Flavour Parametrization of couplings with fermions in the 2HDM: the case with the 4-zero texture mass matrix

Lao-Tse López

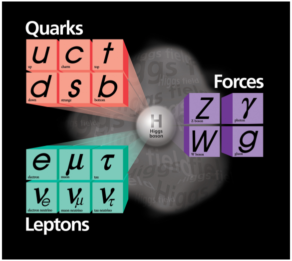
División de Ciencias e Ingeniería - Campus León
Universidad de Guanajuato
XIII Mexican Workshop on Particles and Fields.

October 26, 2011

Outline

1. The 2HDM
    - Motivations for SM extensions in the scalar sector.
    - Features of the 2HDM.
2. Yukawa Couplings
    - Versions with NFC
    - Version III
    - The flavour parametrization

Free parameters of the SM



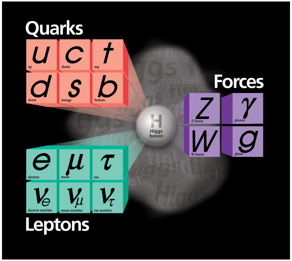
The 19 parameters

- 3 gauge coupling constants
- 6 quarks masses
- 3 charged leptons masses
- 2 gauge boson masses ($W^{\pm}$, Z^0)
- 3 quark mixing angles
- 1 electroweak mixing angle (θ_W)
- 1 CP phase in the CKM matrix

...plus 7 parameters with massive neutrinos

- 3 neutrino masses
- 3 mixing angles
- 1 CP phase in the MPNS matrix

Free parameters of the SM



The 19 parameters

- 3 gauge coupling constants
- 6 quarks masses
- 3 charged leptons masses
- 2 gauge boson masses $(W^{\pm}, Z^0)$
- 3 quark mixing angles
- 1 electroweak mixing angle (θ_W)
- 1 CP phase in the CKM matrix

...plus 7 parameters with massive neutrinos

- 3 neutrino masses
- 3 mixing angles
- 1 CP phase in the MPNS matrix

Main motivations to extent the SM scalar sector

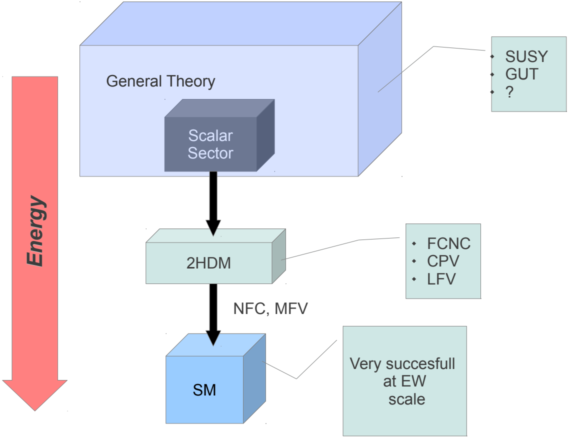
- New sources of CP violation are needed in order to explain matter-antimatter asymmetry
- Biger number of doublets can parametrize, in a simple way, small deviations from SM symmetries(LFV, CPV, FCNC,...).
- Scalar sector of SM is too simple although not experimentally tested
- General models (SUSY, Peccei-Quinn, etc.) have two or more doublets or even more complex scalar sector

We can not make arbitrary extensions of the scalar sector

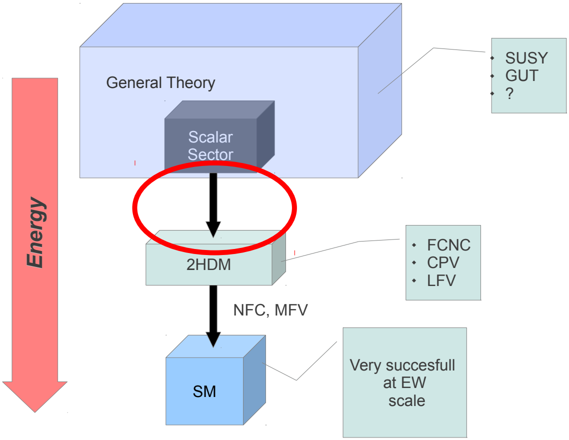
- $\rho = \frac{m_W^2}{m_z^2 \cos^2\theta_W} \simeq 1$
- FCNC must be small ()

