Baryon Properties in Large-N_c Chiral Perturbation Theory

Rubén Flores-Mendieta

Instituto de Física, Universidad Autónoma de San Luis Potosí

San Luis Potosí, S.L.P., Mexico

[In collaboration with M.A. Hernández and C.P. Hofmann]

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Abstract

The static properties of baryons are computed in heavy baryon chiral perturbation theory in the large-N_c limit, where N_c is the number of colors. The analysis is specialized to the baryon axial-vector couplings and magnetic moments. One-loop non-analytic corrections are analyzed in the limit $\Delta \equiv M_T - M_B \to 0$ and compared with the current experimental data. An extra comparison with conventional heavy baryon chiral perturbation theory is carried out for $N_f = N_c = 3$.

Both approaches coincide order by order in the expansion

Outline

1. Introduction
   - ▷ Heavy baryon chiral perturbation theory
   - ▷ The $1/N_c$ expansion of QCD
2. The $1/N_c$ chiral Lagrangian for baryons
3. One loop corrections to the baryon axial current and magnetic moments
4. Comparison with HBCHPT in the degeneracy limit
5. Concluding remarks

Introduction

QCD is an SU(3) gauge theory of quarks and gluons. Despite the progress achieved in the understanding of the strong interactions with QCD, analytic calculations of the spectrum and properties of hadrons are not possible because the theory is strongly coupled at low energies.

Some methods to extract the low-energy consequences of QCD:

- ▷ Chiral perturbation theory
- ▷ The $1/N_c$ expansion

Chiral Perturbation Theory

- CHPT exploits the symmetry of $\mathcal{L}_{\rm QCD}$ under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$ transformations on the flavors u, d, s in the limit $m_q \to 0$.

- Chiral symmetry is spontaneously broken to the vector subgroup $\mathrm{SU}(3) \times \mathrm{U}(1)_V$ by the QCD vacuum, resulting in an octet of pseudoscalar Goldstone bosons, the mesons.

- There is an expansion about the chiral limit in powers of m_q/Λ_χ, or, equivalently, in powers of m_Π^2/Λ_χ^2, where $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking and m_Π is the meson mass.

- Baryons can be incorporated in a systematic way (HBCHPT).

Large-N_c QCD

- The generalization of QCD from $N_c = 3$ to $N_c \gg 3$ colors, known as the large-N_c limit, was proposed[1] to understand the nonperturbative dynamics of hadrons.
- Large-N_c QCD is the SU(N_c) gauge theory of quarks and gluons, where N_c is a parameter of the theory.
- In the large-N_c limit the meson sector consists of a spectrum of narrow resonances[1] and meson-meson scattering amplitudes are suppressed by powers of $1/\sqrt{N_c}$. The baryon sector is more subtle to analyze.[2]
- Physical quantities are considered in this limit, where corrections arise at relative orders $1/N_c$, $1/N_c^2$, ..., the $1/N_c$ expansion.

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[1] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974); B **75**, 461 (1974).

[2] E. Witten, Nucl. Phys. B **160**, 57 (1979).

Heavy Baryon Chiral Perturbation Theory

To lowest order in the derivative expansion, $\mathcal{L}_{\rm baryon}$ is[2]

$$
\begin{aligned}
\mathcal{L}_{\rm baryon} &= i\,{\rm Tr}\,\bar{B}_v (v\cdot\mathcal{D}) B_v - i\,\bar{T}_v^\mu (v\cdot\mathcal{D}) T_{v\mu} + \Delta\,\bar{T}_v^\mu T_{v\mu} \\
&\quad + 2\,D\,{\rm Tr}\,\bar{B}_v S_v^\mu \{\mathcal{A}_\mu, B_v\} + 2\,F\,{\rm Tr}\,\bar{B}_v S_v^\mu [\mathcal{A}_\mu, B_v] \\
&\quad + \mathcal{C}\,(\bar{T}_v^\mu \mathcal{A}_\mu B_v + \bar{B}_v \mathcal{A}_\mu T_v^\mu) + 2\,\mathcal{H}\,\bar{T}_v^\mu S_v^\nu \mathcal{A}_\nu T_{v\mu}\,.
\end{aligned}
$$

B_v and T^μ_{abc} are baryon octet and decuplet fields. Octet meson fields enter into $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$ via

$$
\xi = e^{i\Pi/f}, \qquad \Sigma = \xi^2 = e^{2i\Pi/f}
$$

where $f \approx 93$ MeV is the pion decay constant.

