The infrared fixed point of Landau gauge Yang-Mills theory: A renormalization group analysis

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Gluons

- Euclidean QCD Lagrangian

$$\mathcal{L} = \sum_f \bar{\psi}_f \left(i\gamma_\mu D_\mu + m \right) \psi_f \; + \; \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

- $\mathcal{L}$ invariant under local gauge transformations

$$\psi_f \to \psi_f^U = U \, \psi_f \, , \qquad f = 1, 2, \ldots, N_f$$

$$U(x) = e^{ig\omega(x)} \in SU(N) \, , \qquad \omega = \omega^a T^a \, , \qquad a = 1, 2, \ldots, N^2 - 1$$

- covariant derivative and gauge fields (gluons)

$$D_\mu = \partial_\mu - igA_\mu \, , \qquad A_\mu = A^a_\mu T^a \, , \qquad F_{\mu\nu} = \frac{i}{g} \left[D_\mu, D_\nu \right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \, , \qquad [T^a, T^b] = i f^{abc} T^c \, , \qquad \mathrm{tr}(T^a T^b) = \frac{1}{2} \, \delta^{ab}$$

- $SU(N)$ gauge transformations

$$A_\mu \to A^U_\mu = U \left(A_\mu + \frac{i}{g} \, \partial_\mu \right) U^\dagger \, , \qquad D_\mu \to U D_\mu U^\dagger$$

$$F_{\mu\nu} \to U F_{\mu\nu} U^\dagger \, , \qquad \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} = \frac{1}{2} \, \mathrm{tr}(F_{\mu\nu} F_{\mu\nu})$$

Ghosts

- infinitesimal gauge transformations

$$\delta\psi_f = ig\omega\psi_f \,, \qquad \delta A_\mu = \partial_\mu\omega - ig\,[A_\mu,\omega] = D_\mu\omega \,, \qquad (D_\mu\omega)^a = \partial_\mu\omega^a + gf^{abc}A_\mu^b\omega^c$$

- necessary for quantization: (covariant) gauge fixing

$$\partial_\mu A_\mu = 0$$

- change of variables: $A_\mu = \overline{A}_\mu^U$, $\partial_\mu \overline{A}_\mu = 0$ (suppose $\overline{A}_\mu, U$ unique!)

$$\int D[A] = \int D[U] \int D[\overline{A}] \det \mathcal{J}$$
$$= \int D[U] \int D[A]\, \delta\left(\partial_\mu A_\mu\right) \det\left(-\partial_\mu D_\mu\right)$$
$$= \int D[U] \int D[A] \int D[B] \exp\left(-i \int d^4x\, B^a\, \partial_\mu A_\mu^a\right) \int D[c,\bar{c}]\, \exp\left(\int d^4x\, \bar{c}^a\, \partial_\mu D_\mu^{ab} c^b\right)$$

"Landau gauge", B Nakanishi-Lautrup field, $c, \bar{c}$ ghosts: scalar fermion fields

BRST symmetry

- Faddeev-Popov Lagrangian

$$\int D[\psi_f,\bar{\psi}_f,A]\, \exp\left(-\int d^4x\, \mathcal{L}\right) \propto \int D[\psi_f,\bar{\psi}_f,A,B,c,\bar{c}]\, \exp\left(-\int d^4x\, \mathcal{L}_{FP}\right)$$

$$\mathcal{L}_{FP} = \sum_f \bar{\psi}_f\, (i\gamma_\mu D_\mu + m)\, \psi_f \;+\; \frac{1}{4}\, F^a_{\mu\nu} F^a_{\mu\nu} + iB^a\, \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a\, D^{ab}_\mu c^b$$

