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Secondary electron spectrum in the upper atmosphere

Y. KOMORI[1], T. KOBAYASHI[2], K. YOSHIDA[3] AND J. NISHIMURA[4]
[1]*Kanagawa University of Human Services, Yokosuka, Japan*
[2]*Department of Physics and Mathematics, Aoyama gakuin University, Sagamihara, Japan*
[3]*Department of Electronic Information Systems, Shibaura Institute of Technology, Saitama, Japan*
[4]*Institute of Space and Astronautical Science, JAXA, Sagamihara, Japan*
komori-y@kuhs.ac.jp

Abstract: The secondary electron spectrum in the upper atmosphere($< 10\ \mathrm{g/cm^2}$) is investigated on the basis of the atmospheric gamma ray spectrum above 30 GeV obtained from our emulsion chamber experiments at balloon altitudes. We have to subtract these electrons produced by nuclear interactions from observed electrons to get the primary electron spectrum in the Galaxy. Thus it is required to precise estimates of secondary electron abundance. Both electron and gamma ray intensities are simultaneously solved using the one-dimensional cascade shower theory plus gamma ray production rate derived from observed data. Secondary electrons increases qualitatively as the depth squared. The secondary spectrum has the same index as atmospheric gamma ray index -2.73, namely proton index, while the primary electron index is rather steeper than that. Therefore the ratio of secondary to primary electrons rapidly increases in the TeV region, and the high altitude is essential for TeV electron experiments.

Introduction

In balloon-borne electron experiments, primary electrons coming from interstellar space and secondary electrons produced in the residual atmosphere are observed together. Thus we have to subtract the secondary electrons from the observed electrons for estimating the primary electron spectrum. Secondary electrons above 30 GeV mostly originate in a pair production process of atmospheric gamma rays produced in the hadronic interactions in the atmosphere.

The atmospheric gamma-ray spectrum J_γ has been simultaneously observed in our primary electron experiments as follows[1].

$$J_\gamma(E)|_{4g/cm^2}\ (\mathrm{m^2 \cdot s \cdot sr \cdot GeV})^{-1} = (1.12 \pm 0.13) \times 10^{-4} (\frac{E}{100\ \mathrm{GeV}})^{-2.73 \pm 0.06} \quad (1)$$

This spectrum is normalized at 4.0 g/cm^2 and only includes gamma rays in nuclear interactions since the gamma rays from primary electrons have already been subtracted. Using this observed spectrum, we estimate both intensity of electrons and gamma rays in the upper atmosphere.

Calculations

The one-dimensional cascade shower theory called Approximation A [2], is applicable at the electron energy above 30 GeV, which has a complete screening cross section of radiation and pair creation process and neglects ionization loss. The cross section in this paper agrees with that used in Geant4 simulation code[3] within a few percent. We adopt the radiation length in air, 36.7 $\mathrm{g/cm^2}$.

The number of electrons $\pi(E,t)dE$ and gamma-rays $\gamma(E,t)dE$ satisfy the following simultaneous equations. The notations are seen in the Handbuch der Physik[2].

$$\begin{aligned} \frac{\partial \pi(E,t)}{\partial t} &= -A'\pi(E,t) + B'\gamma(E,t) \\ \frac{\partial \gamma(E,t)}{\partial t} &= C'\pi(E,t) - \sigma_0\gamma(E,t) \\ &\quad +\gamma_{ex}(E,t)\ , \end{aligned}$$

in which we add the gamma ray spectrum $\gamma_{ex}(E,t)dE \propto E^{-\beta-1}dE$ produced by nuclear interactions. $\gamma_{ex}(E,t)$ is estimated from eq. (1) and is assumed to be a constant with the depth in the upper atmosphere($< 10\ \mathrm{g/cm^2}$). The expression with an exponential decrease, $\exp(-t/L)$, is more adequate in the deeper atmosphere, however a constant assumption in this case gives almost same result and the correction is around 5% at $10\mathrm{g/cm^2}$.

The intial values, namely primary electron spectrum[4] and the Galactic diffuse gamma ray spectrum are assumed as

$$\begin{aligned}
\pi(E,0) &= 1.6\times 10^{-4}(\frac{E}{100\ \mathrm{GeV}})^{-3.3} \equiv \pi_0 \\
\gamma(E,0) &= 0\ ,
\end{aligned}$$

where the spectral index is given by $\alpha+1 = 3.3$.