D, F, $\mathcal{C}$, and $\mathcal{H}$ are coupling constants and $\Delta = M_\Delta - M_B$.

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[2] E. Jenkins and A.V. Manohar, Phys. Lett. B **225**, 558 (1991); **259**, 353 (1991).

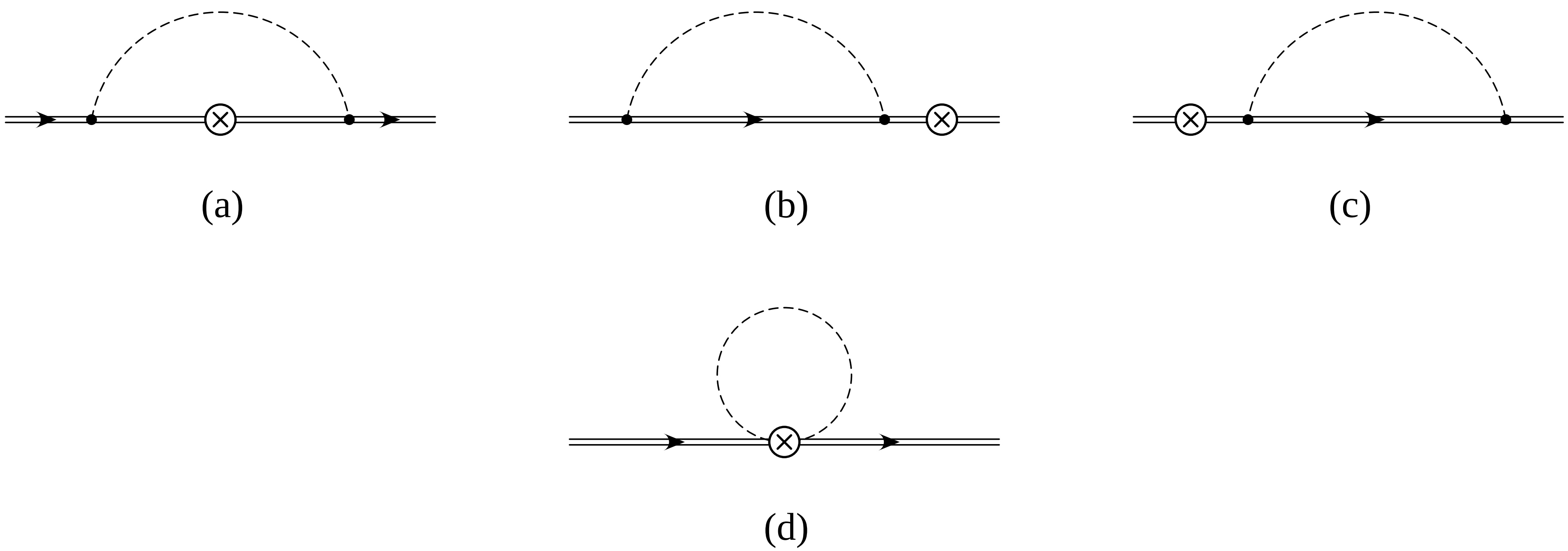

Figure 1: One-loop corrections to the baryon axial vector current.

The renormalized baryon axial vector current is[3]

$$\langle B_i|J_\mu^A|B_j\rangle = \left[\alpha_{ij} - \sum_\Pi \left(\bar\beta_{ij}^\Pi - \bar\lambda_{ij}^\Pi \alpha_{ij}\right) F(m_\Pi, 0, \mu) + \sum_\Pi \gamma_{ij}^\Pi I(m_\Pi, \mu)\right] \times \bar u_{B_i} \gamma_\mu \gamma_5 u_{B_j} \, .$$

- ▷ α_{ij}: lowest order result
- ▷ $\bar\beta_{ij}^\Pi = \beta_{ij}^\Pi + {\beta'}_{ij}^\Pi$: from Fig. 1(a)
- ▷ $\bar\lambda_{ij}^\Pi = \lambda_{ij}^\Pi + {\lambda'}_{ij}^\Pi$: from wave function renormalization, Figs. 1(b,c)
- ▷ γ_{ij}^Π: from Fig. 1(d).

$F(m_\Pi, \Delta, \mu)$ and $I(m_\Pi, \mu)$ are the integrals over the loops.

