Conformal symmetry breaking and degeneracy of high-lying unflavored mesons

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Abstract. We show that though conformal symmetry can be broken by the dilaton, such can happen without breaking the conformal degeneracy patterns in the spectra. Our argumentation goes as follows: We departure from the gauge-gravity duality which predicts on the boundaries of the AdS_5 geometry a conformal theory, associated with QCD at high temperatures, and consider $\mathbf{R}^1 \times S^3$ slicing. The inverse radius, R, of S^3 relates to the temperature of the deconfinement phase transition and has to satisfy, $\hbar c/R \gg \Lambda_{QCD}$. On S^3, whose isometry group is $SO(4)$, we then focus on the eigenvalue problem of the conformal Laplacian there, given by $\frac{1}{R^2}\left(\mathcal{K}^2+1\right)$, with $\mathcal{K}^2$ standing for the Casimir invariant of the $so(4)$ algebra. This eigenvalue problem describes the spectrum of a scalar particle, to be associated with a $q\bar{q}$ system. Such a spectrum is characterized by a $(K+1)^2$-fold degeneracy of its levels, with $K \in [0,\infty)$. We then break the conformal S^3 metric, $ds^2 = d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)$ -in polar χ, θ, and azimuthal φ coordinates- according to, $d\widetilde{s}^2 = e^{-b\chi}\left((1+b^2/4)d\chi^2 + \sin^2\chi(d\theta^2+\sin^2\theta d\varphi^2)\right)$, and attribute the symmetry breaking scale $b\hbar^2c^2/R^2$ to the dilaton. Next we show that the above metric deformation is equivalent to a breaking of the conformal curvature of S^3 by a term proportional to $b\cot\chi$, and that the perturbed conformal Laplacian is equivalent to $\left(\widetilde{\mathcal{K}}^2 + c_K\right)$, with c_K a representation constant, and $\widetilde{\mathcal{K}}^2$ being again an $so(4)$ Casimir invariant, but this time in a representation unitarily nonequivalent to the 4D rotational one. As long as the spectra before and after the symmetry breaking happen to be determined each by eigenvalues of a Casimir invariant of an $so(4)$, no matter whether or not in a representation that generates the orthogonal group $SO(4)$ as a subgroup of the conformal group $SO(2,4)$, the degeneracy patterns remain unaltered though the conformal symmetry breaks at the level of the representation of the algebra. We fit the S^3 radius and the $\hbar^2c^2b/R^2$ scale to the high-lying excitations in the spectra of the unflavored mesons, and observe the correct tendency of the $\hbar c/R = 373$ MeV value to notably exceed Λ_{QCD}. The size of the symmetry breaking scale is calculated as $\hbar c\sqrt{b}/R = 673.7$ MeV.

1. Introduction and background

The five-dimensional Anti-de Sitter space, AdS_5, is presently one of the most intriguing geometries in field theoretical studies due to its relevance in establishing the link between brane theory and QCD. The fact is that according to the gauge-gravity duality conjecture [1], a string theory at the boundary of AdS_5 appears dual to a conformal field theory in (1+3) dimensions, associated with QCD [2]. The AdS_5 geometry is defined as a surface embedded in a (2+4)-dimensional flat ambient space of two time-like (X_0, X_5) and four space-like (X_1, X_2, X_3, X_4) dimensions, according to [3]

$$\begin{aligned} X_0^2 + X_5^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 &= \\ (X_0 + X_4)(X_0 - X_4) + (X_5 + X_3)(X_5 - X_3) &- (X_1 - iX_2)(X_1 + iX_2) = L^2, \end{aligned} \tag{1}$$

where L^2 is a scale. In changing variables to

$$X_0 + X_4 = e^v, \qquad X_\mu = e^v x_\mu, \quad \mu = 5, 1, 2, 3, \tag{2}$$

the equation (1) becomes

$$e^v (X_0 - X_4) + e^{2v}(x_5^2 - x_1^2 - x_2^2 - x_3^2) = L^2. \tag{3}$$

This equation shows that the AdS_5 space contains a flat Minkowski space-time, $\mathbf{R}^{1,3}$, warped by $\exp(2v)$. Dividing now by that very warp factor, and taking the $v \to \infty$ limit, one encounters the Minkowskian light-cone, $\mathcal{C}^{1+3}$ as,

$$\mathcal{C}^{1+3}: \quad \lim_{v\to\infty} \left[\frac{X_0 - X_4}{e^v} + x_5^2 - x_1^2 - x_2^2 - x_3^2 \right] = \lim_{v\to\infty} \frac{L^2}{e^{2v}} \to 0. \tag{4}$$

