Renormalization of a Second Order Formalism for Spin $1/2$ Fermions

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Brief Historical Review of Second Order Formalisms for spin $1/2$

- (1927) V. Fock, Relativistic Quantum Mechanics of spin $1/2$ through a second order differential equation.
- (1928) Dirac, P. A. M.
- (1951,1958) Feynman - Gell-Mann[1] used a two component spinorial field that satisfies ($g = 2, \xi = 0$).

$$[(i\partial_\mu - A_\mu)^2 + \vec{\sigma} \cdot (\vec{B} \pm i\vec{E})]\phi = m^2\phi,$$

  Their main motivation was to describe the weak interactions.
- ...
- (1961) Hebert Pietschmann[2], one loop renormalization of the Feynman-Gell-Mann theory.

Showing the equivalence with the Dirac framework has been always a goal in these works.

[1] Phys. Rev. 84, 108 , 1951; Phys. Rev. 109, 193, 1958

[2] Acta Phys. Austr. 14, 63 (1961)

Motivations

- The NKR second order formalism for massive spin $3/2$ particles is an alternative[3] to the inconsistent Rarita-Schwinger theory of electromagnetic interactions.
- The case of spin $1/2$ is interest by itself e.g. in this theory the gyromagnetic factor g is a free parameter $\Rightarrow$ a low energy effective theory of particles with $g \neq 2$, e. g. proton.
- We expect that this give us a better understanding of the properties of spin $1/2$ particles, e.g. the classical limit[4].
- ¿Generalizations?

In this work we used general principles of QFT to study the quantization and Renormalization. We will only compare with the conventional Dirac results only at the end.

[3] Eur. Phys. J. A29 (2006); Phys. Rev. D77: 014009, 2008

[4] R. P. Feynman Phys. Rev. 84, 108 , 1951.

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Quantum Fields

Quantum theories that satisfy

- special relativity
- cluster descomposition principle

can be built with quantum fields $\phi_l(x)$ defined as

$$\phi_l(x) = \int d\Gamma \left[e^{ip\cdot x} u_l(\Gamma) a^\dagger(\Gamma) + e^{-ip\cdot x} v_l(\Gamma) a(\Gamma) \right],$$

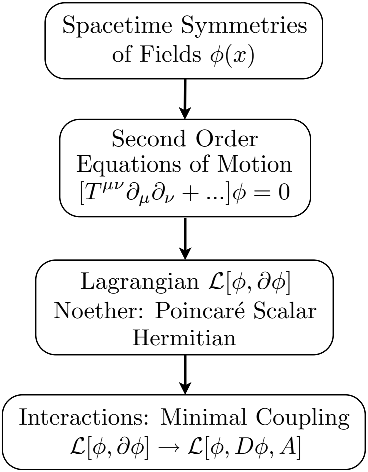
such that under a Poincaré transformation $U(\Lambda, b)$ the fields

$$U(\Lambda, b)\phi_l(x)U(\Lambda, b)^{-1} = D(\Lambda)_{ll'}\phi_{l'}(\Lambda x + b),$$

$$[\phi_l(x), \phi_m(y)]_{\mp} = 0 \quad for \qquad (x-y)^2 > 0,$$

where $D(\Lambda)_{ll'}$ is a representation of $SO(3,1)$.

Scheme of the NKR construction of QFTs



Equations of motion of the NKR formalism

General Idea: To use the Poincare invariants P^2 and W^2 to construct projectors $\mathcal{P}^{(m,s)}$ over spaces of definite mass and spin. Acting these projectos on the fields results in equations of motion.