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[3] E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); **259**, 353 (1991).

Important Results

- ▷ The F/D ratios were found to be close to their SU(6) values, with $F/D \approx 2/3$, value predicted by the nonrelativistic quark model.
- ▷ There were large cancellations in the corrections to the baryon axial vector current between one-loop graphs with intermediate spin-1/2 octet and spin-3/2 decuplet baryon states:
  - □ Corrections $\sim 100\%$ when only octet baryon states are included
  - □ Corrections $\sim 40\%$ when *both* octet and decuplet baryon states are included

Using the $1/N_c$ expansion it can be proved that, for pions,[4]

$$\frac{F}{D} = \frac{2}{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right) .$$

[4] R. Dashen, A.V. Manohar, Phys. Lett. B **315**, 425 (1993), **315**, 438 (1993)

The $1/N_C$ Expansion of QCD

In the large-N_c limit the baryon sector has a contracted spin-flavor symmetry $SU(2N_f)$, with N_f the number of light-quark flavors.

$SU(2N_f)$ decomposes under $SU(2) \times SU(N_f)$ into a tower of baryon states with spins $J = \frac{1}{2}, \ldots, \frac{N_c}{2}$ in the flavor representation.[5]

Any physical operator $\mathcal{O}^{(m)}$ that scales as N_c^m may be written as

$$\mathcal{O}^{(m)} = N_c^m \sum_{n,p,q} c_n \left(\frac{J^i}{N_c}\right)^p \left(\frac{T^a}{N_c}\right)^q \left(\frac{G^{jb}}{N_c}\right)^{n-p-q}$$

The c_n have power series expansions in $1/N_c$ beginning at order unity.

[5] R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993).
J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phy. Rev. D **30**, 1795 (1989).

A QCD operator transforming according to a given $\mathrm{SU}(2) \times \mathrm{SU}(N_f)$ representation can be expanded as[6]

$$\mathcal{O}_{\mathrm{QCD}} = \sum_n c_n \frac{1}{N_c^{n-1}} \mathcal{O}^{(n)}$$

The spin-flavor generators J^i, T^a, and G^{ia} of $\mathrm{SU}(2N_f)$ are

$$\begin{aligned} J^i &= q^\dagger \left(\frac{\sigma^i}{2} \otimes I \right) q\,, & (1,1) \\ T^a &= q^\dagger \left(I \otimes \frac{\lambda^a}{2} \right) q\,, & (0,\mathrm{adj}) \\ G^{ia} &= q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q\,. & (1,\mathrm{adj}) \end{aligned}$$

They satisfy a Lie algebra.

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[6] R. F. Dashen, E. Jenkins, A.V. Manohar, Phys. Rev. D **49**, 4713 (1994).

- Baryon axial vector current operator

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{kc} + \ldots$$

$c = 1 + i2$ for $\Delta S = 0$ transitions and $c = 4 + i5$ for $|\Delta S| = 1$ transitions.

- Baryon magnetic moment operator

$$M^{kc} = m_1 G^{kc} + m_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + m_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + m_4 \frac{1}{N_c^2} \mathcal{O}_3^{kc} + \ldots$$

The magnetic moments are proportional to the quark charge matrix $\mathcal{Q} = \mathrm{diag}(2/3, -1/3, -1/3)$, so they can be separated into isovector and isoscalar components, M^{k3} and M^{k8}, respectively. We thus define the baryon magnetic moment operator as

$$M^k = M^{kQ} \equiv M^{k3} + \frac{1}{\sqrt{3}} M^{k8}$$

Combined Expansion in m_q and $1/N_c$

In the chiral limit $m_q \to 0$

- Mesons become massless Goldstone boson states
- There is an expansion about the chiral limit in powers of m_q/Λ_χ

In the large-N_c limit

- The nucleon and Δ become degenerate, $M_\Delta - M_N \propto 1/N_c \to 0$ and form a single irreducible representation of the contracted spin-flavor symmetry of baryons
- There is an expansion in powers of $1/N_c$ about this limit

Goal: Consider a combined expansion in m_q/Λ_χ and $1/N_c$ about the double limit $m_q \to 0$ and $N_c \to \infty$.