The v variable is known as the *holographic* variable. Therefore, the intersections of the AdS_5 null-ray cone with $X_0 + X_4 = e^v$ hyperplanes, produce slices, Π_v, called branes, which are copies of flat Minkowski space-times in the x_μ coordinates with $\mu = 5, 1, 2, 3$. In terms of the holographic variable, the conformal AdS_5 boundary corresponds to the $v \to \infty$ limit, in which case the warp factor blows up. QCD on $\mathcal{C}^{1+3}$ as it emerges at the AdS_5 boundary, is known as holographic QCD in reference to the description of confinement in terms of an 1D Schrödinger equation with a confining potential in the holographic dimension. A variety of interesting insights into the properties of hadrons have been gained within this approach (see ref. [4] for a recent review). However, $\mathbf{R}^{1,3}$, in combination with an interaction potential of an infinite range, represent a significant obstacle toward the formulation of a finite-temperature field theory and the description of the de-confinement phase transition. The reason is that the absence of a scale presents a serious difficulty in defining the QCD chemical potential. This difficulty has been circumvented [2], [5], [6] in replacing the flat Minkowski space-time by the compactified one, $\mathbf{R}^1 \times S^3$, which equally well describes the conformal AdS_5 boundary, as visible from casting eq. (1) into the form [7]

$$\begin{aligned} &\det U - \det g = L^2, \qquad U = \begin{pmatrix} X_5 + iX_0 & 0 \\ 0 & X_5 - iX_0 \end{pmatrix}, \quad g = \begin{pmatrix} X_4 + iX_3 & iX_1 - X_2 \\ iX_1 + X_2 & X_4 - iX_3 \end{pmatrix}, \\ &X_0 + iX_5 = R_0 e^{i\eta}, \qquad X_1 + iX_2 = R \sin\chi \sin\theta e^{i\varphi}, \\ &X_3 = R\sin\chi\cos\theta, \qquad X_4 = R\cos\chi, \quad R_0^2 - R^2 = L^2. \end{aligned} \tag{5}$$

The proof that $\mathbf{R}^1 \times S^3$ is a conformally compactified flat Minkowski space-time, $\mathbf{R}^{1,3}$, has been elaborated in the literature in great detail [8], [9], and the explicit map that takes the $\mathbf{R}^{1,3}$ metric to the metric of $\mathbf{R}^1 \times S^3$ can be found for example in [7]. The AdS_5 line interval in global coordinates reads,

$$ds^2 = \frac{L^2}{\cos^2\alpha}\left(-d\tau^2 + d\alpha^2 + \sin^2\alpha\left(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)\right)\right), \tag{6}$$

where $\alpha \in [0, \pi/2]$ parametrizes the holographic coordinate. The boundary is approached for $\alpha \to \pi/2$ in which case the compactified Minkowski-spacetime, $\mathbf{R}^1 \times S^3$, emerges. Particle motions on this space have been equally well studied, the ref. [7] providing valuable insights in that regard. In particular, it has been found that the free motion of a scalar particle is described in terms of the eigenvalue problem of the conformal Laplacian, $\mathbf{L}^2_{S^3}$,

$$\mathbf{L}^2_{S^3} = -\frac{1}{R^2}\Delta_{S^3} + \mathcal{R}_{S^3}, \qquad -\frac{1}{R^2}\Delta_{S^3} = -\frac{1}{R^2}\left(\frac{\mathrm{d}^2}{\mathrm{d}\chi^2} + 2\cot\chi\frac{\mathrm{d}}{\mathrm{d}\chi} - \frac{\mathbf{L}^2}{\sin^2\chi}\right) = \frac{1}{R^2}\mathcal{K}^2, \quad (7)$$

where Δ_{S^3} is the Laplace-Beltrami operator on S^3, $\mathcal{R}_{S^3}$ stands for the conformal curvature, and $\mathcal{K}^2$ is the operator of the squared four-dimensional (4D) angular momentum which acts as a Casimir invariant of the isometry algebra $so(4)$ of S^3. The $so(4)$ algebra in the representation chosen in (7) has the property to generate the orthogonal group $SO(4)$. On the unit hypersphere, the spectral problem of $\mathbf{L}_{S^3}$ reads,

$$\begin{aligned}
\mathbf{L}_{S^3}Y_{Klm}(\vec{\Omega}) = (-\Delta_{S^3}+1)Y_{Klm}(\vec{\Omega}) &= \left(\mathcal{K}^2+1\right)Y_{Klm}(\vec{\Omega}) = (K+1)^2 Y_{Klm}(\vec{\Omega}), \quad \vec{\Omega} = (\chi,\theta,\varphi)\\
Y_{Klm}(\vec{\Omega}) = \mathcal{S}^l_K(\chi)Y^m_l(\theta,\varphi), &\qquad \mathcal{S}^l_K(\chi) = \sin^l\chi\,\mathcal{G}^{l+1}_{K-l}(\cos\chi),\\
K\in[0,\infty), \quad l\in[0,K], &\qquad m\in[-l,+l]. \qquad (8)
\end{aligned}$$

Here, $Y_{Klm}(\vec{\Omega})$ are the hyper-spherical harmonics, $\mathcal{G}^\alpha_n(\cos\chi)$ denote the Gegenbauer polynomials, and $Y^m_l(\theta,\varphi)$ stand for the ordinary three-dimensional spherical harmonics. The χ dependent part of the hyper-spherical harmonics, i.e. the $\mathcal{S}^l_K(\chi)$ function, is sometimes referred to as "quasi-radial" function [10], a notation which we occasionally shall make use of. It is obvious, that the spectrum in (8) is characterized by a $(K+1)^2$ fold degeneracy of the levels. Such a spectrum as whole would fit into an infinite unitary representation of the conformal group, $SO(2,4)$, where the group $SO(4)$ appears in the reduction chain,

$$SO(2,4)\subset SO(4)\subset SO(3)\subset SO(2). \qquad (9)$$

It is within this context that $(K+1)^2$-fold degeneracy is associated with conformal symmetry, in parallel to the H atom where $(K+1)$ acquires the meaning of the principal quantum number of the Coulomb potential problem. However, conformal symmetry can be at most an approximate symmetry of QCD because it requires

- massless quarks,
- scale independent strong coupling, and
- a massless dilaton.