For a field $\psi^{(D,m,s)}$ with only one spin sector s in a given representations $D(\Lambda)$ only a projector is necessary $\mathcal{P}^{m,s}$

$$\mathcal{P}^{m,s} = \Big(\frac{P^2}{m^2}\Big)\Big(\frac{W^2}{-s(s+1)P^2}\Big),$$

the action of this projector over the field results in the following equation of motion

$$\big(T^{D\mu\nu}_{ll'} P_\mu P_\nu - \delta_{ll'} m^2\big)\psi^{(D,m,s)}_{l'}(x) = 0,$$

where $T^{D\mu\nu}_{ll'}$ is defined by $W^2 = -\dfrac{1}{s(s+1)} T^{D\mu\nu} P_\mu P_\nu$, it depends on the generators $M^{\mu\nu}$ of the $D(\Lambda)$.

NKR for spin $1/2$ and the representations $(1/2, 0) \oplus (0, 1/2)$

For a field $\psi^{(D,m,s=1/2)}$ in the representation $D \equiv (1/2, 0) \oplus (0, 1/2)$ the NKR equation of motion can be deduced from the following family of *hermitian Poincaré scalar Lagrangians*

$$\mathscr{L} = \partial_\mu \bar{\psi} T^{\mu\nu} \partial_\nu \psi - m^2 \bar{\psi}\psi,$$

where $T^{\mu\nu} = g^{\mu\nu} - igM^{\mu\nu} + \xi\gamma^5 M^{\mu\nu}$.

$M^{\mu\nu}$ are the generators of the $(1/2, 0) \oplus (0, 1/2)$ Lorentz group representation.

$$M^{\mu\nu} = \begin{pmatrix} M^{\mu\nu}_{(1/2,0)} & 0 \\ 0 & M_{(0,1/2)} \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Electromagnetics Interactions

Finally we introduce Electromagnetic interactions are introduced through minimal coupling

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D_\mu\bar{\psi}[g^{\mu\nu} - (ig - \xi\gamma^5)M^{\mu\nu}]D_\nu\psi - m^2\bar{\psi}\psi,$$

$g = 2, \xi = 0$ corresponds to the Feynman-Gell-Mann theory.

The interactions that contains g can be rewritten as

$$\mathscr{L}_i = -\int d^4xeg\,\bar{\psi}M^{\mu\nu}\psi F_{\mu\nu},$$

that includes the interaction $\vec{S}\cdot\vec{B} \Rightarrow$ we recognize g as the gyromagnetic factor.

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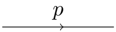
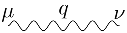
Feynman Rules

$$Z[J_\mu, \bar{\eta}, \eta] = C \int \mathcal{D}A \mathscr{D}\bar{\psi} \mathcal{D}\psi \exp\left[i \int \mathcal{L}_{ef} dx\right],$$

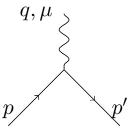
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + D_\mu \bar{\psi} T^{\mu\nu} D_\nu \psi - m^2 \bar{\psi}\psi + J^\mu A_\mu + \bar{\eta}\psi + \bar{\psi}\eta -$$

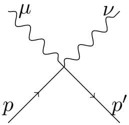




$iS(p) \equiv \frac{i}{p^2 - m^2}$

$i\Delta_{\mu\nu} \equiv \frac{-ig_{\mu\nu}}{q^2 + i\varepsilon}$





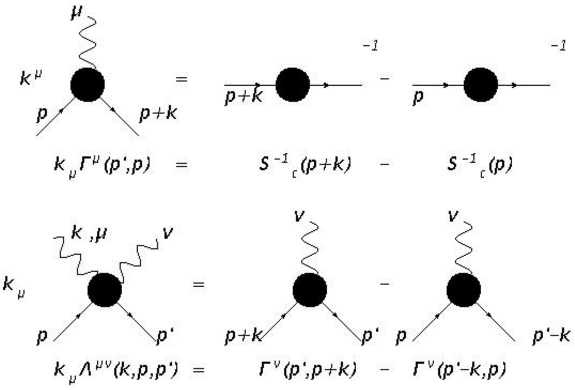
$-ieV_\mu(p, p') = -ie\left[(p' + p)_\mu + (ig + \xi\gamma^5)M_{\mu\nu}(p' - p)^\nu\right]$

