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Electromagnetic moments of elementary particles from a second order formalism

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Ernesto Germán Delgado Acosta (IF-UASLP)
Mauro Napsuciale Mendivil (DCI-UGTO)
Simón Rodríguez Rodríguez (FCFM-UAC)

Electromagnetic moments of elementary particles from a second order formalism

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Introduction

We are interested in the EM interaction of high spin particles, in particular the spin 1 and spin 3/2 cases, however there are issues concerning the conventional description of such particles:

- The Proca equation describes spin 1 massive particles:

$$[(-p^2 + m^2)g_{\mu\nu} + p_\mu p_\nu]A^\nu = 0, \tag{1}$$

  But in the presence of an EM field it does not predict the natural EM moments of a charged spin 1 massive particle such as the W boson.

- Spin 3/2 particles are usually described by the Rarita-Scwhinger equation:

$$(\gamma \cdot p - m)\psi^\alpha = 0, \tag{2}$$

  but it presents the Velo-Zwanziger problem, involving super-luminal propagation of its wave fronts when coupled to an EM field.

We avoid these problems working with a more complete description beyond these formulations, we refer to this alternative method as covariant projector formalism [1].

[1] Napsuciale, Kirchbach, Rodriguez, Eur. Phys. Jour. A 29:289(2006)

Advantages of the covariant projector formalism

- By means of the minimal substitution it allows for the construction of an EM interaction written in terms of undetermined parameters that can be fixed by physical requirements such as a well behaved forward Compton Scattering.
- In the $(1/2, 0) \oplus (0, 1/2)$ representation of the HLG it reproduces, by an appropriate choice of parameters, the EM properties of a spin $1/2$ fermion associated with the Dirac Lagrangian.
- It properly describes the EM interaction of massive spin 1 particles in the $(1/2, 1/2)$ representation of the HLG.
- In the spin $3/2$ case it provides an eq. of motion with causal wave fronts by requiring a gyromagnetic factor $g = 2$ under electromagnetic interactions.

Covariant projector formalism

The method consists in constructing an eq. of motion for a given representation of the Poincaré group as a projection onto the mass and spin of a sate:

$$\mathcal{P}^{(m,s)}\psi^{(m,s)} = \left(\frac{P^2}{m^2}\mathcal{P}^{(s)}\right)\psi^{(m,s)} = \psi^{(m,s)} \tag{3}$$

where the projector $\mathcal{P}^{(s)}$ is usually found in terms of the Casimir operators of the Poincaré group P^2 and W^2. It can be rewritten as:

$$(\Gamma_{\mu\nu}\partial^\mu\partial^\nu + m^2)\psi^{(m,s)} = 0 \tag{4}$$

- This projection is insensitive to the antisymmetric part of $\Gamma_{\mu\nu}$, in order to obtain a complete theory one has to consider the most general antisymmetric part.

This antisymmetric structure is irrelevant in the free case, but it gets activated if one replaces the derivatives by covariant derivatives since they do not commute, in fact:

$$[D^\mu, D^\nu] = -ieF^{\mu\nu}, \tag{5}$$

Electromagnetic interaction

We obtain the electromagnetic interaction from the gauge principle applied to the Lagrangian associated with the equation of motion. The free Lagrangian is:

$$\mathcal{L}_{\text{free}} = \frac{1}{2}(\partial^\mu \overline{\psi})\Gamma_{\mu\nu}\partial^\nu \psi - \frac{1}{2}m^2\overline{\psi}\psi + \text{h.c.}, \tag{6}$$

and by the minimal substitution $\partial^\mu \to D^\mu = \partial^\mu - i\,e\,A^\mu$ we obtain the gauge invariant Lagrangian as:

$$\mathcal{L} = \frac{1}{2}(D^{\dagger\mu}\overline{\psi})\Gamma_{\mu\nu}D^\nu \psi - \frac{1}{2}m^2\overline{\psi}\psi + \text{h.c.}, \tag{7}$$

it can be separated as $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$, where