In the chiral limit $m_q \longrightarrow 0$ with Δ held fixed,

$$F\left(m_\Pi, \Delta, \mu\right) = F_0 + \left(\frac{m_\Pi}{\Delta}\right) F_1 + \left(\frac{m_\Pi}{\Delta}\right)^2 F_2 + \ldots$$

In the $1/N_c \longrightarrow 0$ limit with m_Π held fixed

$$F\left(m_\Pi, \Delta, \mu\right) = \bar{F}_0 + \left(\frac{\Delta}{m_\Pi}\right) \bar{F}_1 + \left(\frac{\Delta}{m_\Pi}\right)^2 \bar{F}_2 + \ldots$$

The difference between the two expansions is referred to as the non-commutativity of the chiral and large-N_c limits.[7]

Conditions for HBCHPT to be valid:

$$m_\Pi \ll \Lambda_\chi \qquad \textbf{and} \qquad \Delta \ll \Lambda_\chi$$

m_Π/Δ is not constrained ($m_\Pi/\Delta \sim 0.5$).

[7] T. D. Cohen, Phys. Lett. **B359**, 23 (1995).

Chiral Lagrangian for Baryons in the $1/N_c$ Expansion

$$\mathcal{L}_{\rm baryon} = $$
$$i\mathcal{D}^0 - \mathcal{M}_{\rm hyperfine} + {\rm Tr}\left(\mathcal{A}^k\lambda^c\right) A^{kc} + \frac{1}{N_c}{\rm Tr}\left(\mathcal{A}^k \frac{2I}{\sqrt{6}}\right) A^k + \ldots ,$$

with[8]

$$\xi(x) = e^{i\Pi(x)/f}, \qquad \Pi(x) = \frac{\pi^a(x)\lambda^a}{2} + \frac{\eta'(x) I}{\sqrt{6}}, \qquad (a = 1, \ldots, 8)$$

For $N_c = 3$

$$\mathcal{M}_{\rm hyperfine} = m_2 \frac{1}{N_c} J^2 ,$$

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} J^k T^c + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{kc}$$

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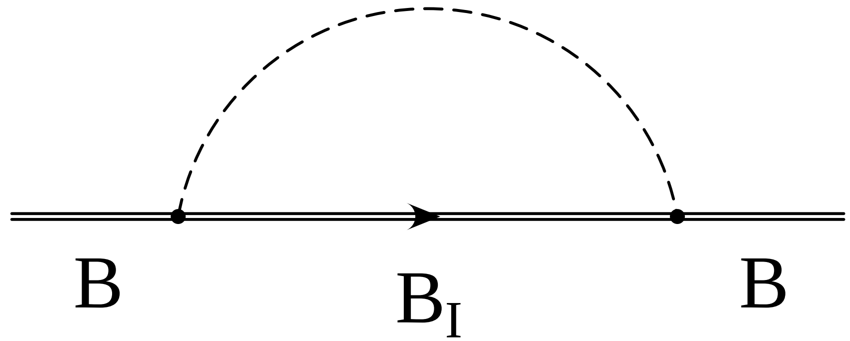
[8] E. Jenkins, Phys. Rev. D **53**, 2625 (1996)

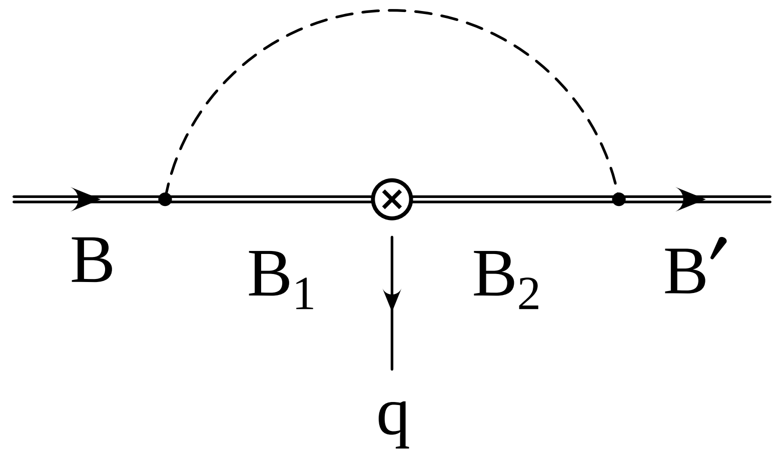
One-loop wave function renormalization graph:



$$iG_B = \sum_{j,k,b,B_I} \frac{i^2}{f^2} \left[A^{kb}\right]_{BB_I} \left[A^{jb}\right]_{B_IB} \\ \times \int \frac{d^4k}{(2\pi)^4} \frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{\left(k^2 - m_b^2\right)\left[(k+p)\cdot v - (M_I - M) + i\epsilon\right]},$$

where $b = 1, \ldots, 9$ or π, K, η, η' labels the intermediate meson.