While the first two conditions seem to be reasonably respected within the unflavored sector of QCD, in which the u and d quark masses can be considered sufficiently small at the scale of the nucleon excitations, which start around 1500 MeV, and in the infrared regime, where the running coupling constant starts walking toward a fixed value [11], the dilaton mass can not be disregarded so far. The breaking of the conformal symmetry is frequently treated in terms of breaking the metric of the Minkowski space-time encountered at the AdS_5 boundary, i.e. by deforming the warp factor (soft-walls) [4]. Notice, the dilaton can be introduced in the action, $\mathcal{S}$, as a background field in an overall exponential according to,

$$\mathcal{S} = \int \mathrm{d}^D X e^{-\Phi(z)}\sqrt{-g}\mathcal{L}, \quad \Phi(z) = (\mu z)^\nu, \quad z = e^{-v}, \qquad (10)$$

where μ sets a mass scale, usually taken as Λ_{QCD} for simplicity. As shown in [12], [13], the dilaton modifies the particle motion and leaves a print in the meson spectra.

We here adapt the essentials of the above scheme to the geometry of the compactified Minkowski space-time, where the holographic dimension can be kept for the time being as a spectator insofar as particle motion on the finite volume S^3 space is automatically confined. We design a pragmatic quark model of conformal symmetry breaking on $\mathbf{R}^1 \times S^3$ through global S^3 metric deformation by an exponential factor of the second polar angle. We then study the impact of this deformation on the spectra of the high-lying unflavored mesons reported by the Cristal Barrel collaboration as incorporated in the compilation of [14]. Our aim is to reveal a possibility to deform the conformal curvature in (6) by a scale, and without removing the conformal degeneracy patterns from the spectra.

The outline of the paper is as follows. In the next section we present the mechanism of conformal symmetry breaking without removing the conformal degeneracy patterns in the spectra. In section 3 we analyze the data on the high-lying unflavored meson spectra listed in [14] with the aim to obtain an estimate for the S^3 radius and the order of magnitude of the conformal symmetry breaking scale which we then attribute to the dilaton. The paper ends with concise conclusions.

2. Breaking the conformal curvature of S^3 by an interaction

Any perturbation of the quantum mechanical motion on S^3 by an interaction potential, $V(\vec{\Omega})$, breaks the conformal curvature as,

$$\hbar^2 c^2 \mathcal{R}_{S^3} \longrightarrow \hbar^2 c^2 \widetilde{\mathcal{R}}_{S^3} = \frac{\hbar^2 c^2}{R^2} + V(\vec{\Omega}), \tag{11}$$

and the perturbed Laplacian is no longer conformal, neither is its spectral problem necessarily exactly solvable. Not so for the potential, given by,

$$V(\chi) = -2B \cot \chi, \tag{12}$$

with $B = b\hbar^2 c^2/R^2$, and a dimensionless b, which gives rise to an exactly solvable $\left(\hbar^2 c^2 \mathbf{L}_{S^3} + V(\chi)\right)$ spectral problem, and one which moreover happens to carry same $(K+1)^2$-fold degeneracy patterns as the spectral problem of the intact conformal Laplacian in (8). The $\cot\chi$ potential is special insofar as it is a harmonic function on S^3, just as is the Coulomb potential in E_3, and can be treated along the line of harmonic analysis and potential theory. Using harmonic functions as potentials brings the advantage of minimizing the Dirichlet functional integral. Occasionally, the cotangent potential on S^3 is referred to as "curved" Coulomb potential. In the present section we wish to take a closer look on the quantum mechanical cotangent potential problem on S^3 from the perspective of conformal Laplacians [15, 16]. The cotangent potential problem on S^3, first considered by Schrödinger [17, 18], is equivalent to the relativistic four-dimensional rigid-rotator problem perturbed by a $\cot\chi$ interaction,

$$\frac{\hbar^2 c^2}{R^2}\left[-\Delta_{S^3} + 1 - 2b\cot\chi\right]\Psi_{K\widetilde{\widetilde{lm}}}(\vec{\Omega}) = \frac{\hbar^2 c^2}{R^2}\left[(K+1)^2 - \frac{b^2}{(K+1)^2}\right]\Psi_{K\widetilde{\widetilde{lm}}}(\vec{\Omega}). \tag{13}$$

