Particle acceleration in relativistic subluminal shock environments

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Abstract: The understanding of the particle spectra resulting from acceleration in relativistic shocks as they occur in extragalactic sources, is essential for the interpretation of the cosmic ray spectrum above the ankle ($E_p > 3 \times 10^{18}$ eV). It is believed that extragalactic sources like Active Galactic Nuclei and Gamma Ray Bursts can produce particle spectra up to $E_p \sim 10^{21}$ eV. In this contribution, subluminal shocks are investigated with respect to different shock boost factors $\Gamma$ and the inclination angle between the shock normal and the magnetic field $\psi$. A correlation between the boost factor and the spectral behavior of the emitted particles is found. The results are compared to Active Galactic Nuclei and Gamma Ray Burst diffuse cosmic ray contribution and the observed cosmic ray spectrum at the highest energies.

Introduction

The observations of electron synchrotron radiation in the radio regime and the X-ray variations from astrophysical environments such as Super Novae (SN), Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRBs), indicate that there is a mechanism with which primary particles (protons or electrons) gain large amounts of energy via acceleration. Connected to this implied mechanism, it is accepted that shocks are the source of the acceleration of the observed cosmic rays. The work of the late 70s by a number of authors such as \cite{1, 2, 3, 4, 5} established the basic mechanism of particle diffusive acceleration in non-relativistic shocks. Later on studies on relativistic shocks appeared (e.g. \cite{6, 7, 8, 9}) and questioned the nature and the limits of the relativistic diffusive acceleration versus the non-relativistic shock mechanism. In this work, we present results on relativistic particle acceleration which are extended to relativistic source models (i.e. AGN, GRBs). These involve subluminal shocks with plasma shock velocity ranging from Lorentz boost factors, $\Gamma = 1 \rightarrow 1000$, using pitch angle and magnetic scattering. In the first section, the Monte Carlo simulations are described, while in the second section the results are presented for different boost factors. In the third section, the Monte Carlo spectra are used to calculate the possible contribution of AGN and GRBs to the observed spectrum of charged cosmic rays. The final section summarizes the results.

Numerical simulations

The Monte Carlo codes simulate relativistic oblique shocks with application to relativistic environments of AGN and GRBs. Initially the particles are considered relativistic, isotropically injected upstream of the shock and they are let to move towards the shock and along the way they collide with the scattering centers. Consequently, as they keep scattering between the upstream and downstream regions of the shock (its width is much smaller than the particle’s gyroradius), they gain a considerable amount of energy. During our simulations, Lorentz relativistic transformations are performed from the local plasma frames to the normal shock frame to check for shock crossings. Particles leave the system if they escape far downstream at a spatial boundary $r_b$, or if they reach a defined maximum energy $E_{\text{max}}$, even though other physics describing particle escape or energy loss would probably need to be taken into account in re-
alistic situations (forthcoming paper). The downstream spatial boundary required can be estimated from the solution of the convection-diffusion equation in a non-relativistic, large-angle scattering approximation in the downstream plasma.

The mean free path is calculated in the respective fluid frames by the formula \( \lambda = \lambda_0 \cdot p_{1,2} \), assuming a momentum dependence to this mean free path for scattering along the field, related to the spatial diffusion coefficient, \( \kappa \). In the shock normal, or in the \( x \) direction, the diffusion coefficient is given by \( \kappa \parallel = \lambda v/3 \) where \( \kappa = \kappa_\parallel \cos^2 \psi \). At the scattering centers the energy (momentum) of the particle is kept constant and only the direction of the velocity vector \( v \) is randomized, using a computational random number generator. Runs were performed with different spatial boundaries to investigate the effect of the size of the acceleration region on the spectrum, as well as to find a region where the spectrum size independent. In the small-angle scattering case, the inherent anisotropy due to the high downstream sweeping effect, may greatly modify this analytical estimate. A splitting technique is used similar to the one used in the Monte Carlo simulations of [10], [11], so that when an energy level is reached such that only few accelerated cosmic rays remain, each particle is replaced by a number of \( N \) particles of statistical weight \( 1/N \), so as to keep roughly the same number of cosmic rays being followed. Workers on the field [12] have demonstrated that small angle scattering (pitch angle diffusion), with \( \delta \theta < 1/\Gamma \), applies with \( \theta \) measured in the upstream fluid frame for scattering in a uniform field or a randomly oriented set of uniform field cells. This arises because particles attempting to penetrate upstream from the shock are swept back into the shock before they can move far in pitch. For this case, we allow particles’ pitch angle diffusion to angles chosen at random, up to an angle \( \sim 10/\Gamma \), where \( \Gamma \) is the upstream gamma, measured in the normal shock frame. For the oblique shock cases that we apply here, provided the field directions encountered are reasonably isotropic in the shock frame, we know that \( \tan \psi_1 = \Gamma^{-1}_1 \tan \psi_\parallel \sim \Gamma^{-1}_1 \sim \psi_1 \) where ‘1’ and ‘\( s \)’ refer to the upstream and normal shock frames respectively. Given the current interest in following shock acceleration in the relativistic flow in test particle regime, we outline a typical approach to such computations. Using the guiding center approximation, a test particle moving a distance, \( d \), along a field line at \( \psi \) to the shock normal, in the plasma frame has a probability of collision within \( d \) given by \( P(d) = 1 - \exp(-d/\lambda) = R \). Where the random number \( R \) is \( 0 \leq R \leq 1 \). Weighting the probability by the current in the field direction, \( \mu \), then \( d = -\lambda \mu \ln R \). Between the ‘i th’ and ‘i+1 th’ scattering, a shock crossing condition is checked, by finding the position and time in the normal shock frame, as