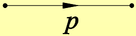
$$\mathcal{L}_{\text{int}} = -j_\mu A^\mu + \frac{1}{2}e^2(\Gamma_{\mu\nu} + \overline{\Gamma}_{\mu\nu})A^\mu A^\nu \tag{8}$$

The electromagnetic transition current is identified in the momentum space as:

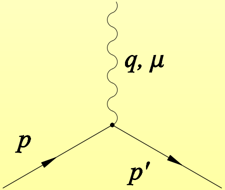
$$j_\mu(p', p) = -\frac{1}{2}e\overline{\psi}(p')(\Gamma_{\nu\mu}p'^\nu + \Gamma_{\mu\nu}p^\nu)\psi(p) + \text{h.c} \tag{9}$$

From the interaction Lagrangian we can extract Feynman rules of first and second order in the charge of the particle.

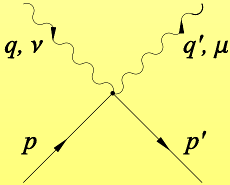
Feynman Rules



$$i\Pi(p) = i(\Gamma_{\mu\nu}p^{\mu}p^{\nu} - m^2)^{-1}$$



$$ie\,(\Gamma_{\nu\mu}p'^{\nu} + \Gamma_{\mu\nu}p^{\nu}) = ie\,V_{\mu\nu}(p', p)$$



$$-ie^2(\Gamma_{\mu\nu} + \Gamma_{\nu\mu})$$

Electromagnetic moments

The electromagnetic moments of a particle are defined by means of a multipole expansion of a current distribution. We obtain such a current from our electromagnetic interaction expressed in the Breit frame:

$$J_B^\mu = \frac{1}{m} j^\mu(p', p), \qquad p' = (\omega/2, \boldsymbol{q}/2), \quad p = (\omega/2, -\boldsymbol{q}/2), \tag{10}$$

The cartesian electromagnetic moments are defined as:

$$Q_E^l = b^{l0}(-i\partial_{\boldsymbol{q}})\varrho_E\Big|_{\boldsymbol{q}=0}, \qquad Q_M^l = \frac{1}{l+1}\, b^{l0}(-i\partial_{\boldsymbol{q}})\varrho_M\Big|_{\boldsymbol{q}=0}, \tag{11}$$

where the electric density ϱ_E and the so called magnetic density ϱ_M read:

$$\varrho_E = j_B^0, \qquad \varrho_M = \partial_{\boldsymbol{q}} \cdot [\boldsymbol{j}_B(\boldsymbol{q}) \times \boldsymbol{q}], \tag{12}$$

the b^{l0} coefficients are obtained from the spherical harmonics as

$$b^{l0}(\boldsymbol{r}) = l!\sqrt{4\pi/(2l+1)}r^l Y_{l0}(\Omega), \tag{13}$$

so that for a monopole $b^{00}(-i\partial_{\boldsymbol{q}}) = 1$, for a dipole $b^{10}(-i\partial_{\boldsymbol{q}}) = -i\partial/\partial q_3$, for a quadrupole $b^{20}(-i\partial_{\boldsymbol{q}}) = 3\partial^2/\partial q_3^2 - \partial^2/\partial \boldsymbol{q}^2$, and for an octupole $b^{30} = 3(\partial/\partial q_3)(5\partial^2/\partial q_3^2 - 3\partial^2/\partial \boldsymbol{q}^2)$, and so on.

$(1/2, 0) \oplus (0, 1/2)$ Representation

We can construct an EOM for a state transforming in the $(1/2, 0) \oplus (0, 1/2)$ representation. The second order equation for a parity conserving particle is:

$$(\Gamma_{\mu\nu} p^{\mu} p^{\nu} - m^2)\psi = 0, \tag{14}$$

$$\Gamma_{\mu\nu} = g_{\mu\nu} - i\, g\, M^{S}_{\mu\nu}, \qquad M^{S}_{\mu\nu} = \sigma_{\mu\nu}/2 = \frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}] \tag{15}$$

where g is an undetermined parameter and after the minimal coupling we get the electromagnetic transition current as:

$$j_{\mu}(p', p) = -e\overline{\psi}(p')(\Gamma_{\nu\mu} p'^{\nu} + \Gamma_{\mu\nu} p^{\nu})\psi(p) = -e\overline{\psi}(p') V_{\mu}(p', p)\psi(p) \tag{16}$$

$$= -e\overline{\psi}(p')\left((p' + p)_{\mu} + igM^{S}_{\mu\nu}(p' - p)^{\nu}\right)\psi(p), \tag{17}$$