[Gunion *et. al.*,1990]

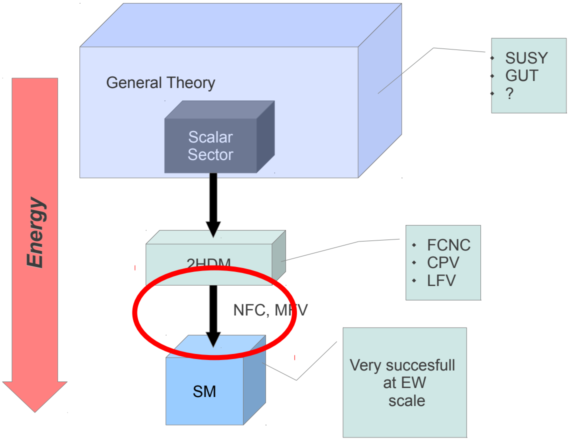
Why study the 2HDM?



Why study the 2HDM?



Why study the 2HDM?



Why doublets?

In general

$$\rho \equiv \frac{\sum_{T,Y}\left[4T(T+1)-Y^2)\right]|v_{T,Y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 |v_{T,Y}|^2}$$

$$\rho \simeq 1$$

$$(2T+1)^2 - 3Y^2 = 1$$

Posibilities

- $T = \frac{1}{2}, Y = \pm 1$
- $T = 3, Y = \pm 4$
- $T = \frac{25}{2}, Y = \pm 15$
- $\ldots$

Yukawa sector of 2HDM($N = 2$)

$$\mathcal{L}_{HF}^{2HDM} = -\overline{Q}_L \sum_{a=1}^{N} (Y_a^d \Phi_a d_R + Y_a^u \tilde{\Phi}_a u_R) - \overline{L}_L \sum_{a=1}^{N} Y_a^l \Phi_a l_R + h.c.$$

$$\Phi_a = \begin{pmatrix} \varphi_a^+ \\ \varphi_a^0 e^{i\theta_a} \end{pmatrix}, \quad \varphi_a^0 = v_a + \frac{\rho_a + i\eta_a}{\sqrt{2}} \quad ; \quad a = 1, ..., N$$

Mass Matrix

$$M_f = \frac{1}{\sqrt{2}}(v_1 Y_1^f + v_2 Y_2^f) \quad , \quad f = u, d, l$$

- The same fermions than SM
- The same SM symmetries and possibly a discrete symmetry
- $4(N-1)-3$ new physical scalars
- New sources of CP violation (θ_a and phases in $Y_a^{u,d,l}$)
- Presence of Flavour Changing Neutral Currents at tree level

Neutral Higgs mixing and new parameters.

$$
\begin{aligned}
H^{\pm} &= -\sin\beta\varphi_1^{\pm} + \cos\beta\varphi_2^{\pm} \\
A^0 &= \sqrt{2}\,(-\sin\beta\eta_1 + \cos\beta\eta_2) \\
H^0 &= \sqrt{2}\,[(\rho_1 - v_1)\cos\alpha + (\rho_2 - v_2)\sin\beta] \\
h^0 &= \sqrt{2}\,[-(\rho_1 - v_1)\sin\alpha + (\rho_2 - v_2)\cos\alpha]
\end{aligned}
$$

With $\beta = \frac{v_2}{v_2}$ and $v^2 = v_1^2 + v_2^2$

Higgs basis

$$
\langle H_1\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad ; \quad \langle H_2\rangle_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
$$

Natural Flavour Conservation (NFC)

PHYSICAL REVIEW D VOLUME 15, NUMBER 7 1 APRIL 1977

Natural conservation laws for neutral currents*

Sheldon L. Glashow and Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 20 August 1976)

We explore the consequences of the assumption that the direct and induced weak neutral currents in an $SU(2)\otimes U(1)$ gauge theory conserve all quark flavors *naturally*, i.e., for all values of the parameters of the theory. This requires that all quarks of a given charge and helicity must have the same values of weak T_3 and $\vec{T}^2$. If all quarks have charge $+2/3$ or $-1/3$ the only acceptable theories are the "standard" and "pure vector" models, or their generalizations to six or more quarks. In addition, there are severe constraints on the couplings of Higgs bosons, which apparently cannot be satisfied in pure vector models. We also consider the possibility that neutral currents conserve strangeness but not charm. A natural seven-quark model of this sort is described. The experimental consequences of charm nonconservation in direct or induced neutral currents are found to be quite dramatic.

