2}]_\mathbf{S}$

$n = 4$:

- $\mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{1_S} \otimes \mathbf{2})]_\mathbf{S}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{2}]_\mathbf{S}$
- $(\mathbf{2} \otimes \mathbf{2})_\mathbf{A} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{A}$
- $(\mathbf{2} \otimes \mathbf{2})_\mathbf{S} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{S}$
- $[(\mathbf{2} \otimes \mathbf{2})_\mathbf{2} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{2}]_\mathbf{S}$

2. Take an explicit convention for the whole theory (Yukawa lagrangian and Higgs potential) of where to place the symmetric and antisymmetric doublet components.

$$H_D = \begin{pmatrix} H_{DA} \\ H_{DS} \end{pmatrix}$$

$$(f_{DA}, f_{DS})^T \otimes (g_{DA}, g_{DS})^T = \frac{1}{\sqrt{2}}(f_{DA}g_{DA} + f_{DS}g_{DS})_{\mathbf{1}_S} \oplus \frac{1}{\sqrt{2}}(f_{DA}g_{DS} - f_{DS}g_{DA})_{\mathbf{1}_A}$$

$$\oplus \frac{1}{\sqrt{2}} \begin{pmatrix} f_{DA}g_{DS} + f_{DS}g_{DA} \\ f_{DA}g_{DA} - f_{DS}g_{DS} \end{pmatrix}_{\mathbf{2}}$$

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  - $(\mathbf{2_w} \otimes \mathbf{2_w})_\mathbf{S} \otimes (\mathbf{2_{w'}} \otimes \mathbf{2_{w'}})_\mathbf{S} = \mathbf{1}_{\mathbf{S_3 \otimes G_{SM}}}$
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- In terms of the Higgs fields:
  - $\frac{1}{2}(H_{1w}^\dagger H_{1w} + H_{2w}^\dagger H_{2w})^2$
  - $\frac{1}{2}[(H_{1w}^\dagger H_{1w})^2 + (H_{2w}^\dagger H_{2w})^2 + (H_{1w}^\dagger H_{2w})^2 + (H_{2w}^\dagger H_{1w})^2]$
  - $\frac{1}{2}[(H_{1w}^\dagger H_{1w})^2 + (H_{2w}^\dagger H_{2w})^2 + (H_{1w}^\dagger H_{2w})^2 + (H_{2w}^\dagger H_{1w})^2]$

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5. Group similar terms and relate their couplings by new parameters.

The most general and renormalizable S_3-invariant Higgs potential is:

$$V = V_{2H} + V_{4H}$$

- $V_{2H} = -\mu_s^2 x_s - \mu_D^2 (x_1 + x_2)$
- $V_{4H} = \mathbf{a} x_s^2 + \mathbf{b}[(y_{S1}^2 + y_{1S}^2 + y_{S2}^2 + y_{2S}^2) + x_s(x_1 + x_2) + y_{S1}y_{1S} + y_{S2}y_{2S}]$ $+\mathbf{c} f_{ijk}(y_{Si}y_{jk} + h.c.) + \mathbf{d}(x_1 + x_2)^2$ $+\mathbf{e}(y_{12} - y_{21})^2 + \mathbf{f}[(x_1 - x_2)^2 + (y_{12} + y_{21})^2]$

where $x_s = H_s^\dagger H_s$, $x_i = H_i^\dagger H_i$ $(i = 1, 2)$, $y_{ij} = H_i^\dagger H_j$ $(i, j = 1, 2, s)$, and $f_{112} = f_{121} = f_{211} = -f_{222} = 1$.

- $a \equiv 3\lambda_1$
- $b \equiv \dfrac{\lambda_2}{\sqrt{2}}$
- $c \equiv \dfrac{3\lambda_3}{2}$
- $d \equiv \dfrac{\lambda_4 + \sqrt{2}\lambda_6}{2}$
- $e \equiv \dfrac{\lambda_4 + \lambda_5 - \sqrt{2}\lambda_6}{2}$
- $f \equiv \dfrac{\lambda_6 + \sqrt{2}\lambda_4}{2\sqrt{2}}$

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- The recipe can be **extended** to the construction of other flavour-invariant Higgs potentials.
- Just **six** self-couplings parameters are needed. **(more predictive power)**

Thanks for your **attention**.

Professor Alfonso Mondragón Ballesteros

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