Vertex Correction



$$\left[\delta A^{ia}\right]^{\text{vertex}}_{B'B} = \sum_{j,k,b,B_1,B_2} -\frac{i}{f^2}\left[A^{kb}\right]_{B'B_2}\left[A^{ia}\right]_{B_2B_1}\left[A^{jb}\right]_{B_1B}\int\frac{d^4k}{(2\pi)^4}$$

$$\times\frac{(\mathbf{k}^k)(-\mathbf{k}^j)}{\left(k^2-m_b^2\right)\left(k\cdot v-\Delta_{M_1M}+i\epsilon\right)\left((k-q)\cdot v-\Delta_{M_2M}+i\epsilon\right)},$$

$q\cdot v=0$ and $q\cdot v=M-M'$ for octet-octet and decuplet-decuplet matrix elements, respectively.

Total Correction From Figs. 1(a,b,c): $\Delta/m_\Pi \to 0$ Limit

$$\delta A^{kc} = \tfrac{1}{2}[A^{ia},[A^{ib},A^{kc}]]\Pi^{ab}$$

where

$$\begin{aligned}\Pi^{ab} = {} & \frac{1}{8}\left[3F(\pi)+4F(K)+F(\eta)\right]\,\delta^{ab} \\ & + \frac{2\sqrt{3}}{5}\left[\frac{3}{2}F(\pi)-F(K)-\frac{1}{2}F(\eta)\right]\,d^{ab8} \\ & + \left[\frac{1}{3}F(\pi)-\frac{4}{3}F(K)+F(\eta)\right]\left(\delta^{a8}\delta^{b8}-\frac{1}{8}\delta^{ab}-\frac{3}{5}d^{ab8}d^{888}\right)\end{aligned}$$

and

$$F(m_b,0,\mu) = -\frac{1}{16\pi^2 f^2}m_b^2\left(\frac{11}{3}+\ln\frac{m_b^2}{\mu^2}\right)$$

Large-N_C Cancellations

For baryons with spins of order unity

$$T^a \sim N_c\,, \qquad G^{ia} \sim N_c\,, \qquad J^i \sim 1$$

Naively

$$\left[A^{ja}, \left[A^{jb}, A^{kc}\right]\right] \sim \mathcal{O}(N_c^3)$$

But from large-N_c consistency conditions and analytic calculations

$$\left[A^{ja}, \left[A^{jb}, A^{kc}\right]\right] \sim \mathcal{O}(N_c)$$

There are large-N_c cancellations provided one sums over all baryon states in a complete multiplet of the large-N_c SU(6) spin-flavor symmetry, *i.e.*, over both the octet and the decuplet, and uses axial coupling ratios given by the large-N_c symmetry.

Some Explicit Calculations

- Singlet contribution

$$[G^{ia},[G^{ia},J^kT^c]]+[G^{ia},[J^iT^a,G^{kc}]]+[J^iT^a,[G^{ia},G^{kc}]]=$$
$$-\frac{2}{N_f}(N_c+N_f)G^{kc}+\left(\frac{9}{4}N_f-\frac{1}{N_f}+2\right)J^kT^c$$
$$\sim \mathcal{O}(N_c)$$

- Octet contribution

$$d^{ab8}[G^{ia},[G^{ib},G^{kc}]]=\left(\frac{3}{8}N_f-\frac{2}{N_f}\right)d^{c8e}G^{ke}+\left(\frac{1}{2}-\frac{2}{N_f^2}\right)\delta^{c8}J^k$$
$$\sim \mathcal{O}(N_c)$$

- **27** contribution

$$[G^{i8},[G^{i8},G^{kc}]] = \frac{1}{4}\left(-f^{c8d}f^{d8e}+2d^{c8d}d^{d8e}\right)G^{ke}+\frac{1}{N_f}\delta^{c8}G^{k8}$$
$$+\frac{1}{2N_f}d^{c88}J^k$$
$$\sim \mathcal{O}(N_c)$$

Thus

$$\delta A^{kc} = \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab} \sim \mathcal{O}(N_c^0)$$

$$\delta A^{kc} \sim \frac{1}{N_c}\times g_A$$

with $g_A \sim \mathcal{O}(N_c)$ and $f \sim \mathcal{O}(\sqrt{N_c})$.