I. INTRODUCTION

It has been known for many years that there are no strangeness-changing neutral-current weak interactions, or none with anything like the strength of the familiar charged-current weak interactions. We see this from the slowness of such decays as $K_L^0 \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$, and even more strongly (and independently of the nature of the lepton couplings) from the size of the K_1^0-K_2^0 mass difference. For this reason, until strangeness-conserving neutral-current weak interactions were discovered

an effective strangeness-changing neutral current of order αG_F. In still other theories, the exchange of Higgs bosons can produce a strangeness-changing neutral current of roughly the same order. In principle these effects may perhaps be eliminated by a retuning of the parameters of the theory (including in the last case the parameters of the Higgs-boson interactions), but we would find a theory much more attractive if the neutral currents conserved strangeness naturally.

In this paper we explore the implications of the condition that the conservation laws obeyed by the

NFC (...or think twice before break a symmetry)

PHYSICAL REVIEW D VOLUME 15, NUMBER 7 1 APRIL 1977

Natural conservation laws for neutral currents*

Sheldon L. Glashow and Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 20 August 1976)

The observed suppression of the strangeness-changing neutral currents is so dramatic numerically that we find it hard to believe that it comes about because the parameters of the theory just happen to take certain values. We would prefer instead to believe that the conservation of strangeness by the neutral currents is *natural*, that is, that it follows from the group structure and representation content of the theory, and does not depend on the values taken by the parameters of the theory.

Invariant potential under $SU(2) \times U(1)$ of the 2HDM

$$
\begin{aligned}
V = & \, m_1 \Phi_1^\dagger \Phi_1 + m_2 \Phi_2^\dagger \Phi_2 + \underline{m_3 \left(e^{i\delta_3} \Phi_1^\dagger \Phi_2 + e^{-i\delta_3} \Phi_2^\dagger \Phi_1 \right)} \\
& + a_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + a_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + a_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\
& + a_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + a_5 \left[e^{i\delta_5} \left(\Phi_1^\dagger \Phi_2 \right)^2 + e^{-i\delta_5} \left(\Phi_2^\dagger \Phi_1 \right)^2 \right] \\
& + a_6 \left(\Phi_1^\dagger \Phi_1 \right) \left(e^{i\delta_6} \Phi_1^\dagger \Phi_2 + e^{-i\delta_6} \Phi_2^\dagger \Phi_1 \right) \\
& + a_7 \left(\Phi_2^\dagger \Phi_2 \right) \left(e^{i\delta_7} \Phi_1^\dagger \Phi_2 + e^{-i\delta_7} \Phi_2^\dagger \Phi_1 \right)
\end{aligned}
$$

Type I

$$
\begin{aligned}
\Phi_1 & \rightarrow -\Phi_1 \\
\Phi_2 & \rightarrow \Phi_2
\end{aligned}
$$

Type II

$$
\begin{aligned}
\Phi_1 & \rightarrow -\Phi_1 \\
\Phi_2 & \rightarrow \Phi_2 \\
d_R^i & \rightarrow -d_R^i
\end{aligned}
$$

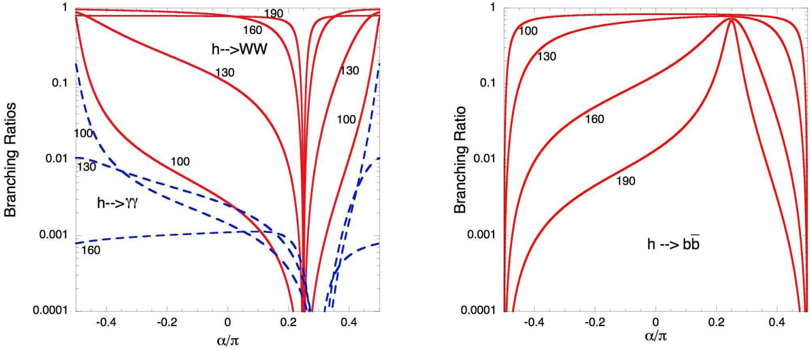
Version	u_R^i	d_R^i	e_R^i
I	Φ_2	Φ_2	Φ_2
II	Φ_2	Φ_1	Φ_1
Lepton-specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