The solutions of equation (13) are well studied and known. Specifically in [19], they have been expressed in terms of real non-classical Romanovski polynomials, $R_n^{\alpha,\beta}(\cot\chi)$ (reviewed in [20]), and in units of $\hbar = 1 = c$, $R = 1$, read,

$$\begin{aligned}
\Psi_{K\widetilde{l}\widetilde{m}}(\vec{\Omega}) &= e^{\frac{\alpha_K \chi}{2}} \psi_K^{\widetilde{l}}(\chi) Y_{\widetilde{l}}^{\widetilde{m}}(\theta, \varphi), \\
\psi_K^{\widetilde{l}}(\chi) &= \sin^K \chi R_{K-\widetilde{l}}^{\alpha_K, \beta_K - 1}(\cot \chi), \\
\alpha_K = -\frac{2b}{K+1}, &\qquad \beta_K = -K.
\end{aligned} \tag{14}$$

The Romanovski polynomials satisfy the following differential hyper-geometric equation,

$$(1+x^2)\frac{\mathrm{d}^2 R_n^{\alpha,\beta}}{\mathrm{d}x^2} + 2\left(\frac{\alpha}{2} + \beta x\right)\frac{\mathrm{d}R_n^{\alpha,\beta}}{\mathrm{d}x} - n(2\beta + n - 1)R_n^{\alpha,\beta} = 0. \tag{15}$$

They are obtained from the following weight function,

$$\omega^{\alpha,\beta}(x) = (1+x^2)^{\beta-1}\exp(-\alpha \cot^{-1} x), \tag{16}$$

by means of the Rodrigues formula,

$$R_n^{\alpha,\beta}(x) = \frac{1}{\omega^{\alpha,\beta}(x)}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left[(1+x^2)^n \omega^{\alpha,\beta}(x)\right]. \tag{17}$$

Recently, it has been shown in [19] that the $\psi_K^{\widetilde{l}}(\chi)$ functions allow for finite decompositions in the $SO(4)$ basis of the quasi-radial functions $S_K^l(\chi)$ in (8). Namely, the following relation has been found to hold valid,

$$\psi_K^{\widetilde{l}}(\chi) = \sum_{\widetilde{l}}^{K} C_l \ \mathcal{S}_K^l(\chi). \tag{18}$$

The explicit expressions for the coefficients C_l defining the full similarity transformation between the invariant spaces of the perturbed and intact Laplacians on S^3 can be consulted in [19]. Upon introducing the notations,

$$\omega_K(\chi) = \frac{\alpha_K \chi}{2} \equiv -\frac{b\chi}{K+1}, \qquad b = -\omega_K'(\chi)(K+1), \tag{19}$$

and insertion of (18) into (14), allows to rewrite (13) to,

$$\begin{aligned}
\left[\mathbf{L}_{S^3} + 2\omega_K'(\chi)(K+1)\cot\chi\right] e^{\omega_K(\chi)}\psi_K^{\widetilde{l}}(\chi) &= \\
& e^{\omega_K(\chi)} \sum_{\widetilde{l}}^{K} C_l \left[(K+1)^2 - {\omega_K'(\chi)}^2\right] S_K^l(\chi).
\end{aligned} \tag{20}$$

Dragging now the exponential factor in the l.h.s. in (20) from the right to the very left, recalling that $\mathbf{L}_{S^3} = \mathcal{K}^2 + 1$, and making use of (18), amounts to

$$\begin{aligned}
e^{\omega_K(\chi)}\left[\mathcal{K}^2 + 1 - {\omega_K'(\chi)}^2 - 2\omega_K'(\chi)\mathcal{D}_{\mathbf{K}}\right]\sum_{\widetilde{l}}^{K} C_l \ \mathcal{S}_K^l(\chi) &= \\
& e^{\omega_K(\chi)} \sum_{\widetilde{l}}^{K} C_l \left[(K+1)^2 - {\omega_K'(\chi)}^2\right] S_K^l(\chi), \\
\mathcal{D}_{\mathbf{K}} &= \frac{\mathrm{d}}{\mathrm{d}\chi} - K\cot\chi.
\end{aligned} \tag{21}$$

Using in the r.h.s. of (21) the relationship, $(\mathcal{K}^2+1)\, S^l_K(\chi) = (K+1)^2 S^l_K(\chi)$, known from (7), allows to equivalently cast eq. (21) as,

$$\left(\widetilde{\mathcal{K}}^2 + 1 - \omega'_K(\chi)^{\,2} \quad - \quad 2\omega'_K(\chi) e^{\omega_K(\chi)} \mathcal{D}_{\mathbf{K}} e^{-\omega_K(\chi)}\right) e^{\omega_K(\chi)} \sum_{\widetilde{l}}^{K} C_l\ \mathcal{S}^l_K(\chi)$$
$$= \sum_{\widetilde{l}}^{K} \left(\widetilde{\mathcal{K}}^2 \ + 1 - \omega'(\chi)^2\right) e^{\omega_K(\chi)} C_l\ \mathcal{S}^l_K(\chi). \tag{22}$$

