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Dependence of the lateral distributions of electrons with a fixed energy on shower parameters

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Abstract: Basing on the EAS simulations with CORSIKA we investigate the lateral distributions of electrons with a fixed energy in large showers. We show how these distributions scale with electron energy, with air density and shower age. We fit some analytical functions to describe them in an easy way. This work is necessary when reconstructing the shower parameters from the light images obtained in EAS experiments basing on the fluorescence technique. The width of a shower track depends not only on the lateral distribution of electrons but also on the lateral distribution of the Cherenkov light accompanying the development of the shower. This light, when scattered in the atmosphere, adds to the fluorescence light and changes the images of showers seen by telescopes.

Introduction

This work is an investigation of the lateral distributions (LD) of electrons (and positrons) in extensive air showers with the information drawn from Monte Carlo simulations with CORSIKA.

We have shown in our earlier papers [1,2] that the angular distribution of electrons with a fixed energy depends only on this energy, i.e. does not depend on the level of shower development (age *s*). Thus, the angular distribution of all electrons at a given age is determined by the energy spectrum of electrons on this level. None of the above distributions depends on the nature of the primary particle or on its energy.

We have also shown [3] that the lateral distribution of all electrons, when expressing lateral distances in the Molière units r_M at the considered level, depends on the shower age s only.

Here we investigate the lateral distributions of electrons with fixed energies and we would like to establish what are the shower parameters that they depend on.

Method of calculations and (in)dependence on primary particle

We have simulated with CORSIKA 6.20. [4] about 90 vertical showers with primary energy $E_0 = 10^{19}$ and 10^{20} eV and primary particles protons and iron nuclei (more that 20 showers for each combination).

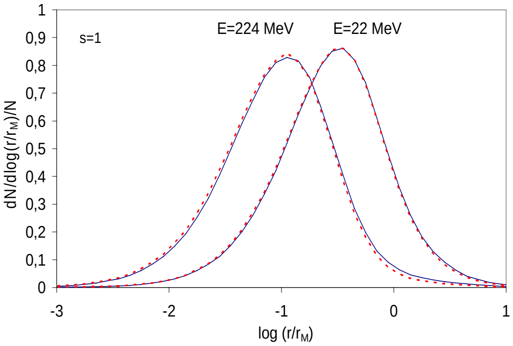

Figure 1: LD of electrons with two different energies, at shower maximum, solid lines – primary iron with $E_0=10^{19}$eV, dotted lines – primary proton with $E_0=10^{20}$eV (averages from 20 showers).

For each shower the depths corresponding to the ages $s = 0.8 - 1.3$ (with $\Delta(s) = 0.1$) were determined. On each of these levels the LD of electrons with energies with

$\log E(\mathrm{GeV}) = -1.7 - (+0.7)$ (from 20 MeV to 2.5 GeV) , in bins $\Delta(\log E) = 0.1$, was calculated.

In this paper we are not dealing with electrons with $E < 20$ MeV, as the ultimate purpose of our investigations is an analysis of Cherenkov light emitted by electrons is EAS (see later) and the threshold energy for this process is larger than that at see level (21 MeV).

First we investigate the dependence of the LD on primary particle. Fig. 1 shows electron LD at shower maximum for two electron energies (smaller and larger than the average). Each curve is an average from ~ 20 showers. To examine a possibly large region of the atmosphere densities where showers develop we show here LD only for primary protons with E_0 larger than that of iron nuclei. Here the distance is expressed in units of the Molière radius r_M (independent of electron energy E), to demonstrate the actual difference of the distributions for the two different values of E. It is evident that LD does not depend on the primary particle. We have also checked that this is true for other shower ages and electron energies .

However, for large s , e.g. s=1.2, and large electron energies (E = 1 GeV) LD is about 20% broader for Fe with 10^{19} eV than that for proton with 10^{20} eV. This is actually not very important as electrons with E>1 GeV at s=1.2 constitute a small fraction of all electrons there.

Lateral distributions of electrons with fixed energies

As the lateral spread of electrons is determined mainly by Coulomb scattering, with the scattering angle inversely proportional to particle energy, we have chosen as a suitable distance scale

$r_E = X_0 \cdot 21\,MeV / E$, where $X_0 = 37\ g/cm^2$ is the cascade unit of the air. Then, LDs do not differ very much from each other and, as we are aiming at fitting some analytical expressions to them , the task becomes easier.

a) $s = 1$

We calculate LD equal to $1/N \cdot dN/d\log(r/r_E)$ for 24 bins of electron energy. Fig.2 illustrates LD for four electron energies. We can see that our choice of r_E introduces a too strong dependence on E, as with increasing energy LD becomes broader. So, when choosing an analytical form of a function describing LD we have to take this into account.