$2ie^2 g^{\mu\nu}$

Ward Identities

As a consequence of gauge invariance there exist identities between the green functions

$$0 = \Big[-\frac{1}{\alpha}\Box(\partial_\mu \frac{\delta}{\delta J^\mu(x)}) - \partial_\mu J^\mu - e(\bar{\eta}\frac{\delta}{\delta\bar{\eta}(x)} + \eta\frac{\delta}{\delta\eta(x)})\Big] Z(J^\mu, \eta, \bar{\eta})$$



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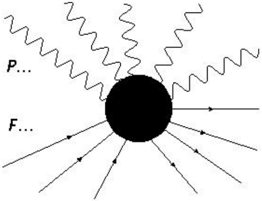
Divergencies in the Second Order Theory

Asking the Lagrangian to be dimensionless one obtains

- $[A] = [\psi] = 1,$
- $[g] = [e] = [\xi] = 0.$

Thus the greater superficial degree of divergency of a process is

$$D \leq 4 - F - P$$



The greater degree of divergency is:

- quadratic for propagators
- linear for 3 lines processes e.g. ffp
- logarithmic for 4 lines processes e.g. $ffpp$

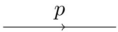
These characteristics are necessary for a theory to be renormalizable QFT.

Free Parameters and Counterterms $(\xi = 0)$

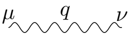
In terms of the bare parameters m_b^2, e_b, g_b the Lagrangian is

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu - ie_bA_\mu)\bar{\psi}[g^{\mu\nu} - ig_bM^{\mu\nu}](\partial_\nu + ie_bA_\nu)\psi - m_b^2\bar{\psi}\psi.$$

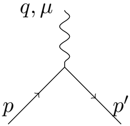
Introducing the *renormalized parameters* m^2, e y g and the renormalized fields $A_r^\mu = Z_1^{-\frac{1}{2}}A^\mu$ y $\psi_r = Z_2^{-\frac{1}{2}}\psi$ there appear the following counterterms



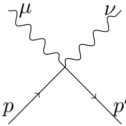
$$i(p^2 - m^2)\delta_{Z_2} - i\delta_m$$



$$-i(g^{\mu\nu}q^2 - q^\mu q^\nu)\delta_{Z_1}$$



$$-ie\left[V^\mu(p', p)\right]\delta_e + egM_{\mu\nu}(p' - p)^\nu\delta_g$$



$$2ie^2g_{\mu\nu}\delta_3$$

Dimensional Regularization

Extend the theory to d dimensions. The natural objects to be extended to d dimension are the Lorentz generators $M^{\mu\nu}$

$$[M^{\alpha\beta}, M^{\mu\nu}] = -ig^{\beta\nu}M^{\alpha\mu} + ig^{\beta\mu}M^{\alpha\nu} - ig^{\alpha\mu}M^{\beta\nu} + ig^{\alpha\nu}M^{\beta\mu}, \text{ with } g^{\mu}_{\ \mu} = d$$

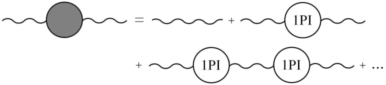
$$\{M^{\mu\nu}, M^{\alpha\beta}\} = \frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}) - \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\gamma^5,$$

e.g. we can use the last expression to calculate to calculate a trace in a fermion loop

$$tr\{M^{\mu\nu}M^{\alpha\beta}\} = \frac{f(d)}{4}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}) \text{ with } \lim_{d\to 4} f(d) = 4,$$

Photon Propagator

As usual one can express the complete photon propagator $i\Delta_c^{\mu\nu}(q)$ as



$$i\Delta_c^{\mu\nu}(q) = i\Delta^{\mu\nu}(q) + i\Delta^{\mu\sigma}[-i\Pi_{\sigma\rho}(q)][i\Delta^{\rho\nu}(q)] + \ldots$$

where $\Pi^{\mu\nu}(q)$ is the vacuum polarization

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\pi(q^2),$$

Then the complete propagator is given by

$$\Delta_c^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu \pi / q^2}{[q^2 + i\epsilon][1 + \pi]}.$$