The solution of electron intensity at the depth t becomes

$$\pi(E,t) = \pi(E,0)\zeta(t) + \gamma_{ex}(E)\xi(t) \qquad (2)$$

$$\begin{aligned}
\zeta(t) &= \frac{[(\lambda_1+\sigma_0)e^{\lambda_1 t} - (\lambda_2+\sigma_0)e^{\lambda_2 t}]}{\lambda_1-\lambda_2} \\
\xi(t) &= \frac{B(\beta)}{\lambda_1-\lambda_2}[\frac{e^{\lambda_1 t}-1}{\lambda_1} - \frac{e^{\lambda_2 t}-1}{\lambda_2}]
\end{aligned}$$

where $\zeta(t)$ represents the energy loss rate of electrons by bremsstrahrung in the atmosphere and $\xi(t)$ represents the pair production rate from gamma rays.

The atmospheric gamma-ray spectrum at depth t is given by

$$\gamma(E,t) = \pi(E,0)\eta_1(t) + \gamma_{ex}(E)\eta_2(t) \qquad (3)$$

$$\begin{aligned}
\eta_1(t) &= \frac{C(\alpha)}{\lambda_1-\lambda_2}[e^{\lambda_1 t} - e^{\lambda_2 t}] \\
\eta_2(t) &= \frac{1}{\lambda_1-\lambda_2}[\frac{(\lambda_1+A(\beta))e^{\lambda_1 t} - A(\beta)}{\lambda_1} \\
&\quad - \frac{(\lambda_2+A(\beta))e^{\lambda_2 t} - A(\beta)}{\lambda_2}] \qquad (4)
\end{aligned}$$

where η_1 represents the gamma ray production rate from primary electrons and η_2 represents the attenuation rate of gamma rays by pair creation. The coefficients λ_1, λ_2 are given by

$$\begin{aligned}
\lambda_1(s) &= \frac{1}{2}[-(A+\sigma_0) \\
&\quad +\{(A+\sigma_0)^2 - 4(A\sigma_0 - BC)\}^{\frac{1}{2}}] \\
\lambda_2(s) &= \frac{1}{2}[-(A+\sigma_0) \\
&\quad -\{(A+\sigma_0)^2 - 4(A\sigma_0 - BC)\}^{\frac{1}{2}}] \\
A(s) &= 1.360\frac{d}{ds}\ln\Gamma(s+2) \\
&\quad -\frac{1}{(s+1)(s+2)} - 0.075 \\
B(s) &= 2(\frac{1}{s+1} - \frac{1.360}{(s+2)(s+3)}) \\
C(s) &= \frac{1}{s+2} + \frac{1.360}{s(s+1)} \\
\sigma_0 &= 0.773
\end{aligned}$$

where s has a value of spectral index $\alpha = 2.3$ or $\beta = 1.73$.

The zenith angle distribution

The intensities of eq.(2) and eq.(3) are integrated over the zenith angle θ. At the depth t , the electron differential spectrum $j_{ob}(E,t)$ and the atmospheric gamma-ray spectrum $\gamma_{ob}(E,t)$ have a unit of $(\mathrm{m^2\cdot sec\cdot GeV})^{-1}$ and are given by

$$\begin{aligned}
\frac{j_{ob}(E,t)}{2\pi} &= \int_0^\theta \pi(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta \\
&= \pi_0\cdot\zeta(\alpha,\theta,t) + \gamma_{ex}\cdot\xi(\beta,\theta,t)
\end{aligned} \qquad (5)$$

$$\begin{aligned}
\frac{\gamma_{ob}(E,t)}{2\pi} &= \int_0^\theta \gamma(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta \\
&= \pi_0\cdot\eta_1(\alpha,\theta,t) + \gamma_{ex}\cdot\eta_2(\beta,\theta,t)
\end{aligned} \qquad (6)$$

The coefficients are calculated in the series as

$$\begin{aligned}
\zeta(s,\theta,t) &= O_1(\alpha,\theta,t) + \sigma_0 O_2(\alpha,\theta,t) \\
\xi(s,\theta,t) &= B(\beta)O_3(\beta,\theta,t) \\
\eta_1(s,\theta,t) &= C(\alpha)O_2(\alpha,\theta,t) \\
\eta_2(s,\theta,t) &= O_2(\beta,\theta,t) + A(\beta)O_3(\beta,\theta,t)
\end{aligned} \qquad (7)$$

where function series are represented by

$$\begin{aligned}
O_1(s,\theta,t) &= \sum_{n=0}^{\infty}\Lambda_n(s)T_n(\theta,t) \\
O_2(s,\theta,t) &= \sum_{n=0}^{\infty}\Lambda_n(s)T_{n+1}(\theta,t)
\end{aligned}$$