This equality implies the following important property of the $\mathcal{D}_K$ operator,

$$\left(\widetilde{\mathcal{K}}^2 - 2\omega'_K(\chi) e^{\omega_K(\chi)} \mathcal{D}_{\mathbf{K}} e^{-\omega_K(\chi)}\right) e^{\omega_K(\chi)} \sum_{\widetilde{l}}^{K} C_l\ \mathcal{S}^l_K(\chi) \quad = \quad \sum_{\widetilde{l}}^{K} \widetilde{\mathcal{K}}^2\, e^{\omega_K(\chi)} C_l\ \mathcal{S}^l_K(\chi),$$
$$\widetilde{\mathcal{K}}^2 \quad = \quad e^{\omega_K(\chi)} \mathcal{K}^2 e^{-\omega_K(\chi)}. \tag{23}$$

Now, the equivalence of eqs. (20)–(21), and (22) allows to draw the final conclusion as

$$\left[\mathcal{K}^2 + 1 + 2\omega'_K(\chi)(K+1)\cot\chi\right] e^{\omega(\chi)} \psi^{\widetilde{l}}_K(\chi) \quad = \quad \sum_{\widetilde{l}}^{K} C_l \left[\widetilde{\mathcal{K}}^2 + 1 - \omega'(\chi)^2\right]\, e^{\omega_K(\chi)} \mathcal{S}^l_K(\chi)$$
$$= \quad \left[(K+1)^2 - \omega'_K(\chi)^{\,2}\right] e^{\omega(\chi)} \psi^{\widetilde{l}}_K(\chi), \tag{24}$$

with $\psi^{\widetilde{l}}_K$ from (18), and $\omega'_K(\chi)$ being a representation constant. In this fashion, the eigenvalue problem of the cotangent-broken conformal Laplacian on S^3 presents itself as the eigenvalue problem of a Casimir invariant of the $so(4)$ algebra in a representation unitarily nonequivalent to the 4D-rotational. As long as the eigenvalues of any Casimir invariant remain unaltered under any well defined similarity transformations, no matter unitary or not, the degeneracy patterns in the spectral problems of the conformal and the cotangent-broken Lapalacians result same. The Killing vectors of this new $so(4)$ algebra,

$$\widetilde{J}_{ik} = e^{\omega_K(\chi)} J_{ik} e^{-\omega_K(\chi)} \quad = \quad e^{\omega_K(\chi)} \left(X_j \frac{\partial}{\partial X_k} - X_k \frac{\partial}{\partial X_j}\right) e^{-\omega_K(\chi)},$$
$$\left[\widetilde{J}_{ik}, \widetilde{J}_{kj}\right] = \widetilde{J}_{ji}, \qquad \chi = \cot^{-1} \frac{X_4}{\sqrt{X_1^2 + X_2^2 + X_3^2}}\ , \tag{25}$$

refer to a surface (call it $\widetilde{S}^3$) which will differ in shape from S^3. In this fashion, the breaking of the conformal symmetry reveals itself through metric deformation. Equivalently, one can say that it reveals itself at the level of the representation functions, in reference to the difference between the hyper-spherical harmonics, $Y_{Klm}(\vec{\Omega})$ in (8), and $\Psi_{K\widetilde{l}\widetilde{m}}(\vec{\Omega})$ in (14). Indeed, the global metric of the S^3 ball is incorporated by the scalar $so(4)$ representation, which, in being the lowest $\mathbf{L}_{S^3}$ eigenstate, is the hyper-spherical harmonic, $Y_{000}(\vec{\Omega})$,

$$S^3: \quad Y_{000}(\vec{\Omega}) \quad = \quad \sum_{i=1}^{4} X_i^2 = \cos^2\chi + \sin^2\chi\left(\cos^2\theta + \sin^2\theta\left(\cos^2\varphi + \sin^2\varphi\right)\right) = 1. \tag{26}$$

Correspondingly, the global metric of the $\widetilde{S}^3$ surface will be implemented by the scalar of the non-unitary $so(4)$ algebra in (23), (25) and is given by the exponentially rescaled hyper-sphere,

$$\widetilde{S}^3: \quad e^{-b\chi} Y_{000}(\vec{\Omega}) \ = \ e^{-b\chi}\left(\cos^2\chi + \sin^2\chi\left(\cos^2\theta + \sin^2\theta\left(\cos^2\varphi + \sin^2\varphi\right)\right)\right) = e^{-b\chi}. \tag{27}$$

The line element on the deformed surface $\widetilde{S}^3$ is then easily calculated as,

$$\mathrm{d}^2\widetilde{s} \;=\; e^{-b\chi}\left((1+b^2/4)\mathrm{d}^2\chi+\sin^2\chi\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}^2\varphi\right)\right). \tag{28}$$

In the following we conjecture that the $\hbar^2c^2b/R^2$ scale of conformal symmetry breaking can be attributed to the dilaton, the only significant source of such a breaking.

Notice that the perturbance of the conformal Laplacian on S^3 by a cotangent interaction does not even produce a conformal map [15], [16], as it could have happened by a different, in general, gradient potential.

> In summary, a subtle mode of symmetry breaking has been identified in which the symmetry breaking happens at the level of the representation functions of the isometry algebra of a given geometric manifold, and reveals itself through a metric deformation, while the spectral problems of the intact and broken Laplacians are characterized by identical degeneracy patterns.