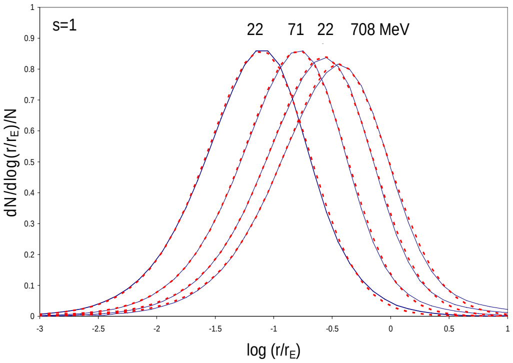

Figure 2: LD of electrons at shower maximum for four electron energies (marked at the curves), solid lines – simulations, dotted lines – best fit with NKG-type function.

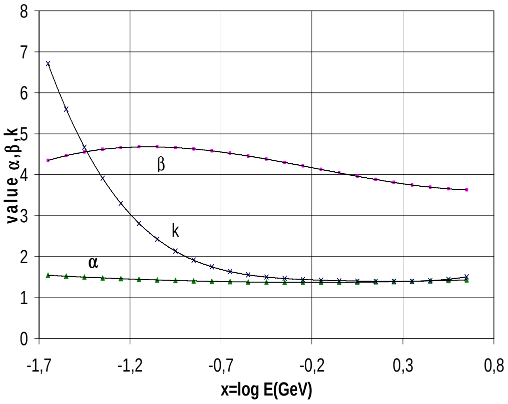

Figure 3: Best fit parameters (formula (1)) as functions of electron energy E.

We chose the NKG-type of the fitting function of the form

$$f_1(x) = 1/N \cdot dN/d\log(r/r_E) = C_1 \cdot (r/r_E)^{\alpha} \cdot (1 + k \cdot r/r_E)^{-\beta} \qquad (1)$$

where C_1 is the normalization constant and equals

$$C_1 = \ln(10) \cdot k^{\alpha} \frac{\Gamma(\beta)}{\Gamma(a)\Gamma(\beta-\alpha)} \tag{2}$$

For each of the 24 LD we have fitted the above functional form (1) and the best fit parameters are presented in Fig.3 as functions of electron energy E. It can be seen that for $E >> E_{cr}$ (~80 MeV for air) the parameters do not change much with E. In particular, the scaling factor k becomes constant, so that LD at these energies depends on E rather weekly (β does change a little) when expressed in r/r_E.

At energies $E \sim E_{cr}$ or smaller one should not expect r_E to be a good scaling distance , as an electron looses ~80 MeV for ionisation losses after traversing one cascade unit X_0 .Thus, the scaling distance (if any) should correspond to a higher energy than that calculated for energy E an electron has at the end of the path X_0. This is seen in the increasing of k with decreasing E.

We have found analytical functions (polynomials of $x=log(E)$) describing best the dependence of the parameters α, β and k on $log(E)$. They are

$$\begin{aligned} \alpha &= 0.083 \cdot x^2 + 0.051 \cdot x + 1.377 \\ \beta &= 0.334 \cdot x^3 + 0.276 \cdot x^2 - 0.783 \cdot x + 3.919 \\ k &= 0.651 \cdot x^4 + 0.256 \cdot x^3 + 0.15 \cdot x^2 - 0.071 \cdot x + 1.397 \end{aligned} \tag{3}$$

Now, we want to check how the lateral distribution functions (LDF) , with parameters calculated from the formulas (3), fit to the actual LD (from simulations). The comparison is illustrated in Fig.4 where the two distributions are shown for 4 electron energies. The agreement is only slightly worse (as it must be) than that of simulations with LD with parameters fitted separately to each energy.

So, we can conclude that LDF (1) and (2), with parameters depending on energy according to (3), describes quite well LD of electrons at a shower maximum.

b) $s \neq 1$

We have shown in [2] that the angular distribution of electrons with a fixed energy depends only on this energy, i.e. does not depend on age s. One would have thought that the same might be true for the lateral distribution of electrons with fixed energy. This would be the case for an homogeneous medium.

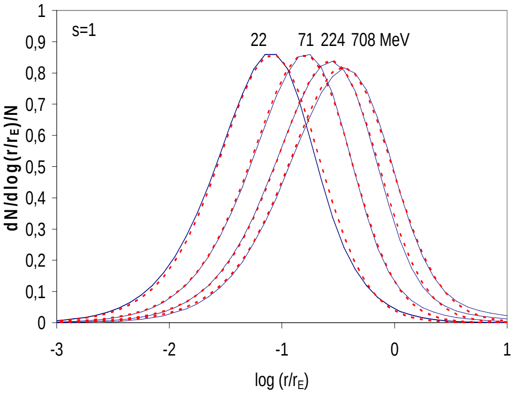

Figure 4: LD of electrons at shower maximum for four electron energies, solid lines – simulations, dotted lines LDF (2) with parameters calculated from analytical expressions (3).