The first condition of renormalization is that the photon doesn't acquired mass due to the radiative corrections, i.e.

$$\pi(q^2 \to 0) = 0.$$

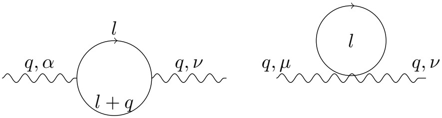
Vacuum polarization to one loop

It has the following contributions

$$-i(g^{\mu\nu}q^2 - q^\mu q^\nu)\pi(q)^2 = -i(g^{\mu\nu}q^2 - q^\mu q^\nu)\pi^*(q^2) - i(g^{\mu\nu}q^2 - q^\mu q^\nu)\delta_{Z_1}$$

The first renormalization conditions requires

$$\delta_{Z_1} = -\pi^*(q^2 = 0),$$



Finally, imposing the renormalization condition the physical vacuum polarization is

$$\pi(q^2) = \frac{2e^2}{(4\pi)^2}\int_0^1 dx \ln\left[\frac{m^2 - q^2x(1-x)}{m^2}\right]\left[(1-2x)^2 - \frac{g^2}{4}\right],$$

for $g = 2$ one recovers the one loop vacuum polarization of the conventional Dirac formalism.

Charge running in the Ultrarelativistic limit

Due to the quantum effects the classical Coulomb potential modifies as

$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{-e^2}{|\vec{q}|^2[1+\pi(-|\vec{q}|^2)]},$$

in the ultrarelativistic domain one has an effective charge given by

$$e^2_{eff} = \frac{e^2}{1+\pi(q^2 >> m^2)} = e^2 \Big/ \Big[1 - \frac{e^2}{12\pi^2}\big(1 - \frac{3}{2}[1 - \frac{g^2}{4}]\big) \ln \frac{-q^2}{Am^2}\Big],$$

$$\text{where } A \equiv \exp\left\{ \frac{5}{3} \frac{1 - \frac{9}{5}[1 - \frac{g^2}{4}]}{1 - \frac{3}{2}[1 - \frac{g^2}{4}]} \right\},$$

Which means that the gyromagnetic factor g impacts the running of the fine structure constant $\alpha(q^2)$!

Fermion Propagator

Analogously the complete fermion propagator $iS_c(p)$ could be expressed as

$$iS_c(p) = iS(p) + iS(p)[-i\Sigma(p)]iS(p) + ...$$

where $-i\Sigma(p2)$ is the fermion self energy. Adding up the series

$$S_c(p) = \frac{1}{p^2 - m^2 - \Sigma(p) + i\epsilon},$$

Second renormalization condition: m represents the physical mass of the particle, i.e. the complete propagator has a simple pole at $p^2 = m^2$

$$\Sigma(p = m^2) = 0, \qquad \frac{\partial\Sigma(p)}{\partial p^2}\Big|_{p^2=m^2} = 0.$$

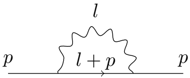
Fermion Self Energy to one loop

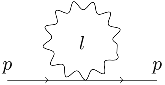
The contributions up to one loop are

$$-i\Sigma(p^2) = -i\Sigma^*(p^2) + i(p^2 - m^2)\delta_{Z_2} - i\delta_m,$$

The second renormalization conditions requires

$$i\delta_m = -i\Sigma^*(p^2 = m^2) \qquad \delta_{Z_2} = \frac{\partial \Sigma^*(p)}{\partial p^2}\Big|_{p^2=m^2},$$