It seems opportune to refer to this symmetry breaking mechanism as "symmetry breaking camouflaged by degeneracy", or, shortly, "camouflaged symmetry breaking".

3. Degeneracy of high-lying unflavored mesons

In the following we employ the cotangent-broken conformal Laplacian on S^3 in (13) in the description of the spectra of the high-lying unflavored mesons as compilation in [14] and adjust the R and b potential parameters. The cotangent potential derives its utility for employment in the description of hadron spectra from its close relationship to the Cornell potential, predicted by Lattice QCD [21], as visible from its Taylor series decomposition,

$$-2b\cot\frac{\overset{\frown}{r}}{R}=-\frac{2bR}{\overset{\frown}{r}}+\frac{4b}{3R}\,\overset{\frown}{r}+..., \quad \text{for}\quad \chi=\frac{\overset{\frown}{r}}{R}, \tag{29}$$

where $\overset{\frown}{r}$ denotes the geodesic distance on S^3, and R was the S^3 radius.

The data on the spectra of the high-lying unflavored mesons, and specifically the Crystal Barrel data, show well pronounced conformal degeneracy patterns in the region above ≈ 1200 MeV. Our case is that

- these patterns emerge in consequence of the breaking of the conformal $SO(2,4)$ group symmetry at the level of the representation of its algebra, and in accordance with eqs. (24), (28),
- such a breaking may be attributed to the dilaton.

In Figs. (1–4) we present the spectra of the mesons of interest. Tables 1 and 2 contain the results of the least square parameter fit.

Table 1. The potential parameters from the least square fit to the meson spectra by means of eq. (13).

Isospin	Lowest spin	R [fm]	$(\hbar^2 c^2 b)/R^2$[GeV^2]
I=0	$J_{\rm min}$=0^-	0.5157279	0.459666
I=0	$J_{\rm min}$=1^-	0.5293829	0.425489
I=1	$J_{\rm min}$=0^-	0.5443879	0.522783
I=1	$J_{\rm min}$=1^-	0.5268850	0.411833

Table 2. The meson masses, $M^{\rm fit}$, from the least square fit to the experimental values, $M^{\rm expr}$, by means of eq. (13). The label K stands for the four-dimensional angular momentum characterizing a degenerate level in accordance with eq. (13). The fourth column contains the averaged mass of the experimentally observed states as displayed in the respective plots on the Figs. 1-4.

I	$J_{\rm min}$	K	$M^{\rm expr}$[MeV]	$M^{\rm fit}$ [MeV]
0	0^-	1	1278.33	1349.60
		2	1692.5	1659.48
		3	1970.66	1961.83
		4	2274.22	2280.1
1	0^-	1	1328.33	1404.89
		2	1696.66	1708.67
		3	2031	1985.04
		4	2253.75	2273.69
0	1^-	1	1325	1416.19
		2	1653.33	1697.43
		3	1994.14	1979.19
		4	2269.77	2279.69
1	1^-	1	1343.33	1385.85
		2	1682.5	1670.28
		3	1825.85	1957.16
		4	2256.66	2262.75

4. Discussion and conclusions

The Tables 1 and 2 show that our least square fits to on the data on the masses of the high-lying unflavored mesons by means of the cotangent broken conformal Laplacian on S^3 in eq. (13) provide quite a reasonable description of the observed degeneracies in the spectra under investigation and in support of our hypothesis that the breaking of the conformal symmetry can occur at the level of the representation of the conformal algebra and without affecting the degeneracies. The b parameter is obtained as

$$b \;=\; 3.2793 \pm 0.0697. \qquad (30)$$

We furthermore observe that to a very good approximation, both the S^3 radius and the $\hbar^2 c^2 b/R^2$ scale are isospin and parity independent, as should be for a conformal symmetry breaking scale due to the dilaton. The mean value of the inverse radius obtained from our fits corresponds to a temperature of $T = \hbar c/R$= 373 MeV which reasonably fulfills the requirement to be notably larger than $\Lambda_{QCD} = 175$ MeV. In order to extract the scale parameter of the conformal symmetry