$$O_3(s,\theta,t) = \sum_{n=0}^{\infty} \Lambda_n(s) T_{n+2}(\theta,t)$$

$$T_0 = \frac{1-\cos^2\theta}{2}, \quad T_1 = t(1-\cos\theta),$$

$$T_2 = \frac{t^2}{2!}\log(\frac{1}{\cos\theta}), \quad T_3 = \frac{t^3}{3!}(\frac{1}{\cos\theta}-1),$$

$$\cdots \quad T_n = \frac{t^n}{n!}(\frac{1}{\cos^{n-2}\theta}-1) \quad \cdots$$

$$\Lambda_0 = 1, \quad \Lambda_1 = \lambda_1 + \lambda_2,$$

$$\Lambda_2 = (\lambda_1+\lambda_2)\Lambda_2 - \lambda_1\lambda_2\Lambda_1, \cdots$$

$$\Lambda_n = (\lambda_1+\lambda_2)\Lambda_{n-1} - \lambda_1\lambda_2\Lambda_{n-2}, \quad \cdots$$

In the upper atmosphere($< 10\ \mathrm{g/cm^2}$), Each first term gives the following qualitative expression

$$\begin{aligned} \zeta &\sim \frac{1-\cos^2\theta}{2} = const \\ \xi &\sim B(\beta)\cdot\frac{t^2}{2}\log\frac{1}{\cos\theta} \propto t^2 \\ \eta_1 &\sim C(\alpha)\cdot t\,(1-\cos\theta) \propto t \\ \eta_2 &\sim t\,(1-\cos\theta) \equiv \check{\eta}_2 \propto t\,. \end{aligned}$$

The calculation using these terms are expressed by a dotted line in Figure 1, and we need the exact calculations using eq. (7).

Gamma ray production rate

The gamma ray production rate $\gamma_{ex}(E,t)$ is calculated from eq. (3), $\gamma(E,t)/\eta_2(t)|_{t=4\mathrm{g/cm^2}}$, and the values of eq. (1) and eq. (4) are substituted. We have obtained the result of

$$\begin{aligned} \gamma_{ex}(E) &= 1.07\times10^{-3}(\frac{E}{100})^{-2.73} \\ &\quad (\mathrm{m^2\cdot sr\cdot sec\cdot GeV\cdot c.u.})^{-1} \end{aligned}$$

Actually, the vertical spectrum should be estimated from the spectrum including zenith angle distribution as

$$2\pi\eta_2(1.7,\theta,4\mathrm{g/cm^2})\cdot\gamma_{ex}(E) = \Omega\cdot J_\gamma(E)|_{4\mathrm{g/cm^2}}$$

where $\Omega = 2\pi(1-\cos\theta)$. The value estimated from zenith angle within 60°, however, only gives the correction within 2%.

Results

The secondary electron spectrum at the depth t is given by

$$j_{sec}(E,t) = 2\pi\xi\cdot\gamma_{ex}(E)\ (\mathrm{m^2\cdot s\cdot GeV})^{-1}. \quad (8)$$

The vertical secondary spectrum($= j_{sec}/\Omega$) is shown in Figure 1.

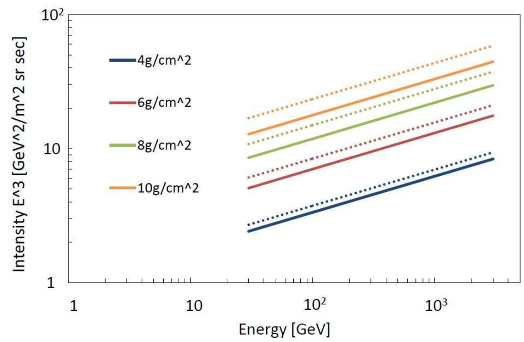

Figure 1: Secondary spectrum at different depths 4, 6, 8, 10 $\mathrm{g/cm^2}$. Dotted lines show the approximate expression of $j_\perp \sim B(1.73)\cdot t^2/2\cdot\gamma_{ex}(E)$ $= 2.33\times10^{-7}(t\ \mathrm{g/cm^2})^2 \times (E/100\ \mathrm{GeV})^{-2.73}$.

