Proceedings of the 30th International Cosmic Ray Conference
Rogelio Caballero, Juan Carlos D'Olivo, Gustavo Medina-Tanco,
Lukas Nellen, Federico A. Sánchez, José F. Valdés-Galicia (eds.)
Universidad Nacional Autónoma de México,
Mexico City, Mexico, 2008
Vol. 1 (SH), pages 113–116

30TH INTERNATIONAL COSMIC RAY CONFERENCE

Acceleration of solar cosmic rays in stochastic non-Gaussian electric fields

IVAN ZIMOVETS[1,2]
[1]*Institute of Space Research, Profsoyuznaya Street, 84/32, Moscow, Russia, 117997*
[2]*Moscow Institute of Physics and Technology (State University), Institutsky pereulok, 9, Dolgoprudny, Russia, 141700*
ivanzim@mail.ru

Abstract: Acceleration of charged particles in stochastic electric fields is considered. The fractional symmetric Fokker-Planck equation is derived by method is introduced in [1] based on the Langevin equations and with an assumption that the fluctuations of electric fields are subordinated by the Levy stable laws. Asymptotic solution of non-friction spatial homogeneous equation of an ensemble of charge particles in the case of a non-Brownian motion has a power law dependence rather than Maxwellian-type one. The concerned stochastic process is discussed in the frame of acceleration of solar cosmic rays in flares or in term of post-eruptive acceleration. Possible application of Levy statistic for turbulent electromagnetic field in active solar atmosphere is discussed briefly.

Introduction

Charged particles can be readily accelerated to relativistic energy during some special processes of solar magnetic fields restructure. There're several possible acceleration mechanisms against different types of solar and particle events which in turn depend on modes of magnetic energy release. One class of charged particles acceleration processes is an energy gain via stochastic propagation through a region of multiple magnetic discontinuities induced by motion of magnetic field lines in solar atmosphere. Such particle transport essentially results in second-order stochastic Fermi-type acceleration. In several papers [2,3,4,5,6] particle acceleration was really shown by computer simulation of their trajectory in man-made turbulent electromagnetic fields and in near real coronal fields stressed by photospheric motion [7]. Similar result was obtained in model with fractal distribution of bulb-like current-sheets [8].

It was shown [6] that existing large current sheet can be destroyed into smaller filaments under magnetic fluctuations during several tens of seconds. Filament structure has fractal dimension depending on dissipation time. Also it was shown that resistive part of electric field which is important for particle acceleration has power-law magnitude distribution function. But in simulation electrons are accelerated to MeV energies and simultaneously collect a half of stored magnetic energy during small fraction of second and can not fill process of current dissipation in real time.

Formation of highly fragmented electric fields also noted in [7].

To provide conditions for Petscheck-type reconnections in solar flares it is necessary to have anomalous resistivity that in turn demands current sheet thickness on the order of ion Larmor radius. Such small thickness can be achieved in the case of fractal current sheet [9].

Spatial intermittencies, especially small scale, are very crucial for charged particles acceleration in current sheets. It was shown [5] that longitudinal electric field fluctuations in coronal loops are inversely as fluctuations length. If distribution of character lengths of clusters from which fractal current sheet consists submitted to a power law and if charged particle rushes through all clusters of reasonable size for time small compared to time of considerable energy change we can expect that we will deal with a Levy statistic of particles energy gain in contrast to Gaussian one. If it is so we must use mathematical instruments of fractional kinetic for analytic description of charged particles motion in stochastic electromagnetic fields, though we can't do without numerical simulation for detailed study of charged particles motion in turbulent electromagnetic fields with specified geometry in solar atmosphere.

Fractional Fokker-Planck kinetic equation

We use method [1] for derivation of fractional Fokker-Plank equation for probability density function (PDF) $f(x,v,t)$ of charged particles based on Langevin equations [10] in 1D case

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = F_0(t) - v\eta \qquad (1)$$

which contain for simplicity only fluctuation force component and friction term with friction coefficient η. For the theory of Brownian motion to be valid [10] significant changes of particle energy must occur on time scale τ much longer than time scale of force fluctuations $\tau >> t$. It is true for current sheets during fractal reconnection. Let assume generated electric field is

$$E \approx \frac{v_A B}{c} = \frac{B^2}{c(4\pi n m_i)^{0.5}} \approx 10^4 B^2 n^{-0.5} [Volt/m],$$

where v_A - Alfven speed, m_i - proton mass, n - plasma density, and for coronal loop $n \approx 10^{-7}[kg/m^3]$, $B \sim 10^{-2}[Tesla]$ - coronal magnetic field. Scale of energy gain in one element of current sheet is $\Delta\varepsilon = eEr_L$, where $r_L \approx \frac{m_i v_{thermal}}{eB} \sim 10^{-6}\frac{T^{0.5}}{B}[m]$ - ion Larmor radius, where $TK \sim 10[^7]$ - plasma temperature. Thereby $\Delta\varepsilon \sim 10^{-3}[eV]$. But thermal energy of proton is about 100 eV in coronal temperature. We can see that thermal proton has to undergo a lot of interaction with current sheet element to gain considerable energy. Such multiple interactions can be treated as force fluctuations. Of course, this remains valid for relativistic particles because they collide with filaments of current sheet more frequently.