Correction From Fig. 1(d)

$$\delta A^{kc} = -\frac{1}{2}\left[T^a\left[T^b, A^{kc}\right]\right]\Pi^{ab},$$

where Π^{ab} is now a function of $I(m_\Pi)$.

- Singlet piece $[T^a,[T^a,A^{kc}]] = N_f A^{kc} \sim \mathcal{O}(N_c)$
- Octet piece $d^{ab8}[T^a,[T^b,A^{kc}]] = \frac{N_f}{2} d^{c8e} A^{ke} \sim \mathcal{O}(N_c)$
- **27** piece $[T^8,[T^8,A^{kc}]] = f^{c8d} f^{8de} A^{ke} \sim \mathcal{O}(N_c)$

Thus

$$\delta A^{kc} \sim \frac{1}{N_c} \times g_A$$

Comparison Between the two Approaches

In the limit $\Delta/m_{\Pi} = 0$

$$\delta A^{kc}_{\rm deg} = \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab}_{(F)} - \frac{1}{2}\left[T^{a}\left[T^{b},A^{kc}\right]\right]\Pi^{ab}_{(I)}$$

so the renormalized current is

$$A^{kc}_{\rm ren} = A^{kc} + \delta A^{kc}_{\rm deg}$$

The matrix elements of the space components of the axial vector current between baryon states are

$$\langle B_j|\bar{\psi}\gamma^k\gamma_5 T^c\psi|B_i\rangle = \left[A^{kc}_{\rm ren}\right]_{B_j B_i}$$

B_i and B_j are baryons in the lowest-lying irreducible representation of contracted SU(6) spin-flavor symmetry.

The correction within HBCHPT to the axial current can be decomposed into flavor singlet, octet and **27** contributions in terms of flavor singlet, octet, and **27** linear combinations of $F(m_\Pi)$ and $I(m_\Pi)$:

$$\langle B_j | J_\mu^A | B_i \rangle = \Big[\alpha_{B_j B_i} + b_{\mathbf{1}}^{B_j B_i} F_{\mathbf{1}} + b_{\mathbf{8}}^{B_j B_i} F_{\mathbf{8}} + b_{\mathbf{27}}^{B_j B_i} F_{\mathbf{27}} \\ + c_{\mathbf{1}}^{B_j B_i} I_{\mathbf{1}} + c_{\mathbf{8}}^{B_j B_i} I_{\mathbf{8}} + c_{\mathbf{27}}^{B_j B_i} I_{\mathbf{27}} \Big] \bar{u}_{B_j} \gamma_\mu \gamma_5 u_{B_i},$$

with

$$b_{\mathbf{1}}^{B_j B_i} = -(a^\pi_{B_j B_i} + a^K_{B_j B_i} + a^\eta_{B_j B_i}),$$
$$a^\Pi_{B_j B_i} = \bar{\beta}^\Pi_{B_j B_i} - \bar{\lambda}^\Pi_{B_j B_i} \alpha_{B_j B_i}$$

and

$$F_{\mathbf{1}} = \frac{1}{8} \left[3F(\pi) + 4F(K) + F(\eta) \right],$$

with similar expressions for the remaining coefficients.[9]

[9] R.F.M. and C.P. Hofmann, hep-ph/0609120

For $N_c = 3$, there is a one-to-one correspondence between the contributions of $[A^{kc}_{\rm ren}]_{B_jB_i}$ and $\langle B_j|J^A_\mu|B_i\rangle$.