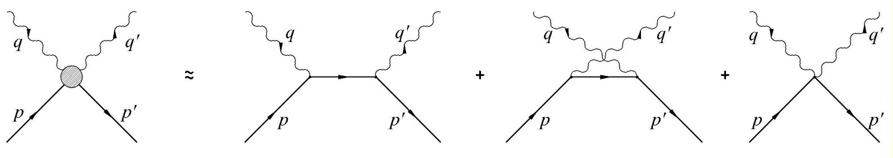
The EM moments associated with this current for a particle of positive parity are found to be:

$$\langle Q^{0}_{E}\rangle = -e, \qquad Q^{1}_{M} = -\frac{ge}{2m}\langle S_3\rangle, \tag{18}$$

higher moments being zero. This results match the electron moments when $g = 2$. We can also calculate Compton scattering in this formalism.

Compton scattering

We calculate Compton scattering at the tree level using states with well defined parity as:



The result of the differential cross section is given in terms of the g parameter, the photon energy $\eta = \omega/m$ and the scattering angle in the lab system $x = \cos\theta$. In particular we have independently of the g parameter:

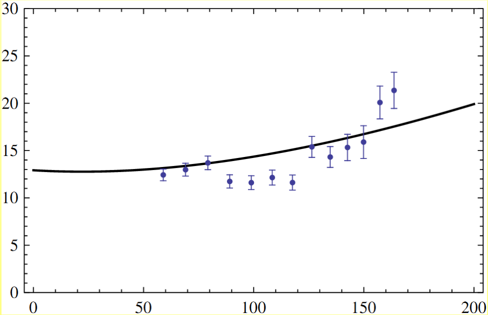
$$\left[\frac{d\sigma}{d\Omega}\right]_{x=1} = r_0^2, \qquad \left[\frac{d\sigma}{d\Omega}\right]_{\eta=0} = \frac{1}{2}(x^2+1)r_0^2, \qquad \left[\frac{d\sigma}{d\Omega}\right]_{\eta\to\infty} = 0, \tag{19}$$

where $r_0^2 = e^2/(4\pi m) = \alpha/m$ is the so called classical radius of the particle. In other directions and at other energies, the differential cross section is written in terms of the factor g,

- As a result we obtain the Compton scattering cross section for a particle with an arbitrary dipole magnetic moment, the Dirac cross section is obtained in the particular case g=2. [2].

[2] Delgado, Napsuciale, Rodriguez, Phys. Rev. D83, 073001 (2011).

Diferential cross section with $g = g_P = 5.58$



$$\frac{d\sigma(\omega)}{d\Omega}$$

Differential cross section (in nb) as a function of the photon energy ω (in MeV) at the fixed angle θ=107°. With the experimental data obtained by the TAPS experiment [3]

[3] V. Olmos de Leon et al., Eur. Phys. J. A 10 , 207(2001).

$(1/2, 1/2)$ Representation

Now the equation of motion reads:

$$(\Gamma_{\alpha\beta\mu\nu} p^\mu p^\nu - m^2 g_{\alpha\beta})\eta^\beta = 0, \tag{20}$$

with:

$$\Gamma_{\alpha\beta\mu\nu} = g_{\alpha\beta} g_{\mu\nu} - \frac{1}{2}(g_{\alpha\nu} g_{\beta\mu} + g_{\alpha\mu} g_{\beta\nu}) - i\,(g - 1/2)\ [M^V_{\mu\nu}]_{\alpha\beta}, \tag{21}$$

$$[M^V_{\mu\nu}]_{\alpha\beta} = i(g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}), \tag{22}$$