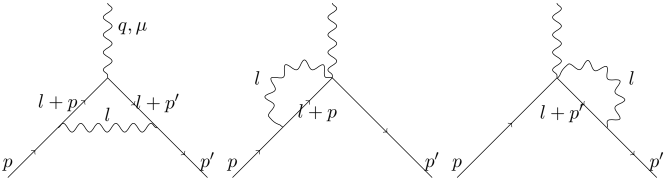


$$\Sigma(p^2) = \frac{\alpha}{\pi} p^2 \int_0^1 dx(x-1)\ln\left[\frac{m^2x - p^2x(1-x)}{m^2}\right] - \frac{3\alpha m^2}{2\pi} - \frac{\alpha}{\pi}[p^2 - m^2]\int_0^1 \frac{dx}{x}.$$

ffp Vertex

The contributions to the one particle irreducible ffp vertex $\Gamma^{\mu}(q \equiv p' - p, r \equiv p' + p)$ are

$$-ie\Gamma^{\mu}_{c}(p',p) = -ieV^{\mu}(p',p) - ie\Gamma^{*\mu}(p',p) - ieV^{\mu}(p',p)\delta_e - ie[igM_{\mu\nu}(p'-p)^{\nu}]\delta_g,$$



Evaluating on mass shell

$$\Gamma^{*\mu}(p^2 = p'^2 = m^2, q^2 = 0) = \frac{e^2}{(4\pi)^2}\Big\{\Big[-2[\frac{1}{\epsilon} - \gamma + \ln 4\pi] + 2\ln\frac{m^2}{\mu^2} - 4\int\frac{dx}{x}\Big]V^{\mu}(r,q)$$

$$+\Big[2 + [1 - \frac{g^2}{4}][\frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln\frac{m^2}{\mu^2}]\Big]igM^{\mu\nu}q_{\nu} - \frac{e^2}{(4\pi m)^2}igM^{\beta\alpha}r_{\beta}q_{\alpha}r^{\mu}\Big\}.$$

There is a divergency for $g \neq 2$, this can only be removed assuming that the gyromagnetic factor must be renormalized.

Renormalization of the ffp Vertex

The tensor decomposition of the sum of contributions is

$$-ie\Gamma_c^{\mu}(q,r) = -ie\mathbb{E}q^{\mu} - ie\mathbb{F}r^{\mu} - ie\mathbb{G}igM^{\mu\nu}q_{\nu} - ie\mathbb{H}igM^{\mu\nu}r_{\nu} - ie\mathbb{I}igM^{\beta\alpha}r_{\beta}q_{\alpha}r^{\mu} - ie\mathbb{J}igM^{\beta\alpha}r_{\beta}q_{\alpha}q^{\mu}$$

Where $\mathbb{E}$, $\mathbb{F}$, ..., $\mathbb{J}$ are scalar functions.

The renormalization conditions over the ffp vertex are:

- e is the electric charge on mass shell, this requires that the form factor $\mathbb{F}$ satisfies

$$\mathbb{F}(p^2 = p'^2 = m^2, q^2 = 0) = 1,$$

- That the effective gyromagnetic factor on mass shell is equal to g plus a finite correction Δg, this requires that the form factor $\mathbb{G}$ satisfies

$$g\mathbb{G}(p^2 = p'^2, q^2 = 0) = g + \Delta g,$$

Renormalized ffp vertex

These renormalizations conditions determine the value of the remaining counterterms

$$\delta_e = \frac{e^2}{(4\pi)^2}\Big[2(\frac{1}{\epsilon} - \gamma + \ln 4\pi) - 2\ln\frac{m^2}{\mu^2} + 4\int_0^1 dx/x\Big],$$
$$\delta_g = \frac{e^2}{(4\pi)^2}[\frac{g^2}{4} - 1][\frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln\frac{m^2}{\mu^2}],$$

the first expression implies $e = \sqrt{Z_1} e_d$.