- gauge symmetry is broken by the gauge fixing, but $\mathcal{L}_{FP}$ is invariant under BRST transformations

$$s\psi_f = igc^a T^a \psi_f\,, \qquad sA^a_\mu = D^{ab}_\mu c^b$$

$$sc^a = \frac{1}{2}\, gf^{abc} c^b c^c\,, \qquad s\bar{c}^a = iB^a\,, \qquad sB^a = 0$$

s is nilpotent: $s^2 = 0$

Perturbative beta function

- perturbation theory: one-loop beta function

$$\beta(g_R) = \mu^2 \frac{d}{d\mu^2} g_R(\mu) = \frac{g_R^3}{(4\pi)^2} \left(\frac{N_f}{3} - \frac{11N}{6} \right)$$

  negative for $N_f < 11N/2$ ($= 33/2$ for proper QCD)

- running coupling constant

$$\frac{g_R^2(\mu)}{4\pi} = \frac{2\pi}{(11N/6 - N_f/3) \ln(\mu^2/\Lambda_{\rm QCD}^2)}$$

  asymptotic freedom for $\mu \gg \Lambda_{\rm QCD}$, Landau pole at $\mu = \Lambda_{\rm QCD}$

- gluons are responsible for asymptotic freedom and Landau pole (also for quark confinement: lattice calculations), from now on put $N_f = 0$: no dynamical quarks, "quenched QCD" or $SU(N)$ Yang-Mills theory

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Gribov copies

- problem with the Faddeev-Popov procedure: $\bar{A}_\mu^U \neq \bar{A}_\mu$ with $\partial_\mu \bar{A}_\mu^U = 0 = \partial_\mu \bar{A}_\mu$ exist, "Gribov copies"
- for infinitesimal gauge transformations

$$\partial_\mu \bar{A}_\mu^U = \partial_\mu \bar{A}_\mu + \partial_\mu D_\mu \omega$$

  Gribov copies exist (at least) when $(-\partial_\mu D_\mu)$ has zero modes
- Gribov (1978) suggests to restrict the integration over $\bar{A}$ to the (first) Gribov region Ω where $(-\partial_\mu D_\mu)$ is positive definite

$$\int D[A] \to \int_\Omega D[A]$$

  boundary of Ω: the (first) Gribov horizon $\partial\Omega$, $\det(-\partial_\mu D_\mu) = 0$
- no Landau pole can arise, restriction to Ω breaks BRST symmetry (softly)

Horizon function

- possible implementation: Zwanziger (1994)

$$\int_\Omega D[A] = \int D[A]\, \theta(-\partial_\mu D_\mu) = \int D[A] \exp\left(-\gamma^4 \int d^4x\, h(x)\right)$$
$$h(x) = g^2 f^{abc} f^{cde} A^b_\nu \left[(-\partial_\mu D_\mu)^{-1}\right]^{ad} A^e_\nu$$

with the horizon condition (to fix γ^2)

$$\langle h(x) \rangle = 4(N^2-1)$$

- local formulation

$$\mathcal{L}_{FP} \to \mathcal{L}_{GZ} = \mathcal{L}_{FP} - \partial_\mu \bar\phi^{ac}_\nu D^{ab}_\mu \phi^{bc}_\nu + \partial_\mu \bar\omega^{ac}_\nu D^{ab}_\mu \omega^{bc}_\nu - g f^{abc} \partial_\mu \bar\omega^{ad}_\nu D^{be}_\mu c^e \phi^{cd}_\nu$$
$$- \gamma^2 \left[g f^{abc} A^a_\mu \phi^{bc}_\mu + g f^{abc} A^a_\mu \bar\phi^{bc}_\mu + 4(N^2-1)\gamma^2 \right]$$

and $\quad \langle g f^{abc} A^a_\mu (\phi^{bc}_\mu + \bar\phi^{bc}_\mu) \rangle = 8(N^2-1)\gamma^2$

with bosonic and fermionic auxiliary fields ϕ^{ab}_μ and ω^{ab}_μ: “Gribov-Zwanziger framework”

Propagators and condensates

- tree-level propagators: gluon propagator IR suppressed, ghost propagator IR enhanced

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \frac{p^2}{p^4 + \lambda^4} \, \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) , \qquad \lambda^4 = 2 g^2 N \gamma^4$$

$$\langle c^a(p) \bar{c}^b(-p) \rangle \propto \frac{1}{p^4} \, \delta^{ab} \quad \text{for} \quad p^2 \ll \Lambda^2_{\mathrm{QCD}}$$