$$
\begin{aligned}
-\mathcal{L}_Y^{2HDM} = & \sum_{f=u,d,l} \left(\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H - i \xi_A^f \bar{f} \gamma_5 f A \right) \\
& + \left[\frac{\sqrt{2} V_{ud}}{v} \bar{u} \left(m_u \xi_A^u P_L + m_d \xi_d \xi_A^d P_R \right) d H^+ + \frac{\sqrt{2} m_\ell \xi_A^\ell}{v} \bar{\nu}_L \ell_R H^+ + \text{H.c.} \right]
\end{aligned}
$$

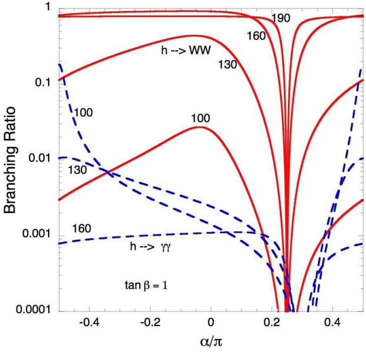
M. Aoki, *et.al*, Phys.Rev.D**80**(2009)

	Type I	Type II	Lepton-specific	Flipped
ξ_h^u	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$
ξ_h^d	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$
ξ_h^ℓ	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$-\sin\alpha / \cos\beta$	$\cos\alpha / \sin\beta$
ξ_H^u	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$
ξ_H^d	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$
ξ_H^ℓ	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\cos\alpha / \cos\beta$	$\sin\alpha / \sin\beta$
ξ_A^u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_A^d	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
ξ_A^u	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$

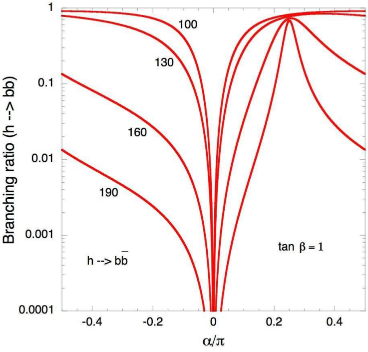
Version I


h-->WW
h-->γγ
Branching Ratios
α/π
190
160
130
100
h --> bb̄
Branching Ratio
α/π

Version II

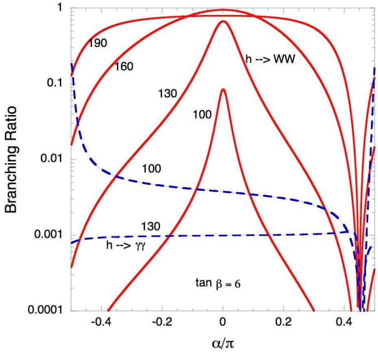


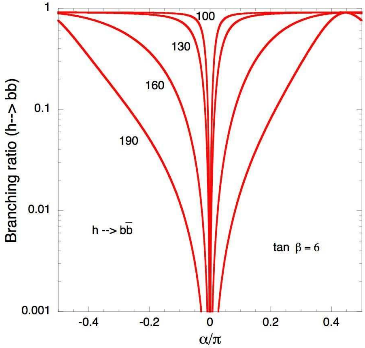
1
0.1
0.01
0.001
0.0001
Branching Ratio
190
160
h --> WW
130
100
100
130
160
h --> γγ
tan β = 1
-0.4
-0.2
0
0.2
0.4
α/π



1
0.1
0.01
0.001
0.0001
Branching ratio (h --> bb)
100
130
160
190
tan β = 1
h --> bb
-0.4
-0.2
0
0.2
0.4
α/π

Version II


Branching Ratio
1
0.1
0.01
0.001
0.0001
190
160
130
100
h --> WW
100
130
h --> γγ
tan β = 6
-0.4
-0.2
0
0.2
0.4
α/π


Branching ratio (h--> bb)
1
0.1
0.01
0.001
100
130
160
190
h --> bb̄
tan β = 6
-0.4
-0.2
0
0.2
0.4
α/π

Why a Version III?