Introducing these expressions one obtains the ffp vertex at arbitrary momentum (q, r)

$$\begin{aligned}-ie\Gamma_c^\mu(q,r) = &- ie\mathbb{E}q^\mu - ie\mathbb{F}r^\mu - ie\mathbb{G}igM^{\mu\nu}q_\nu - ie\mathbb{H}igM^{\mu\nu}r_\nu \\ &- ie\mathbb{I}igM^{\beta\alpha}r_\beta q_\alpha r^\mu + \mathbb{J}igM^{\beta\alpha}r_\beta q_\alpha q^\mu\end{aligned}$$

Form Factors

$$\mathbb{F}(r^2,q^2,r\cdot q,m) = 1+\frac{\alpha}{4\pi}\Big\{\int_0^1 dx(2-x)\Big[\ln\frac{\Delta_1(p,mx^{\frac{1}{2}},x)}{m^2}+\ln\frac{\Delta_1(p',mx^{\frac{1}{2}},x)}{m^2}\Big]$$
$$+\int_0^1\int_0^{1-x} dxdy\Big[2\ln\frac{m^2}{\Delta_2(q,r,m,x,y)}+\frac{q^2[(\frac{g^2}{4}-1)(x+y)+1]+r^2[2(x+y)-(x+y)^2-1]}{\Delta_2(q,r,m,x,y)}$$
$$+\frac{r\cdot q[y-x+x^2-y^2]}{\Delta_2(q,r,m,x,y)}+\frac{4}{(x+y)^2}\Big]\Big\},$$

$$\mathbb{G}(r^2,q^2,r\cdot q,m) = 1+\frac{\alpha}{4\pi}\Big\{\int_0^1 dx(\frac{g^2}{4}-1)\ln\frac{\Delta_1(q,m,x)}{m^2}$$
$$+\int_0^1 dxx\Big[\ln\frac{\Delta_1(\frac{q+r}{2},mx^{\frac{1}{2}},x)}{m^2}+\ln\frac{\Delta_1(\frac{q-r}{2},mx^{\frac{1}{2}},x)}{m^2}\Big]+\int_0^1\int_0^{1-x}\frac{4dydx}{(x+y)^2}$$
$$+\int_0^1\int_0^{1-x} dxdy\Big[-2\ln\frac{m^2}{\Delta_2(q,r,m,x,y)}+\frac{r^2[(x+y)-1]+(1-\frac{g}{2})(r\cdot q)(y-x)+q^2}{\Delta_2(q,r,m,x,y)}\Big]\Big\}.$$

...

Finite correction to the gyromagnetic factor

The effective gyromagnetic factor on mass shell is given by

$$-ie\Gamma_c^{\mu} = -ie[\mathbb{G}(r^2 = 4m^2, q^2 = r \cdot q = 0)igM^{\mu\nu}q_{\nu}] + ...,$$

$$\mathbb{G}(r^2 = 4m^2, q^2 = r \cdot q = 0) = 1 + \frac{\alpha}{2\pi}.$$

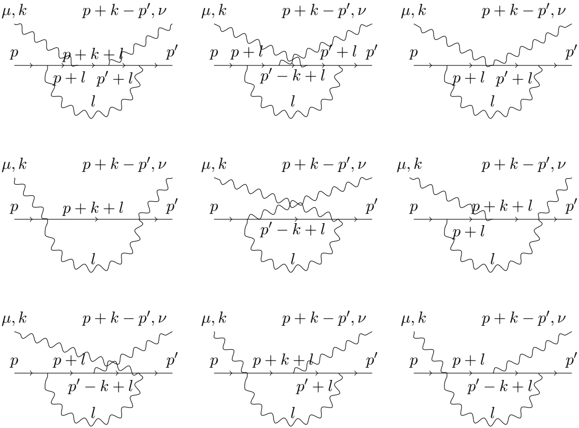
This equation shows that *the finite correction to the gyromagnetic factor to one loop is*

$$\Delta g = \frac{g}{2}\frac{\alpha}{\pi},$$

for $g = 2$ this is just the the conventional result $\Delta g = \dfrac{\alpha}{\pi}$!