- however, the auxiliary fields can form a condensate $\langle \bar{\phi}^{ab}_\mu \phi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \rangle \neq 0$, then the linear coupling of ϕ^{ab}_μ and $\bar{\phi}^{ab}_\mu$ to A^a_μ in $\mathcal{L}_{GZ}$ generates a gluon mass term: "refined Gribov-Zwanziger framework" by Dudal, Sorella, Vandersickel, Verschelde et al. (2008)
- consequence for the tree-level propagators: finite suppression and enhancement

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \frac{p^2 + M^2}{p^4 + M^2 p^2 + \lambda^4} \, \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$\frac{p^2 + M^2}{p^4 + M^2 p^2 + \lambda^4} = \frac{1}{p^2 + M^2_{\mathrm{eff}}(p^2)} , \qquad M^2_{\mathrm{eff}}(p^2) = \frac{\lambda^4}{p^2 + M^2}$$

$$\langle c^a(p) \bar{c}^b(-p) \rangle \propto \frac{1}{p^2} \, \delta^{ab} \quad \text{for} \quad p^2 \ll \Lambda^2_{\mathrm{QCD}}$$

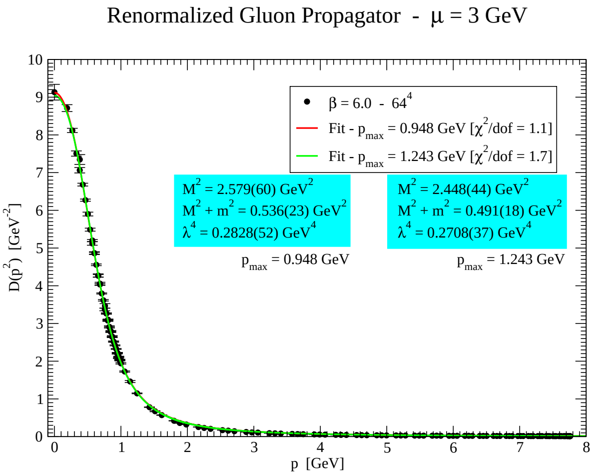
Present status

- excellent agreement in the IR with lattice calculations in the (absolute) Landau gauge upon including a further condensate $\langle A^a_\mu A^a_\mu \rangle$

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4 + M^2 m^2} \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

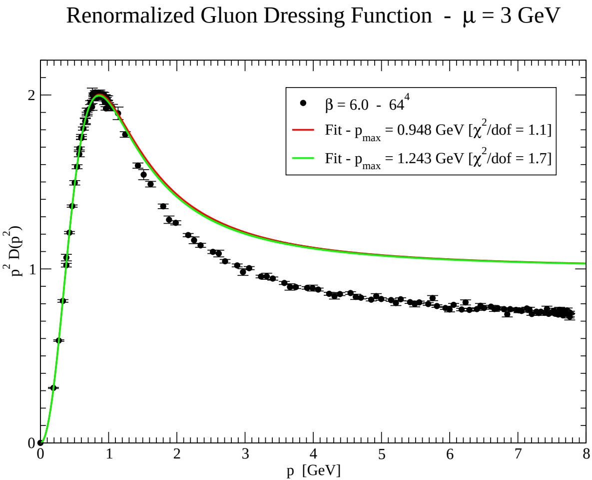
  and fitting the parameters M^2 and m^2
- complete calculation of the condensates and one-loop corrections to the propagators very demanding

Gluon propagator



Dudal, Oliveira and Vandersickel (2010)

Gluon dressing function



Dudal, Oliveira and Vandersickel (2010)

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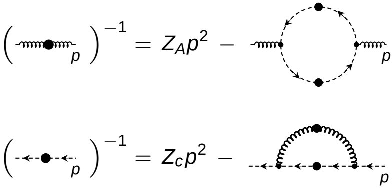
General setup