We compare the number of secondary electrons with that of gamma rays. Atmospheric gamma ray spectrum is also given by

$$\gamma_{ob}(E,t) = 2\pi\eta_2\cdot\gamma_{ex}(E)\ (\mathrm{m^2\cdot s\cdot GeV})^{-1},$$

neglecting the contribution from primary electrons. Therefore the ratio of secondary electrons to atmospheric gamma rays is given by ξ/η_2, and

$$\xi/\eta_2 \sim 0.01\cdot t[\mathrm{g/cm^2}]$$

when zenith angle is 60°. Thus the secondary electron number is roughly t% of gamma rays at the same depth t.

Next, we study the secondary/primary electron ratio, $\xi(2.3,\theta,t)/\zeta(1.73,\theta,t)$, which varies with $\sim t^2$. Figure 2 shows that for electron measurements in a few TeV region with the large zenith angle of 60°, we require a higher altitude than $8\ \mathrm{g/cm^2}$.

Finally, we estimate the most important quantity, that is the secondary electron contribution to the observed electrons. The electron spectrum at the

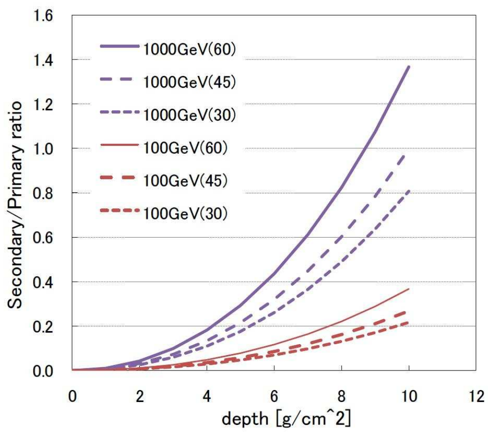

Figure 2: Secondary/Primary electron ratio: The curves are shown in the case of electron energy of 100 GeV and 1 TeV with the zenith angle of 30°, 45°, 60° respectively.

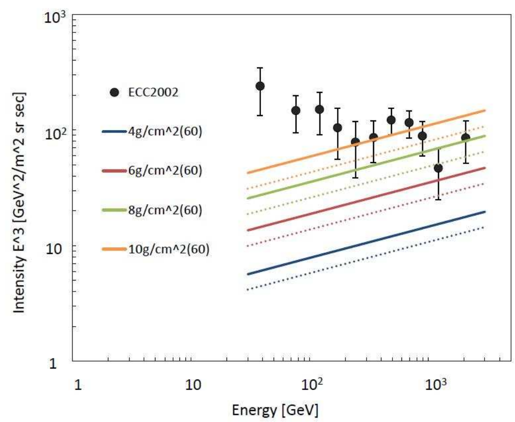

Figure 3: The contribution of secondary electron spectrum at the top of the atmosphere. The straight line($\theta = 60°$) and dotted line($\theta = 45°$) show the spectrum at the depth of 4, 6, 8, 10g/cm^2.

top of the atmosphere is determined from eq. (5) as

$$\pi(E, 0) = \frac{j_{ob}(E, t)}{2\pi\zeta} - \gamma_{ex}(E)\,\frac{\xi}{\zeta}\,. \qquad (9)$$

The second term in the right side represents the contribution of secondary electrons,

$$\hat{j}_{\mathrm{sec}}(E, t) \equiv \gamma_{ex}(E)\frac{\xi(\beta, \theta, t)}{\zeta(\alpha, \theta, t)} \qquad (10)$$
$$(\mathrm{m}^2 \cdot \mathrm{sr} \cdot \mathrm{sec} \cdot \mathrm{GeV})^{-1}$$

and is shown in Figure 3 with our observed result of primary electrons[5].

Discussions

In the balloon experiments of electrons, the floating altitude varies with time. We have to take into account the distribution of altitude for estimating the value of ξ/ζ. It becomes larger than that from the average altitude in proportion to the dispersion of altitude. The total secondary electrons ($E_1 < E < E_2$) of all flights with the parameters ξ_i/ζ_i and $(S\Omega T)_i$ at the ith flight becomes

$$N_{sec}(E_1, E_2) = \int_{E_1}^{E_2} dE\,\gamma_{ex}(E)\sum_i \frac{\xi_i}{\zeta_i}(S\Omega T)_i\,.$$

The correction of energy reduction by bremsstrahrung is given by the parameter ζ, as shown in eq. (9). We do not know which observed electrons are primary, so that all of them are firstly corrected by bremsstrahrung loss energy and next subtracted secondary electrons statistically. The balloon experiments of primary electrons in a few TeV region have to be performed at high altitude ($< 8\ \mathrm{g/cm^2}$), as shown in Figure 2 and Figure 3. In the next, we will perform the comparison of these analytic results with Monte Carlo Simulations.

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