For Figs. 1(a,b,c) one has

$$\left[\frac{1}{2}[A^{ia},[A^{ia},A^{kc}]]\right]_{B_jB_i} = b_{\mathbf{1}}^{B_jB_i},$$

$$\left[\frac{1}{2}d^{ab8}[A^{ia},[A^{ib},A^{kc}]]\right]_{B_jB_i} = b_{\mathbf{8}}^{B_jB_i},$$

$$\left[\frac{1}{2}[A^{i8},[A^{i8},A^{kc}]]\right]_{B_jB_i} = b_{\mathbf{27}}^{B_jB_i},$$

and similar expressions occur for Fig. 1(d)

For example, for the process $n \to pe\bar{\nu}_e$ the singlet piece is

$$\left[\frac{1}{2}[A^{ia},[A^{ia},A^{kc}]]\right]_{pn} = \frac{115}{144}a_1^3 + \frac{7}{48}a_1^2 b_2 + \frac{19}{48}a_1 b_2^2 - \frac{31}{432}a_1^2 b_3 - \frac{11}{12}a_1^2 c_3 + \frac{7}{144}b_2^2 + \frac{169}{216}a_1 b_2 b_3 - \frac{37}{36}a_1 b_2 c_3 + \ldots, \qquad (1)$$

whereas from HBCHPT

$$b_{\mathbf{1}}^{pn} = -2(F+D)^3 - \frac{2}{9}(F+D)^2\mathcal{C}^2 - \frac{50}{81}\mathcal{H}\mathcal{C}^2 \qquad (2)$$

Equations (1) and (2) are found to be the same under

$$D = \tfrac{1}{2}a_1 + \tfrac{1}{6}b_3\,, \qquad \mathcal{C} = -a_1 - \tfrac{1}{2}c_3\,,$$

$$F = \tfrac{1}{3}a_1 + \tfrac{1}{6}b_2 + \tfrac{1}{9}b_3\,, \qquad \mathcal{H} = -\tfrac{3}{2}a_1 - \tfrac{3}{2}b_2 - \tfrac{5}{2}b_3\,.$$

Both approaches yield the same results order by order

Some Numerical Values

Including corrections to order $\mathcal{O}(1/N_c^2)$ to g_A one has[10]

Table 1: Values of g_A for various semileptonic processes.

Process	Total value	Tree level	Singlet piece	Octet piece	**27** piece
$n \to p e^- \overline{\nu}_e$	1.272	1.031	0.279	−0.040	0.002
$\Sigma^+ \to \Lambda e^+ \nu_e$	0.653	0.542	0.168	−0.057	0.000
$\Sigma^- \to \Lambda e^- \overline{\nu}_e$	0.624	0.542	0.113	−0.031	−0.000
$\Lambda \to p e^- \overline{\nu}_e$	−0.904	−0.720	−0.134	−0.055	0.005
$\Sigma^- \to n e^- \overline{\nu}_e$	0.375	0.298	0.080	−0.002	−0.001
$\Xi^- \to \Lambda e^- \overline{\nu}_e$	0.139	0.178	−0.034	−0.004	−0.001
$\Xi^- \to \Sigma^0 e^- \overline{\nu}_e$	0.869	0.729	0.128	0.014	−0.002
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}_e$	1.312	1.031	0.246	0.041	−0.006

Calculation of higher-order corrections is rather involved.

[10] R.F.M., C.P. Hofmann, hep-ph/0609120

Work in Progress: Full Dependence on Δ/m_Π

Expanding the function $F(m, \Delta, \mu)$ in a power series yields,

$$\delta A^{kc} = \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab}_{(1)} - \frac{1}{2}\left\{A^{ja},\left[A^{kc},\left[\mathcal{M},A^{jb}\right]\right]\right\}\Pi^{ab}_{(2)}$$
$$+\frac{1}{6}\Big(\left[A^{ja},\left[\left[\mathcal{M},\left[\mathcal{M},A^{jb}\right]\right],A^{kc}\right]\right]$$
$$-\frac{1}{2}\left[\left[\mathcal{M},A^{ja}\right],\left[\left[\mathcal{M},A^{jb}\right],A^{kc}\right]\right]\Big)\Pi^{ab}_{(3)} + \ldots$$

where the tensor $\Pi^{ab}_{(n)}$ is written in terms of

$$F^{(n)}(m_\Pi, \Delta, \mu) \equiv \frac{\partial^n F(m_\Pi, \Delta, \mu)}{\partial \Delta^n}$$

We expect to get a more stable fit by improving the convergence of the series.

Conclusions

- An alternative approach to write one-loop corrections in heavy baryon chiral perturbation theory has been proposed, including the functional dependence in $\Delta \equiv M_\Delta - M_N$.
- There are large cancellations in loops containing intermediate octet and decuplet baryon states.
- These cancellations arise naturally in this approach and not as a numerical cancellation at the end of the calculation.
- The one-loop correction is very sensitive to the deviations of the axial coupling ratios from their SU(6) values.