- observation: restriction to Ω leaves Dyson-Schwinger equations unchanged (and hence the perturbative expansion), no contributions from $\partial\Omega$ since $\det(-\partial_\mu D_\mu) = 0$ there
- however, BRST symmetry is (softly) broken, (re)normalization conditions may be different
- assume IR suppressed gluon propagator and IR enhanced ghost propagator (check self-consistency a posteriori), "ghost dominance": diagrams with maximal number of ghost propagators dominate in the IR
- Taylor's non-renormalization theorem for the ghost-gluon vertex
- simple closed set of Dyson-Schwinger equations for the IR-propagators



Results

- numerical solutions over the whole momentum range (see Pietro Dall'Olio's talk)
- Zwanziger (2002): analytical solutions in the IR approximation

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \frac{G(p^2)}{p^2}\,\delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \qquad \langle c^a(p)\bar{c}^b(-p)\rangle = \frac{F(p^2)}{p^2}\,\delta^{ab}$$

$$G(p^2) \propto (p^2)^{-\alpha_G}, \qquad F(p^2) \propto (p^2)^{-\alpha_F}$$

for dimensions $2 \leq D \leq 4$

- scaling solutions, first found by von Smekal, Hauck and Alkofer (1997) in $D = 4$ dimensions

$$\text{sum rule} \qquad \alpha_G + 2\alpha_F = \frac{D}{2} - 2$$

$$\text{solution 1:} \qquad \alpha_F(D) = \frac{D-2}{2}, \qquad \alpha_G(D) = -\frac{D}{2}$$

$$\text{solution 2:} \qquad \alpha_F(D) \approx \frac{D-1}{5}, \qquad \alpha_G(D) \approx -\frac{16-D}{10}$$

$\alpha_F > 0$, $1 + \alpha_G < 0$, except for solution 1 at $D = 2$: $\alpha_F = 1 + \alpha_G = 0$

Scaling vs. decoupling solutions

- dimensionless running coupling constant (from Taylor's theorem)

$$g_R^2(p^2) = (p^2)^{(D-4)/2}\, G(p^2)\, F^2(p^2)\, g^2$$

  goes to a finite IR fixed point value (goes to zero for solution 1 at $D = 2$)
- decoupling solution: $\alpha_F = 1 + \alpha_G = 0$ for any dimension D, $g_R^2(p^2) \to 0$ for $p^2 \to 0$, consistent with lattice calculations and the refined Gribov-Zwanziger framework (except at $D = 2$ where lattice calculations show scaling behavior)
- from the Dyson-Schwinger equations alone one cannot decide which one of the solutions is physically realized
- it is unclear how the results could be systematically improved

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A gluon mass term

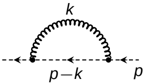
- Dyson-Schwinger results reminiscent of critical phenomena, most successful tool: renormalization group equations of Callan-Symanzik type in an epsilon expansion, systematic approach with analytical (perturbative) input
- (soft) BRST breaking: introduce a mass m for the gluons in $\mathcal{L}_{FP}$, the only perturbatively relevant parameter that can arise

$$\mathcal{L}_{FP}^{m} = \mathcal{L}_{FP} + \frac{1}{2} A_\mu^a m^2 A_\mu^a$$

- for the IR physics, the new mass term dominates over the $(A_\mu^a p^2 A_\mu^a)$-contribution from $\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a$

Example

- one-loop ghost self-energy

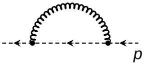


$$= Ng^2 \delta^{ab} \int \frac{d^D k}{(2\pi)^D}\, p_\mu \,\frac{1}{(p-k)^2}\,(p_\nu - k_\nu)\,\frac{1}{k^2+m^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$= -\frac{1}{2}\frac{Ng^2}{4\pi}\,\delta^{ab}\,\frac{p^2}{m^2}\left[\ln\frac{p^2}{m^2} - 1 - \frac{1}{2}\frac{p^2}{m^2} + \mathcal{O}\left(\left(\frac{p^2}{m^2}\right)^2\right)\right]$$

for $p^2 \ll m^2$, $D = 2 + \epsilon$

- with the gluon propagator $\dfrac{1}{m^2}\left(\delta_{\mu\nu} - \dfrac{k_\mu k_\nu}{k^2}\right)$ instead



$$= -\frac{1}{2}\frac{Ng^2}{4\pi}\,\delta^{ab}\,\frac{p^2}{m^2}\left[\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln\frac{p^2}{\kappa^2}\right]$$