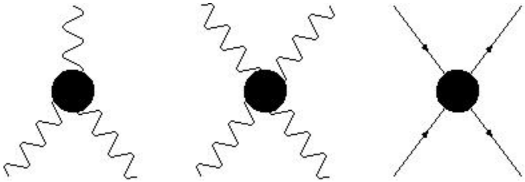
$ffpp$ Vertex

Calculating the $ffpp$ vertex one observes that the divergencies are removed by the past renormalization conditions



Perspectives

The rest of superficially divergent processes are (with 3 and 4 external lines)



These processes must be finite if the theory is renormalizable to one loop.

We expect that the first process to be zero due to charge conjugation symmetry.

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Conclusions

- We studied the one loop renormalization using path integral quantization, obtaining the Feynman rules and showing the Ward identities to all orders, they were verified to one loop.
- It was shown that the coupling constants are adimensional and that the superficial degree of divergency of a given process is bounded by the number of external lines.
- By imposing renormalization conditions (that identified the renormalized couplings) it was shown that the divergencies corresponding to the propagators, ffp and $ffpp$ vertexes are removed for all g.
- It is remarkable that the Dirac gamma matrixes γ^μ are not necessary but natural objects are the Lorentz generators $M^{\mu\nu}$.

Conclusions

- Vacuum polatizations to one loop: is gauge invariant, for $g = 2$ we recover the conventional result. However in general it depends on g which means that the running of the fine structure constant $\alpha(q^2)$ depends of it. The fermion self energy is independent of g at one loop level.
- Divergencies corresponding to the ffp vertex for $g \neq 2$ are only removed assuming that the gyromagnetic factor must be renormalized.
- The finite correction to the gyromagnetic factor which depends on g, and in the case of $g = 2$ one recovers the correct Schwinger correction.

Perspectives

- To finish the study of the one loop renormalization for $1/2$,
- Tenormalization of the NKR formalism for spin $3/2$.
- ¿Generalizations?

Thanks

The Reduction Formula S

Consider the S matrix elements

$$S_{\alpha\beta} = \langle k^{\mu}_{1'}, \sigma_{1'}, ..., k^{\nu}_{n'}, \sigma_{n'}; \beta, out | p^{\kappa}_{1}, \sigma_{1}, ..., p^{\theta}_{m}, \sigma_{m}; \alpha, in \rangle$$

reduction formulas allow us to simplify

$$\begin{aligned} S_{\alpha\beta} = & Z_{1'}^{\frac{1}{2}} ... Z_{m}^{\frac{1}{2}} \sum_{l_i l_{i'}} \int dx_{1'} ... dx_m \left[u_{l_{1'}}(x_{1'}, p_{1'}, \sigma_{1'}) ... u_{l_{n'}}(x_{n'}, p_{n'}, \sigma_{n'}) \right] \\ & \langle 0 | T(\phi_{l_{1'}}(x_{l_{1'}}) ... \phi_{l_m}(x_{l_m})) | 0 \rangle \left[u_{l_1}(x_1, p_1, \sigma_1) ... u_{l_m}(x_m, p_{n'}, \sigma_m) \right] \end{aligned}$$

- $\langle 0 | T(\phi_{l_{1'}}(x_{l_{1'}}) ... \phi_{l_{n'}}(x_{l_{n'}}) \phi_{l_1}(x_{l_1}) ... \phi_{l_m}(x_{l_m})) | 0 \rangle$.
- $\phi_i(x_i)$ with quantum numbers $\{p^{\nu}_i, \sigma_i\}$ corresponding to *in* or *out*,
- Z_i field strength renormalization of ϕ^i,
- $u_i(x_i, p_i, \sigma_i)$ differential operators acting in $\phi_i(x_i)$.

To study the renormalization we focus on calculating $\langle 0 | T(\phi ...) | 0 \rangle$.