with g an arbitrary parameter. The electromagnetic transition current is:

$$\begin{aligned} j_\mu(p', p) &= -e\overline{\eta}^\alpha(p')(\Gamma_{\alpha\beta\nu\mu} p'^\nu + \Gamma_{\alpha\beta\mu\nu} p^\nu)\eta^\beta(p) = -e\overline{\eta}^\alpha(p') V_{\alpha\beta\mu}(p', p)\eta^\beta(p) \\ &= -e\overline{\eta}^\alpha(p')\left(g_{\alpha\beta}(p' + p)_\mu + ig[M^V_{\mu\nu}]_{\alpha\beta}(p' - p)^\nu\right)\eta^b(p), \end{aligned} \tag{23}$$

The EM moments associated with this current are found to be:

$$\langle Q^0_E \rangle = -e, \qquad \langle Q^1_M \rangle = -\frac{ge}{2m}\langle S_3 \rangle, \qquad \langle Q^2_E \rangle = -\frac{(1-g)e}{m^2}\langle 3S_3^2 - \boldsymbol{S}^2 \rangle. \tag{24}$$

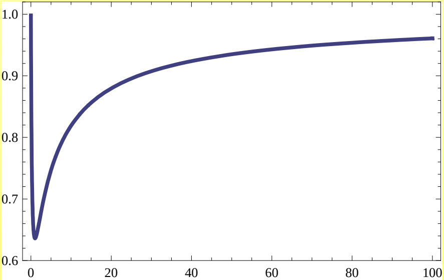
higher moments being zero. This results match the W boson moments when $g = 2$.

Compton Scattering

In this case we have:

$$\left[\frac{d\sigma}{d\Omega}\right]_{x=1} = \left(\frac{1}{24}(g-2)^4\eta^2 + 1\right) r_0^2, \qquad \left[\frac{d\sigma}{d\Omega}\right]_{\eta=0} = \frac{1}{2}(x^2+1)r_0^2, \qquad (25)$$

In the general case, the differential cross section is written in terms of g and grows with the energy, except for $g = 2$, in this case the cross section looks like [3]:



$$\frac{1}{\sigma_T}\sigma(g = 2, \eta)$$

Cross section for Compton scattering off spin 1 particles with $\eta \sim 100$, normalized to $\sigma_T = (8/3)\pi r_0^2$

And it grows with the energy when $g \neq 2$. So that we fix $g = 2$ as the only value preventing the cross section to grow indefinitely at high energy. This requirement fixes the EM moments to those of the W boson.

[3] Napsuciale, Rodriguez, Delgado, Kirchbach, Phys. Rev. D77, 014009 (2008).

$(1/2,1/2)\otimes[(1/2,0)\oplus(0,1/2)]$ Representation

The equation of motion for this representation is:

$$(\Gamma_{\alpha\beta\mu\nu}p^{\mu}p^{\nu}-m^{2}g_{\alpha\beta})\psi^{\beta}=0, \tag{26}$$

$$\begin{aligned}\Gamma_{\alpha\beta\mu\nu}=&-\frac{1}{3}g_{\alpha\nu}g_{\beta\mu}-\frac{i}{6}\sigma_{\alpha\nu}g_{\beta\mu}-\frac{1}{3}g_{\alpha\mu}g_{\beta\nu}+\frac{2}{3}g_{\alpha\beta}g_{\mu\nu}\\&+\frac{i}{3}g_{\mu\nu}\sigma_{\alpha\beta}-\frac{i}{6}g_{\beta\nu}\sigma_{\alpha\mu}+\frac{1}{6}g_{\alpha\nu}\sigma_{\beta\mu}+\frac{i}{6}g_{\alpha\mu}\sigma_{\beta\nu}\\&-\frac{i}{2}g_{S}g_{\alpha\beta}\sigma_{\mu\nu}+g_{V}(g_{\alpha\mu}g_{\beta\nu}-g_{\beta\mu}g_{\alpha\nu})+ic(g_{\alpha\mu}\sigma_{\beta\nu}-g_{\alpha\nu}\sigma_{\beta\mu})\\&+id(g_{\beta\nu}\sigma_{\alpha\mu}-g_{\beta\mu}\sigma_{\alpha\nu})+if\gamma^{5}\epsilon_{\alpha\beta\mu\nu},\end{aligned} \tag{27}$$