- same result for $p^2 \ll m^2$ after renormalizing the $(\bar{c}^a p^2 c^a)$-term in $\mathcal{L}^m_{FP}$

New scaling dimensions

- for the IR analysis, neglect the $(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu})$-term in the action $\mathcal{L}^m_{FP}$
- scaling analysis: invariance under $x \to x/s$, $s > 1$, of the non-interacting part of $\mathcal{L}^m_{FP}$ ($g = 0$) implies

$$A^a_\mu(x) \to s^{D/2} A^a_\mu(sx)$$

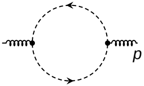
  the scaling (canonical) dimension of A^a_μ changes, equivalent to writing the action in terms of $\tilde{A}^a_\mu = m A^a_\mu$
- consequence: ghost-gluon coupling is relevant only for $D < 2$, three- and four-gluon couplings become irrelevant, also $(A^a_\mu p^2 A^a_\mu)$ becomes an irrelevant local operator
- keep only the relevant terms in the Lagrangian

$$\mathcal{L}^m_{FP} = \frac{1}{2} A^a_\mu m^2 A^a_\mu + iB^a\, \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a\, D^{ab}_\mu c^b$$

  do an epsilon expansion around the upper critical dimension $D = 2$, ghost dominance arises from the irrelevance of three- and four-gluon vertices

Implementing the ϵ-expansion

- calculate gluon and ghost self-energies to one-loop order in $D = 2 + \epsilon$ dimensions from $\mathcal{L}^m_{FP}$



$$= \frac{1}{2}\frac{N\bar{g}^2}{4\pi}\,\delta^{ab} m^2 \left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln\frac{p^2}{\kappa^2} - 2 \right) \delta_{\mu\nu} + 2\,\frac{p_\mu p_\nu}{p^2} \right]$$

with the dimensionless coupling constant $\bar{g}$

- introduce renormalized fields $A^a_\mu = Z_A^{1/2} A^a_{R,\mu}$, $c^a = Z_c^{1/2} c^a_R$, fix Z_A, Z_c through normalization conditions

$$\langle A^a_{R,\mu}(p) A^b_{R,\nu}(-p) \rangle \big|_{p^2=\mu^2} = \frac{1}{m^2}\,\delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$\langle c^a_R \bar{c}^b_R \rangle \big|_{p^2=\mu^2} = \frac{1}{\mu^2}\,\delta^{ab}$$

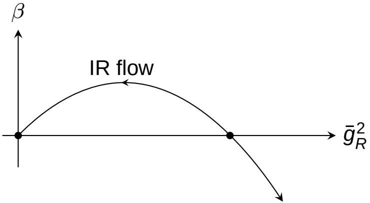
at the renormalization scale μ

The beta function

- use the renormalized proper ghost-gluon vertex at the symmetric point for the definition of the dimensionless renormalized coupling constant $\bar{g}_R(\mu)$
- crank the handle: beta function to first order in ϵ (and $\bar{g}_R^2$)

$$\beta(\epsilon, \bar{g}_R) = \mu^2 \frac{d}{d\mu^2} \bar{g}_R(\mu) = \frac{1}{2} \bar{g}_R \left(\frac{\epsilon}{2} - \frac{1}{2} \frac{N \bar{g}_R^2}{4\pi} \right)$$

- for $\epsilon > 0$



- two fixed points: $\bar{g}_R^2 = 0$ (IR-stable), $\dfrac{N \bar{g}_R^2}{4\pi} = \epsilon$ (IR-unstable)