Free Parameters and Counterterms $(\xi = 0)$

In terms of the bare parameters m_d^2, e_d, g_d the Lagrangian is

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu_d}F_{d\mu\nu} + (\partial_\mu - ie_d A_{d\mu})\bar{\psi}_d[g^{\mu\nu} - ig_d M^{\mu\nu}](\partial_\nu + ie_d A_{d\nu})\psi_d - m_d^2\bar{\psi}_d\psi_d.$$

Introducing the *renormalized parameters* m_r^2, e_r y g_r and the renormalized fields $A_r^\mu = Z_1^{-\frac{1}{2}} A_d^\mu$ y $\psi_r = Z_2^{-\frac{1}{2}}\psi_d$ the Lagrangian is

$$\begin{aligned}\mathscr{L} =& -\frac{1}{4}F_r^{\mu\nu}F_{r\mu\nu} - \frac{1}{2}(\partial^\mu A_{r\mu})^2 - \frac{1}{4}F_r^{\mu\nu}F_{r\mu\nu}\delta_{Z_1} - \frac{1}{2}(\partial^\mu A_{r\mu})^2\delta_{Z_1} \\ &+ \partial^\mu\bar{\psi}_r\partial_\mu\psi_r - m_r^2\bar{\psi}_r\psi_r + [\partial^\mu\bar{\psi}_r\partial_\mu\psi_r - m^2\bar{\psi}_r\psi_r]\delta_{Z_2} - \delta_m\bar{\psi}_r\psi_r \\ &- ie_r[\bar{\psi}_r T_{r\nu\mu}\partial^\mu\psi_r - \partial^\mu\bar{\psi}_r T_{r\mu\nu}\psi_r]A_r^\nu - ie_r[\bar{\psi}_r T_{r\nu\mu}\partial^\mu\psi_r - \partial^\mu\bar{\psi}_r T_{r\mu\nu}\psi_r]A_r^\nu\delta_e \\ &- ie_r[\bar{\psi}_r(-ig_r M_{\nu\mu})\partial^\mu\psi_r - \partial^\mu\bar{\psi}_r(-ig_r M_{\mu\nu})\psi_r]A_r^\nu\delta_g + e_r^2\bar{\psi}_r\psi_r A_r^\mu A_{r\mu} \\ &+ e_r^2\bar{\psi}_r\psi_r A_r^\mu A_{r\mu}\delta_3,\end{aligned}$$

where

$$\delta_{Z_1} \equiv Z_1 - 1 \qquad \delta_{Z_2} \equiv Z_2 - 1 \qquad \delta_m \equiv Z_2[m_d^2 - m_r^2],$$

$$\delta_e \equiv \frac{e_d}{e_r}Z_1^{\frac{1}{2}}Z_2 - 1 \qquad \delta_g \equiv \frac{e_d}{e_r}Z_1^{\frac{1}{2}}Z_2[\frac{g_d}{g_r} - 1], \quad \delta_3 \equiv \frac{e_d^2}{e_r^2}Z_1 Z_2 - 1$$

Dimensional Regulatization

We could use the conventional extension

$$\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} \text{ with } g^\mu_{\ \mu} = d,$$
$$tr\{\gamma^\mu\} = 0, \;\; trI = f(d) \text{ with } \lim_{d\to 4} f(d) = 4,$$
$$M^{\mu\nu} = i/4[\gamma^\mu, \gamma^\nu].$$

but the gammas γ^μ are *not* necessary we could use instead only the Lorentz generators $M^{\mu\nu}$

$$[M^{\alpha\beta}, M^{\mu\nu}] = -ig^{\beta\nu}M^{\alpha\mu} + ig^{\beta\mu}M^{\alpha\nu} - ig^{\alpha\mu}M^{\beta\nu} + ig^{\alpha\nu}M^{\beta\mu}, \text{ con } g^\mu_{\ \mu} = d$$
$$\{M^{\mu\nu}, M^{\alpha\beta}\} = \frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}) - \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\gamma^5 \text{ con } tr\gamma^5 = 0, (\gamma^5)^2 = 1,$$
$$trM^{\mu\nu} = 0, \;\; tr\{M^{\mu\nu}M^{\alpha\beta}\} = \frac{f(d)}{4}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}) \text{ con } \lim_{d\to 4} f(d) = 4,$$