where c, d, f, g_S, g_V are arbitrary parameters for a parity conserving particle. Eliminating the spin 1/2 sector coupling, and identifying the gyromagnetic factor as $g=2f+g_V$ the current reduces to

$$j_{\mu}(p',p)=-e\,\overline{\psi}^{\alpha}(p')(g_{\alpha\beta}(p'+p)_{\mu}+ig[M_{\mu\nu}]_{\alpha\beta}(p'-p)^{\nu})\psi^{\beta}(p). \tag{28}$$

where

$$[M_{\mu\nu}]_{\alpha\beta}=M^{S}_{\mu\nu}g_{\alpha\beta}+[M^{V}_{\mu\nu}]_{\alpha\beta} \tag{29}$$

are the generators of the $(1/2,1/2)\otimes[(1/2,0)\oplus(0,1/2)]$ representation of the HLG.

Electromagnetic moments

The explicit form of the current is

$$j_\mu = -e\overline{\psi}^\alpha(g^{\alpha\beta}(p'+p)_\mu + ig(M^S_{\mu\nu}g_{\alpha\beta} + [M^V_{\mu\nu}]_{\alpha\beta})(p'-p)_\nu)\psi^\beta \quad (30)$$
$$= -e\overline{\psi}^\alpha(g\, m\gamma_\mu g_{\alpha\beta} - (g-2)(p+p')_\mu g_{\alpha\beta} + 2g(p_\alpha g_{\beta\mu} + p'_\beta g_{\alpha\mu}))\overline{\psi}^\beta,$$

it differs from the one of the RS formalism:

$$j^{RS}_\mu = -e\overline{\psi}^\alpha(2m\gamma_\mu g_{\alpha\beta})\psi^\beta, \quad (31)$$

The EM moments of a particle of well defined parity of charge $Q^0_E = -e$, are:

$$\langle Q^1_M\rangle = -\frac{ge}{2m}\langle S_3\rangle, \qquad \langle Q^2_E\rangle = -\frac{(1-g)e}{m^2}\langle\mathcal{A}\rangle, \qquad \langle Q^3_M\rangle = -\frac{g\,e}{2m^3}\langle\mathcal{B}\rangle, \quad (32)$$
$$\langle \mathcal{Q}^1_M\rangle = -\frac{2}{3}\frac{e}{2m}\langle S_3\rangle, \qquad \langle \mathcal{Q}^2_E\rangle = -\frac{e}{m^2}\langle\mathcal{A}\rangle, \qquad \langle \mathcal{Q}^3_M\rangle = -\frac{2e}{2m^3}\langle\mathcal{B}\rangle, \quad (33)$$

Where we have used $\mathcal{Q}$ for the RS results and:

$$\mathcal{A} = \frac{1}{3}(3S_z^2 - \boldsymbol{S}^2), \qquad \mathcal{B} = S_3\left(15S_z^2 - \frac{41}{5}\boldsymbol{S}^2\right). \quad (34)$$

In order to find g in the second order formalism we can use Compton Scattering.

Compton Scattering

The differential cross section for this process is found in terms of the g parameter and grows with the energy in every direction for an arbitrary g. This behavior can only be prevented in the forward direction:

$$\left.\frac{d\sigma}{d\Omega}\right|_{x=1} = r_0^2 + \frac{r_0^2}{81}(g-2)(5(g-2)^3 - 18(g-2) - 36)\eta^2 + \frac{8r_0^2}{81}g^2(g-2)\eta^4, \quad (35)$$