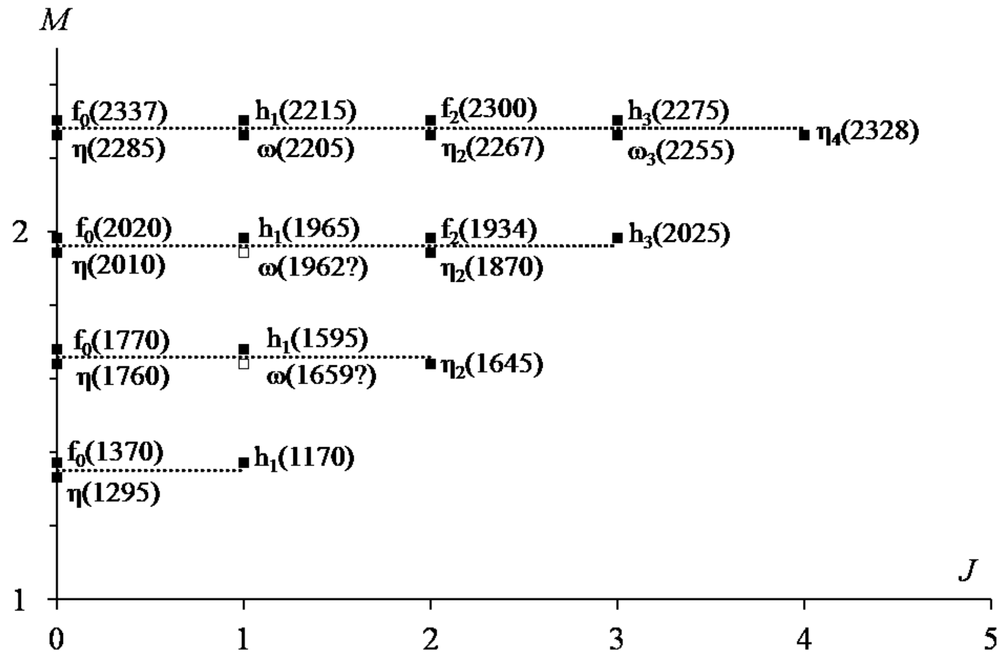

Figure 1. High-lying excitation masses, M [GeV], of the isosinglet pseudoscalar η meson according the data compilation of [14]. On the one side, the quantum numbers of the mesons in the four groups displayed here fit well into the respective $K = 1, 2, 3$, and 4 eigenstates of the conformal Laplacian on S^3 in (8). At the same time, they fit equally well into the eigenstates of the broken Laplacian in (24), which shows that the breaking of the conformal group symmetry at the level of the representation of its algebra by the $\hbar^2c^2b/R^2$ mass scale does not leave a print in the degeneracies in the spectrum. Empty squares denote "missing" resonances. These are additionally marked by a question-mark inside the round brackets, while the accompanying number is our prediction for the respective mass. Notice that not only the conformal degeneracy patterns are well pronounced, also practically all the states with spins lower than the maximal, in a level, ($J_{\rm max} = K$), appear parity duplicated, a phenomenon that is indicative of a manifest realization of the chiral symmetry in the Wigner-Wyle mode at that scale, or, remains of it after a moderate chiral symmetry breaking.

breaking within our approach, we make use of eq. (29), i.e. of $\chi = \frac{\widehat{r}}{R}$, and rewrite the $\exp(-b\chi)$ factor equivalently as,

$$\exp(-b\chi) \quad = \quad \exp\left(-\left[\left(\frac{\hbar c\sqrt{b}}{R}\right)^2\left(\frac{\sqrt{\widehat{r}\ R}}{\hbar c}\right)^2\right]\right)\equiv \exp(-\mu^2 r^2),$$

$$\mu = \frac{\hbar c\sqrt{b}}{R}, \qquad r = \frac{\sqrt{\widehat{r}\ R}}{\hbar c}. \tag{31}$$

Evaluating this scale with the numbers listed in Table 1 amounts to a mean value of $\mu = 673.7$ MeV, which is closer to the dilaton mass than to $\Lambda_{QCD} = 175$ MeV in (10). We conclude that the data on high-lying unflavored mesons support our model of conformal symmetry breaking by a mass scale at the level of the representation of the underlying $so(4)$ algebra, and without loosing the degeneracies in the spectra.

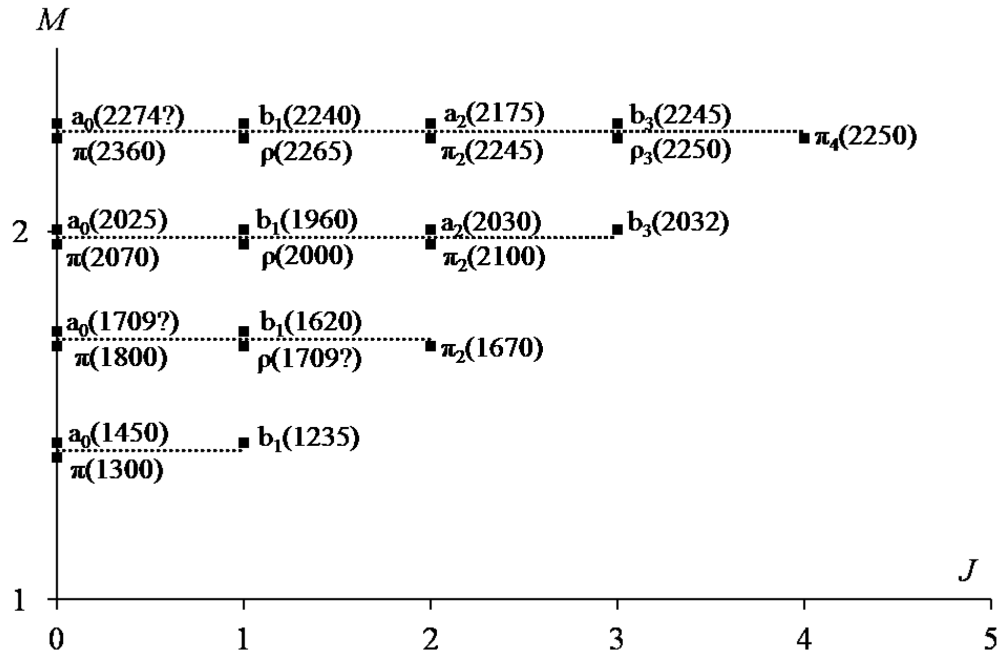

Figure 2. High-lying excitations of the isotriplet pseudoscalar π meson according to the data compilation of [14]. For notations and clarifying comments see Fig. 1.