- Discrete symmetries imposed on the Higgs doublets are too restrictive that it not possible to describe deviations from the SM symmetries.
- The magnitude of couplings have the same order of magnitude for every generation.
- Relax the restrictions on the Higgs doublets introduce FCNC at tree level
- The FCNC need a mechanism to control them.

At the Higgs basis

$$
\begin{aligned}
\mathcal{L}_Y &= \eta^u \bar{Q}_L \tilde{H}_1 u_R + \eta^d \bar{Q}_L H_1 d_R + \eta^\ell \bar{L}_L H_1 \ell_R \\
&+ \hat{\xi}^u \bar{Q}_L \tilde{H}_2 u_R + \hat{\xi}^d \bar{Q}_L H_2 d_R + \hat{\xi}^\ell \bar{L}_L H_2 \ell_R + \text{H.c.}
\end{aligned}
$$

Cheng and Sher *Ansatz*

- First try to control FCNSI was $m_H \sim \mathcal{O}(\text{TeV})$
- A Fritzsch form mass matrix can reproduce V_{CKM} at the quark sector (remember last saturday's talk of Prof. Mondragón)

$$M_f = \begin{pmatrix} 0 & C_f & 0 \\ C_f^* & D_f & B_f \\ 0 & B_f^* & A_f \end{pmatrix}$$

- FCNSI inherits the hierarchy of mass matrix

$$\xi_{ij}^f = \frac{\sqrt{2}}{v} \underbrace{\sqrt{m_i^f m_j^f}}_{\sim M_{ij}^f} \tilde{\chi}_{ij}^f$$

- In order to preserve the hierarchy $|\tilde{\chi}_{ij}^f| \sim 1$

Mass matrix diagonalization

$$\bar{M}_f \equiv \mathrm{diag}(m_1^f, m_2^f, m_3^f) = U_L^\dagger M_f U_R$$

$$\begin{aligned} H_f &\equiv M_f M_f^\dagger = U_L^f \bar{M}_f^2 U_L^{f\dagger} \\ I_r &\equiv M_f^\dagger M_f = U_R^f \bar{M}_f^2 U_R^{f\dagger} \end{aligned}$$

Hermitian case $M_f = M_f^\dagger$; $I_f = H_f$;$U_R^f = U_L^f$

Invariants equations

$$\mathrm{Det}(H_f) = \mathrm{Det}(\bar{M}_f^2)$$

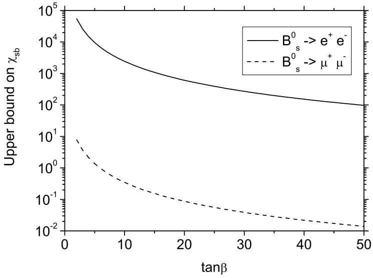
$$\mathrm{Tr}(H_f) = Tr(\bar{M}_f^2)$$

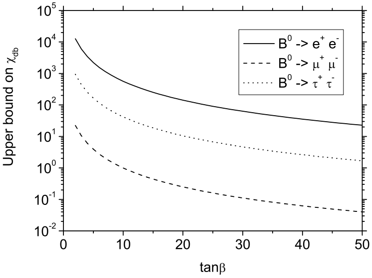
$$\mathrm{Tr}^2(H_f) - \mathrm{Tr}(H_f^2) = \mathrm{Tr}^2(\bar{M}_f^2) - \mathrm{Tr}\left((\bar{M}_f^2)^2\right)$$

$$\begin{aligned} A_f &= A_f(m_1^f, m_2^f, m_3^f, x_{CKM}, \phi) \\ B_f &= B_f(m_1^f, m_2^f, m_3^f, A_f) \\ C_f &= C_f(m_1^f, m_2^f, m_3^f, A_f) \\ D_f &= D_f(m_1^f, m_2^f, m_3^f, A_f) \end{aligned}$$

Some problems with Version III

Bounds with leptonic B decays ($m_{A^0} = 300\text{GeV}$)





- Phys.Rev.D55,3156 reported for B, D, K systems at some scenarios

$$(\chi_{ds}, \chi_{uc}, \chi_{bd}, \chi_{bs}) \leq (0.1, 0.2, 0.06, 0.06)$$

- Phys.Rev.D67,075011 reported for $\mu \to e\gamma$ with internal fermion τ, $\chi_{e\tau}\chi_{\mu\tau} < 0.04$

Is there another way to introduce the hierarchy of masses in the scalar interactions?