Results

- unstable (nontrivial) fixed point gives exactly the DSE scaling solution 1

$$\langle A^a_{R,\mu}(p) A^b_{R,\nu}(-p) \rangle = \frac{1}{m^2} \left(\frac{p^2}{\mu^2} \right)^{\epsilon/2} \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$\langle c^a_R(p) \bar{c}^b_R(-p) \rangle = \frac{1}{p^2} \left(\frac{\mu^2}{p^2} \right)^{\epsilon/2} \delta^{ab}$$

- approach to the stable (trivial) fixed point: running coupling constant

$$\frac{N \bar{g}^2_R(\mu)}{4\pi} = \frac{(\mu^2/\Lambda^2)^{\epsilon/2}}{1 + (\mu^2/\Lambda^2)^{\epsilon/2}} \, \epsilon$$

  with a reference scale Λ
- IR-behavior of the propagators

$$\langle A^a_{R,\mu}(p) A^b_{R,\nu}(-p) \rangle = \frac{1}{m^2} \frac{1 + (p^2/\Lambda^2)^{\epsilon/2}}{1 + (\mu^2/\Lambda^2)^{\epsilon/2}} \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$\langle c^a_R(p) \bar{c}^b_R(-p) \rangle = \frac{1}{p^2} \frac{1 + (\mu^2/\Lambda^2)^{\epsilon/2}}{1 + (p^2/\Lambda^2)^{\epsilon/2}} \delta^{ab}$$

Discussion

- stable fixed point produces decoupling solutions for $D > 2$, in qualitative agreement with lattice calculations at $D = 3, 4$, with $\beta = 0$ for the gluon propagator at $D = 4$
- adding the irrelevant operator $(A^a_\mu p^2 A^a_\mu)$ to the action (no anomalous dimension at one-loop level) gives for the gluon propagator at $D = 4$

$$\langle A^a_{R,\mu}(p) A^b_{R,\nu}(-p) \rangle \propto \left(p^2 + m^2 \, \frac{1 + \mu^2/\Lambda^2}{1 + p^2/\Lambda^2} \right)^{-1}$$

  exactly the form of the gluon propagator in the refined Gribov-Zwanziger framework (without the $\langle A^a_\mu A^a_\mu \rangle$-condensate) with effective mass

$$M^2_{\text{eff}}(p^2) = \frac{m^2(\mu^2 + \Lambda^2)}{p^2 + \Lambda^2}$$

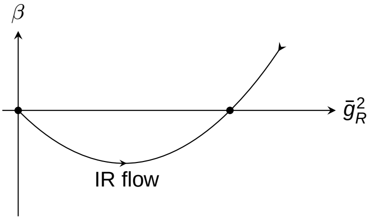
- at $D = 2$, the only fixed point $\bar{g}^2_R = 0$ becomes IR-unstable, coincident with lattice calculations which find a scaling solution and not the decoupling solution for $D = 2$

Horizon condition

- motivated by the (unrefined) Gribov-Zwanziger framework, one may implement the "horizon condition" $F(p^2) \to \infty$ for $p^2 \to 0$ as a normalization condition, replacing the $(\bar{c}^a p^2 c^a)$-term in $\mathcal{L}_{FP}^m$ with
$$\frac{1}{b^2}\,\partial_\mu \bar{c}^a(-\partial^2)\partial_\mu c^a$$
(isotropic) Lifshitz point for the ghost fields
- the scaling analysis about $g = 0$ now yields a relevant ghost-gluon coupling for $D < 6$, the three- and four-gluon couplings and the operator $(A_\mu^a p^2 A_\mu^a)$ remain irrelevant
- to implement the epsilon expansion around the upper critical dimension $D = 6$, calculate the (one-loop) gluon and ghost self-energies in $D = 6 - \epsilon$ dimensions with the bare ghost propagator b^2/p^4
- proceeding as before yields the beta function
$$\beta(\epsilon, \bar{g}_R) = \mu^2 \frac{d}{d\mu^2}\,\bar{g}_R(\mu) = -\frac{1}{2}\,\bar{g}_R \left(\frac{\epsilon}{2} - \frac{1}{2}\,\frac{N\bar{g}_R^2}{4\pi} \right)$$
for the dimensionless renormalized coupling constant $\bar{g}_R(\mu)$