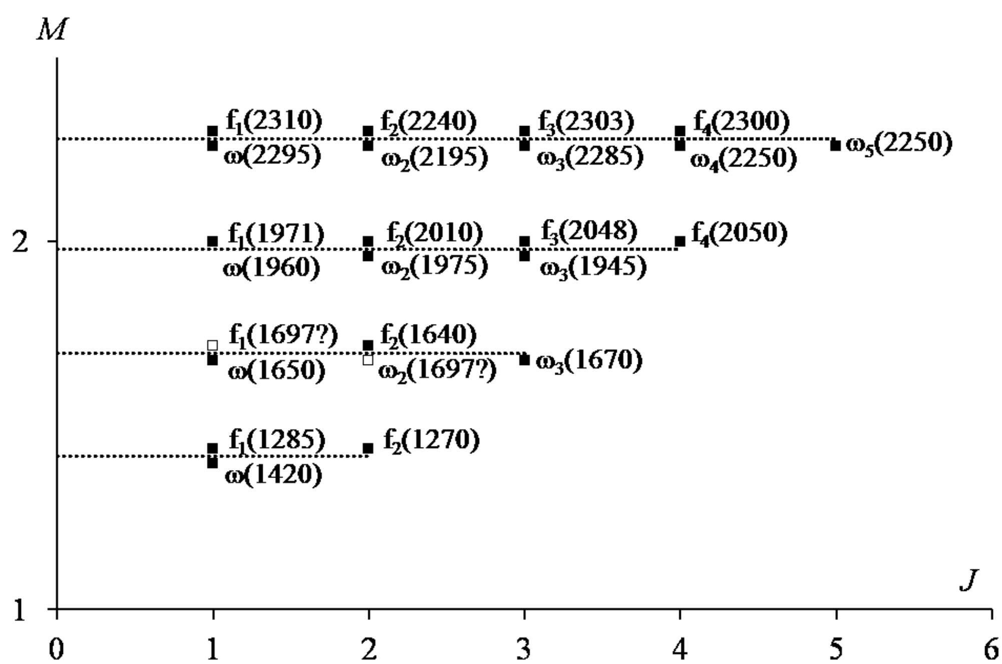

Figure 3. High-lying excitations of the isosinglet vector ω meson according to the data compilation of [14]. For notations and clarifying comments see Fig. 1.

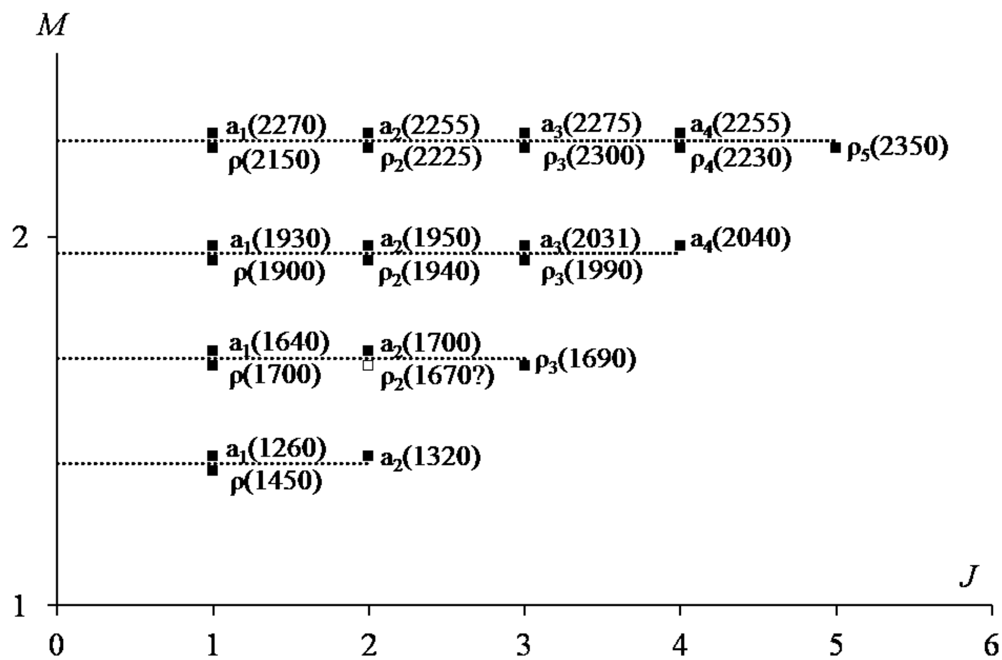

Figure 4. High-lying excitations of the isotriplet vector ρ meson according to the data compilation of [14]. For notations and clarifying comments see Fig. 1.

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