so that the only value leading to a well behaved forward Compton scattering is

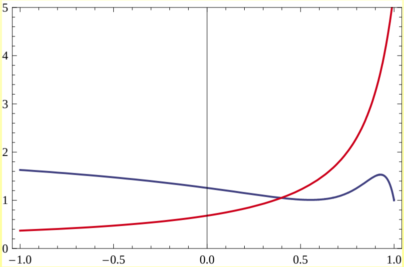
$$g \to 2, \quad (36)$$

The EM moments associated with this requirement are:

$$\langle Q_M^1 \rangle = -2 \times \frac{e}{2m}\langle S_3 \rangle, \quad \langle Q_E^2 \rangle = +\frac{e}{m^2}\langle \mathcal{A} \rangle, \quad \langle Q_M^3 \rangle = -2 \times \frac{e}{2m^3}\langle \mathcal{B} \rangle, \quad (37)$$

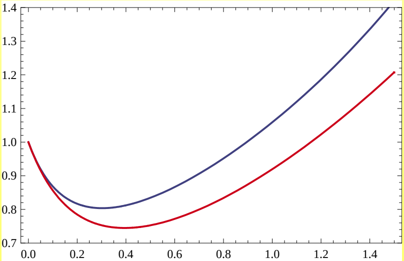
Unlike the RS moments, this EM moments are related with an energy independent forward Compton scattering.

Compton scattering



$$\frac{1}{r_0^2}\frac{d\sigma(\eta,x)}{d\Omega}$$

Differential cross section at $\eta = 1.5$. The $g = 2$ result is in blue, the RS result is in red.



$$\sigma(\eta)/\sigma_T$$

Total cross section for Compton Scattering, the $g = 2$ result is in blue, the RS result is in red.

[4] Delgado, Napsuciale, Phys. Rev. D80, 054002 (2009).

Summary

- Second order formalism describes spin 1/2 particles in the $(1/2,0)\oplus(0,1/2)$ with an arbitrary dipole magnetic moment,

$$\langle Q_E^0\rangle = -e, \qquad Q_M^1 = -\frac{ge}{2m}\langle S_3\rangle, \tag{38}$$

  Forward Compton scattering is insensitive to the g parameter. Dirac result corresponds to the particular case $g=2$.

- For spin 1 in the $(1/2,1/2)$ representation we find:

$$\langle Q_E^0\rangle = -e, \qquad \langle Q_M^1\rangle = -\frac{ge}{2m}\langle S_3\rangle, \qquad \langle Q_E^2\rangle = -\frac{(1-g)e}{m^2}\langle 3S_3^2 - \boldsymbol{S}^2\rangle, \tag{39}$$

  Forward Compton scattering requires $g=2$, and so one gets the moments of the W boson as predicted by the SM.

- In the case of spin $3/2$ in the $(1/2,1/2)\otimes[(1/2,0)\oplus(0,1/2)]$ for a particle of charge $Q_E^0=-e$ we find:

$$\langle Q_M^1\rangle = -\frac{ge}{2m}\langle S_3\rangle, \quad \langle Q_E^2\rangle = -\frac{(1-g)e}{m^2}\langle \mathcal{A}\rangle, \quad \langle Q_M^3\rangle = -\frac{ge}{2m^3}\langle \mathcal{B}\rangle, \tag{40}$$

  Forward Compton scattering requires $g=2$, so that the resultant moments are also associated with causality.

Conclusions

- The covariant projector formalism allows for a complete description of electromagnetic interactions in a given representation of the Poincaré group.
- We have shown that the Dirac equation is not the only way to properly describe spin $1/2$ fermions.
- By considering a general antisymmetric part in the equation of motion, the covariant projector formalism in the vector case is free of the problems exhibited by the Proca equation in an EM environment.
- Rarita-Scwhinger equation fails to properly describe the complete structure of a spin 3/2 particle in the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$, this can be seen in the Gordon-like decomposition of the electromagnetic current.
- We have found that Compton scattering from a second order formalism exhibits a cross section that grows with the energy except in the forward direction when $g = 2$, a condition also required for the causality of the theory.
- The EM moments of a spin $3/2$ particle in the second order formalism differ from those of the RS formalism and are related with an energy independent forward Compton scattering, same as the moments of lower spin particles.