IR-stable nontrivial fixed point

- for $\epsilon > 0$



- here the nontrivial fixed point $\dfrac{N\bar{g}_R^2}{4\pi} = \epsilon$ is IR-stable and leads to the propagators

$$\langle A^a_{R,\mu}(p) A^b_{R,\nu}(-p)\rangle = \frac{1}{m^2}\left(\frac{p^2}{\mu^2}\right)^{\epsilon/12} \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$\langle c^a_R(p)\bar{c}^b_R(-p)\rangle = \frac{b^2}{p^4}\left(\frac{p^2}{\mu^2}\right)^{5\epsilon/24} \delta^{ab}$$

Correspondence to Dyson-Schwinger solution

- in terms of the anomalous dimensions α_F and α_G

$$\alpha_F(D) = \frac{5D-6}{24}\,, \qquad \alpha_G(D) = -\frac{18-D}{12}$$

  to be compared to the scaling solution 2 of the Dyson-Schwinger equations

$$\alpha_F(D) \approx \frac{5D-5}{25}\,, \qquad \alpha_G(D) \approx -\frac{16-D}{10}$$

- the values are very close: exact coincidence at $D=6$ (also with the trivial fixed point), largest deviation in the range $2 \leq D \leq 4$ for $D=2$:

$$\left(\alpha_F = \frac{1}{6}\,, \quad \alpha_G = -\frac{8}{6}\right) \qquad \text{vs.} \qquad \left(\alpha_F = \frac{1}{5} \quad \alpha_G = -\frac{7}{5}\right)$$

- however, the fixed point is unstable with respect to perturbations of the local operator $(\bar{c}^a p^2 c^a)$, even more when one-loop corrections to the latter are taken into account
- since there is no reason (any more) to implement the “horizon condition” of the unrefined Gribov-Zwanziger framework, even the IR-stable Lifshitz fixed point has to considered as unstable

Final comments

- the point of departure is the existence of a gluon mass term which is a natural consequence of the BRST symmetry breaking, and implies ghost dominance
- the (Callan-Symanzik) renormalization group equations generate all the IR solutions of the Dyson-Schwinger equations in a completely analytic way (in an epsilon expansion)
- in addition, it is possible to discuss the IR-stability of the solutions
- as a result, only the decoupling solution is IR-stable and hence physical in dimensions $D > 2$
- the IR results of the refined Gribov-Zwanziger framework (without the $\langle A^a_\mu A^a_\mu \rangle$-condensate) and the lattice calculations are successfully reproduced for $D = 3, 4$ (and 2)
- the analytic calculations can be systematically improved by calculating to higher loop order in perturbation theory
- for a description of the complete momentum range, the crossover from the UV to the IR fixed point has to be described

Contents

Conclusions

- Yang-Mills theory in Landau gauge: restrict the integration over the gluon field to the (first) Gribov region to avoid gauge copies
- implementation via horizon function: local formulation with auxiliary fields, absence of the Landau pole, IR suppressed gluon propagator, IR enhanced ghost propagator
- “refined Gribov-Zwanziger scenario”: condensates of auxiliary (and gluon) fields lead to decoupling solutions in agreement with latest lattice results, calculation beyond tree-level technically demanding
- Dyson-Schwinger equations (and perturbative expansion) unchanged by restriction to the Gribov region, (soft) BRST symmetry breaking: introduce a gluon mass term
- renormalization group analysis of the IR fixed point with Callan-Symanzik equations in an epsilon expansion: scaling and decoupling solutions, decoupling solutions are IR stable in $D = 3, 4$ dimensions, unstable in $D = 2$ dimensions
- Outlook: inclusion of local composite operators, description of the crossover from UV to IR fixed point, condensates, quarks