For $\tau >> t$ we replace equations (1) by

$$\Delta x = v\tau, \quad \Delta v = F(\tau) - v\eta\tau \qquad (2)$$

where $F(\tau) = \int_t^{t+\tau} F_0(\xi)d\xi$ - some stochastic process. In the case of power law type force field fluctuations effecting on moving particle we can use Levy stable processes $L(t)$ instead of Brownian ones for which PDF is

$$W(F(\tau)) = \frac{1}{\sqrt{4\pi D_0\tau}} \exp\left(-\frac{(F(\tau))^2}{4D_0\tau}\right).$$

Using characteristic function for symmetric stable law

$$\hat{W}_L(k,\tau) = \exp(-D_0|k|^\alpha \tau)$$

with characteristic Levy index ($0 < \alpha < 2$), equations (2), appropriate integral equation for PDF of the Markovian stochastic process in phase space and assumed that $\tau \to 0$ we can finally obtain fractional Fokker-Planck equation (FFPE) [1]

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \eta\frac{\partial}{\partial v}(vf) + D\frac{\partial^\alpha}{\partial|v|^\alpha} f, \qquad (3)$$

where $D = D_0\left(\frac{e}{m}\right)^\alpha$ if particle is under electric field force,

$$\frac{d^\alpha}{d|v|^\alpha}Q(v) = -\frac{1}{2\cos(0.5\pi\alpha)}\left(D_+^\alpha Q(v) + D_-^\alpha Q(v)\right).$$

For $\alpha \geq 1$ Liouville fractional derivatives

$$D_\pm^\alpha Q(v) = \frac{(\pm 1)^n}{\Gamma(n-\alpha)}\frac{d^n}{dv^n}\int_0^\infty \xi^{n-1-\alpha} Q(v \mp \xi)d\xi,$$

$n = aliquot(\alpha) + 1$.

For $0 < \alpha < 1$

$$D_+^{\alpha}Q(v) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dv}\int_{-\infty}^{v}\frac{Q(\xi)d\xi}{(x-\xi)^{\alpha}}$$

$$D_-^{\alpha}Q(v) = \frac{-1}{\Gamma(1-\alpha)}\frac{d}{dv}\int_{v}^{\infty}\frac{Q(\xi)d\xi}{(\xi-x)^{\alpha}}.$$

Asymptotic solution of spatial homogeneous FFPE with zero friction term

In the case of spatial homogeneity and with an assumption of absence of particle friction we can rewrite (3) as

$$\frac{\partial f}{\partial t} = D\frac{\partial^{\alpha}}{\partial |v|^{\alpha}} f . \quad (4)$$

Equation (4) in Fourier domain has the form

$$\frac{\partial \hat{f}}{\partial t} = -D|k|^{\alpha}\hat{f} , \quad (5)$$

which has an analytical solution in terms of Fox's H-functions [11]. Asymptotic solution for initial conditions in the form $f(v,0)=\delta(v), f(k,0)=1$ has a form

$$f(v,t) \sim \frac{Dt}{|v|^{1+\alpha}} \quad (6)$$

for $|v|^{\alpha}/(Dt) >> 1$ or for kinetic energy K in non-relativistic case

$$f(K,t) \sim \frac{Dtm^{\frac{1+\alpha}{2}}}{K^{\frac{1+\alpha}{2}}} . \quad (7)$$

We can see that $\langle v^2(t)\rangle \to \infty$ but in real situation there are lower and upper limits of particle speed modulus, which are zero and light speed corresponding. Power law type asymptotic of particle PDF leads to Levy flights and superdiffusion in velocity space.

It should be noted that we considered only simplest case of particle motion in fluctuating field in the frame of FFPE. Author just steals up on this interesting and powerful mathematical apparatus. For detailed analytical investigation of charged particles transport in electromagnetic fields with fluctuations by solving initial-boundary problem of appropriate FFPE we have to know fields geometry, spatial boundary conditions, seed particle distributions, fluctuations properties. Under real conditions of solar flaring active regions, we have to solve non-stationary spatial-impulsive coupled FFPE.