However, for the inhomogeneous atmosphere, where higher parts are less dense than lower ones, this is not quite true. LD of parent particles (rather large due to small air density) does affect LD of daughter particles, because the latter, although with lower energies, do not spread much more due to a denser medium. The inhomogeneity of the atmosphere does not influence the angular distributions, of course.

Fig.5a,b show LD for our extreme values of age (s = 0.7 and 1.3) and for two electron energies E = 22 MeV and 280 MeV; at s = 0.7 (1.3) there are ~ 20% (5%) of particles above 280 MeV.

We can see that LD (as expressed in r/r_E) does depend on age: it is broader at larger s . However, the dependence is not very strong. Moreover, what is fortunate for someone who tries to fit analytical formulae, the LD(s) is almost the same as LD(s=1) if one rescales the distance axis, what is equivalent to shifting LD along the logarithmic scale of r/r_E. In Fig.5a,b there are also drawn LD(s=1) shifted so as to best fit the LD(s=0.7 and 1.3). One can see that the agreement is quite satisfactory. For ages closer to 1 it is, of course, better and it is there where it is more important.

Thus, the analytical form of LDF is obtained from the relation $f_s(x) = f_1(x - \Delta)$, where $x=log(r/r_E)$ and Δ depends on s. We get

$$f_s = 10^{-\alpha\cdot\Delta} \cdot C_1 \cdot (r/r_E) \cdot (1 + 10^{-\Delta} \cdot k \cdot r/r_E)^{-\beta} \qquad (4)$$

Δ *(s)* (shown in Fig.6) is almost a linear function

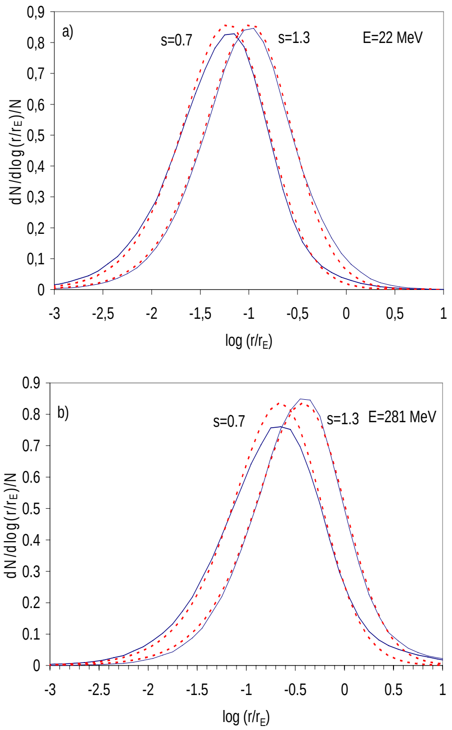

Figure 5: LD for our extreme values of age (s = 0.7 and 1.3) and two electron energies – solid lines. Dotted lines – LD(s=1) shifted to best fit solid lines.

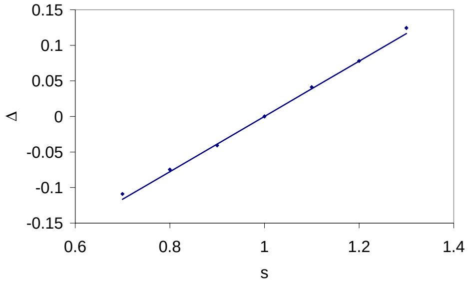

Figure 6: Dependence of the shift of the LD(s=1) to fit LD(s≠1)

and can be approximated by:

$\Delta(s)$= 0.3884·s - 0.3884

We can see that the rescaling is not dramatic – for any change of s by + 0.1, r/r_E has to be multiplied by $10^{0.0.3884}$ = 1.0935

Conclusions

We have shown that lateral distributions (LD) of electrons with a fixed energy at given level in a shower , when expressed as functions of r/r_E , depend on shower age at this level only. They can be quite well described as NKG- type functions of three parameters depending on electron energy. The dependence on age can be allowed for by rescaling the lateral distance (different k in (4)).

This dependence is caused by the inhomogeneity of the atmosphere, in particular by the density gradient directed downwards. For an homogeneous medium LD should not depend on s, as do not the angular distributions of electrons [2].The knowledge of LD for fixed electron energies is useful for predicting widths of optical images registered in the EAS fluorescence experiments. The image width (particularly below shower maximum) is determined not only by fluorescence light but also by scattered Cherenkov light [5]. To calculate the latter for showers developing at different heights one has to know LD for $E > E_{th}(h)$, which can be obtained by integrating calculated here LD(E) above Cherenkov threshold $E_{th}(h)$.

Acknowledgments

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