- Aligned 2HDM

$$Y_2 = aY_1$$

- Partially Aligned 2HDM

$$Y_2^f = \begin{pmatrix} 0 & c_2 C_f & 0 \\ c_2^* C_f^* & d_2 D_f & b_2 B_f \\ 0 & b_2^* B_f^* & A_f \end{pmatrix}$$

$$\tilde{\chi}_{ij}^f = \tilde{\chi}_{ij}^f(b_2, c_2, d_2, a_2, \phi_C, \phi_B)$$

Flavour Transformation

$$\xi^f = a' \cdot A_L M_f A_R$$

Orthogonal transformation $A_L = \mathcal{O}^T$; $A_R = \mathcal{O}$

$$\xi_f = a' \cdot R^T(\theta_1, \theta_2, \theta_3) M_f R(\theta_1, \theta_2, \theta_3)$$

- Same physical content than mass matrix
  Tr$(\xi^f) = a'$Tr(M_f), Det$(\xi^f) = a' \cdot$ Det(M_f)
- Establish relations between χ_{ij} reducing the number of free parameters.
- Suitable parameters of rotations can reproduce the version I, II and the A-2HDM

The infinitesimal rotation preserves at approximately the hierarchy of the mass matrix

$$\xi^f = \begin{pmatrix} -2(\theta_1+\theta_3)|C_f| & |C_f|\left(1-(\theta_1+\theta_3)\frac{D_f}{|C_f|}\right) & \theta_2|C_f| - (\theta_1+\theta_3)|B_f| \\ |C_f|\left(1-(\theta_1+\theta_3)\frac{D_f}{|C_f|}\right) & |D_f| + 2(\theta_1+\theta_3)|C_f| - 2\theta_2|B_f| & |B_f| + \theta_2(D_f - A_f) \\ \theta_2|C_f| - (\theta_1+\theta_3)|B_f| & |B_f| + \theta_2(D_f - A_f) & A_f + 2\theta_2|B_f| \end{pmatrix}$$

$N = 3$

$$\xi_f = a' U^\dagger M_f U$$

$$U = \sum_a C_a \lambda_a \qquad ; \qquad C_{ab} \equiv C_a^* \cdot C_b$$

- Again the mass matrix inherit its physical content on ξ
- In the hermitian case: $C_{ab} = C_{ba}^*$
- There is a connection between the experimental measurements and the group parameters.

$$\mathrm{Tr}(\lambda_a \xi^f \lambda_b) = \sum_{c,d} C_{cd} \mathrm{Tr}(\lambda_a \lambda_c M_f \lambda_d \lambda_b)$$

- The texture of the mass matrix is preserved if U have only contribution of λ_3, λ_8

$$M_f = \sum_a \tilde{m}_a \lambda_a$$

The system can be solved if we define

$$\mathcal{V} = \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{88} \end{pmatrix}; \quad \mathcal{W} = \begin{pmatrix} \mathrm{Tr}(\lambda_1 \xi^f \lambda_1) \\ \mathrm{Tr}(\lambda_1 \xi^f \lambda_2) \\ \vdots \\ \mathrm{Tr}(\lambda_8 \xi^f \lambda_8) \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} \mathrm{Tr}(\lambda_1 \lambda_1 M_f \lambda_1 \lambda_1) & \ldots & \mathrm{Tr}(\lambda_1 \lambda_8 M_f \lambda_8 \lambda_1) \\ \vdots & \ddots & \vdots \\ \mathrm{Tr}(\lambda_8 \lambda_1 M_f \lambda_1 \lambda_8) & \ldots & \mathrm{Tr}(\lambda_8 \lambda_8 M_f \lambda_8 \lambda_8) \end{pmatrix}$$

$$\mathcal{V}_\mu = \mathcal{M}^{-1}_{\mu\nu} \mathcal{W}_\nu$$

Final comments

- 2HDM is good model to parametrize possible new physics effects as tree FCNC at tree level.
- The different version are enough different in order to distinguish them.
- The flavour parametrization allows a reduction of free parameters in the orthogonal parametrization.
- At the unitary parametrization many parameters are present but is a good way to analyse the structure of group

Thank You

Thank You