Post-eruptive acceleration in dissipating current sheets

During and after eruption of coronal mass ejection magnetic fields of solar atmosphere are very disturbed and can make appropriate conditions for charged particles acceleration. There are some evidences that such acceleration occurs [12] and produced solar cosmic rays contributes to interplanetary flux [13]. Charge particle can be accelerated to relativistic energy in reconnection current sheets [14]. But they dissipate quickly and form very discontinuous medium which can acts as fluctuating force on moving charged particle. Such force can effective accelerate it.

For example during solar energetic event of 20 January, 2005 there were two populations of cosmic rays injected into interplanetary medium. The former had an exponential energy spectrum and the latter had power law one [15]. We can indentify the latter populations with acceleration by bow shock of coronal mass ejection but also with post-eruptive acceleration in turbulent coronal fields. Such acceleration mechanism also can give power law spectrum.

Conclusion

We fluently touched on non-Gaussian statistic concerning charged particle acceleration in turbulent solar electromagnetic fields and considered one simple case of fractional Fokker-Planck equations in impulsive space and their stationary solution. We did not consider more or less real field turbulence in this work. We just attempted to realize that classical Gaussian statistic and therefore classical Fokker-Planck equation can be found inappropriate to solar corona turbulence. One possible mechanism of particles energization whose description might need fractional kinetic apparatus is a post-eruptive particle acceleration in disturbed coronal magnetic fields.

Acknowledgments

This work is partially supported by the Russian Foundation for basic Research (grants 06-02-17256-a, 07-02-00319-a, SS 5359.2006.2) and by the theme of the Presidium of Russian Academy of Science "Young scientists support". I also thank the LOC for local hospitality.

References

[1] A. V. Chechkin and V. Yu. Gonchar. Linear relaxation processes governed by fractional symmetric kinetic equations. JETP, 91, 635, 2000.
[2] P. Dmitruk, W. H. Matthaeus, N. Seenu and M. R. Brown. Test particle acceleration in three-dimensional magnetohydrodynamic turbulence. ApJ, 597, L81-L84, November 2003.
[3] K. Arzner and L. Vlahos. Particle acceleration in multiple dissipation regions. ApJ, 605, L69-L72, April 2004.
[4] K. Arzner, B. Knaepen, D. Carati, N. Denewet, L. Vlahos. The effect of coherent structures on stochastic acceleration in MHD turbulence. ApJ, 637, 322-332, January 2006.
[5] N. Decamp and F. Malara. Particle acceleration and intermittent turbulence in coronal loops. ApJ, 637, L61-L64, January 2006.
[6] M. Onofri, H. Isliker and L. Vlahos. Stochastic acceleration in turbulent electric fields generated by 3D reconnection. Phys. Rev. Lett., 96, 151102, April 2006.
[7] R. Turkmani, L. Vlahos, K. Galsgaard, P. J. Cargill and H. Isliker. Particle acceleration in stressed coronal magnetic fields. ApJ, 620, L59-L62, February 2005.
[8] L. Vlahos, H. Isliker and F. Lepreti. Particle acceleration in an evolving network of unstable current sheets. ApJ, 608, 540-553, June 2004.
[9] K. Shibata and S. Tanuma. Plasmoid-induced-reconnection and fractal reconnection. Earth Planets Space, 53, 473-482, 2001.
[10] S. Chandrasekhar. Stochastic problems in physics and astronomy. Reviews of Modern Physics, V. 15, No 1, 1-89, 1943.
[11] B. J. West, P. Grigolini, R. Metzler, T. F. Nonnenmacher. Fractional diffusion and Levy stable processes. Phys. Rev. E, 55, 1, 99-106, 1997.
[12] K.-L. Klein, S. Krucker, G. Trottet, S. Hoang. Coronal phenomena at the release of solar energetic electron events. A&S, 431, 1047-1060, 2005.
[13] G. A. Bazilevskaya, A. I. Sladkova, V. V. Fomichev and I. M. Chertok. Possible relation between solar proton fluxes in interplanetary space and the formation of a post-flare system of loops.
[14] Yu. E. Litvinenko and B. V. Somov. Particle acceleration in reconnecting current sheets. SoPh, 146, 127L, 1993.
[15] E. V. Vashenyuk, Yu. V. Balabin, B. B. Gvozdevsky, S. N. Karpov, V. G. Yanke, E. A. Eroshenko, A. V. Belov and R. T. Gushchina. Relativistic solar cosmic rays in January 20, 2005 event on the ground based observations. ICRC29, 1, 209-212, 2005.