\[
\Delta x_{\parallel}^{i+1} = \Gamma_1 (\Delta x_{\parallel}^{i} + V_1 \frac{\Delta x_{\parallel}^{i} \sec \psi_1}{v_1 \mu_1})
\]

\[
\Delta t_{\parallel}^{i+1} = \Gamma_1 (\frac{\Delta x_{\parallel}^{i} \sec \psi_1}{v_1 \mu_1}) + V_1 \Delta x_{\parallel}^{i}
\]

where ‘\( i \)’ refers to the quantities that are calculated in the upstream fluid frame. For further details on relativistic transformation calculations see [13]. We note that for the description of the physical quantities necessary throughout the simulations we use the fluid rest frames, the normal shock frame and the de Hoffman-Teller frame where \( E=0 \). Furthermore, as the theory of the oblique shocks postulates, [14], from the conservation of the first adiabatic invariant, one can find the new pitch angle in the downstream frame and similar transformations allow the particle scattering to be followed in this frame.

**Results**

**Energy spectra**

As discussed above, Monte Carlo calculations have been performed for different parameter settings. Three different shock inclinations have been simulated, i. e. \( \psi = 23^\circ, 33^\circ \) and \( 43^\circ \), where \( \psi \) is the angle between the shock normal and the magnetic field. For each of the angles, nine different boost factors have been used, \( \Gamma = 10, 20, 30, 100, 300, 500, 700, 900 \) and \( 1000 \). Results will be presented in detail in [13]. Furthermore, each single spectrum is fit by a single power-law approximation, \( dN/dE \propto E^{-\alpha_p} \), in order to get an estimate of the spectral index \( \alpha_p \). At the lowest energies, particle injection features are excluded. Since AGN and GRBs
are likely to accelerate particles up to $\sim 10^{21}$ eV, this value is used as the maximum energy. This way, each shock boost factor $\Gamma$ is associated with a spectral index $\alpha_p(\Gamma)$. A linear correlation between $\alpha_p$ and $\Gamma$ is found as it is displayed in Fig. 1. The correlation appears to be linear.

![Figure 1: Correlation between the shock boost factor $\Gamma$ and the spectral index, for shock inclination of $\psi = 43^\circ$. The correlation appears to be linear.](image)

This implies three main things for relativistic, subluminal shocks:

1. The result is basically independent of the inclination angle between shock normal and magnetic field, as long as the shocks are oblique.

2. Mildly-relativistic shocks have spectral indices of around $\alpha_p \sim 2.0 - 2.4$.

3. The more relativistic the shock, the flatter the spectrum. Highly-relativistic shocks with $\Gamma \sim 1000$ can produce spectra as flat as $\alpha_p \sim 1.5$.

The above findings seem to support similar simulation results and observational evidences regarding irregular and flat spectra from many GRBs (further details in [13]).

**Contribution to the charged Cosmic Ray spectrum**

In this section, the spectra of different boost factors are associated to different source classes in order to calculate the possible contribution to the diffuse spectrum of charged cosmic rays. Specifically, it is assumed that the cosmic ray spectrum above the ankle, $E > 10^{18.5}$ eV, is produced by protons accelerated in AGN and GRBs sources. For AGN, a fix boost factor of $\Gamma = 10$ is used.

In the case of GRBs, it is assumed that $10\%$ of all GRBs have $\Gamma = 100$, $80\%$ have $\Gamma = 300$ and the remaining $10\%$ have $\Gamma = 1000$. It is further assumed that the observed spectrum of charged cosmic rays is produced by a linear combination of AGN, $dN_{AGN}/dE$, and GRBs, $dN_{GRB}/dE$.

$$
\frac{dN_{CR}}{dE} = \alpha \cdot \frac{dN_{AGN}}{dE} + (1 - \alpha) \cdot \frac{dN_{GRB}}{dE},
$$

with $0 \leq \alpha \leq 1$. Source evolution has been taken into account by using the source density given by the observation of X-ray AGN [15]. This source evolution is also used in the case of GRBs, assuming that GRBs also follow the Star Formation Rate. The energy density of the observed cosmic ray spectrum $j_E$ is then used to normalize the calculated Monte Carlo spectra

$$
j_e := \int_{E_{\text{min}}=10^{18.5} \text{ eV}}^{E} \frac{dN}{dE} E \, dE \\
\approx 10^{-7} \text{GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}.
$$

The final result is shown in Fig. 2. The upper line represents a pure AGN spectrum, $\alpha = 1$, the middle line represents a mixture of $50\%$ AGN and $50\%$ GRBs, $\alpha = 0.5$, and the lower line is a pure GRB spectrum, $\alpha = 0$. Since the cosmic ray spectrum above $E = 10^{18.5}$ eV needs to be explained by extragalactic sources, the best fit results from a pure AGN spectrum. With a significant contribution, the spectrum becomes too flat.
Conclusions

In this work we presented Monte Carlo simulation studies on the acceleration of test particles in relativistic, subluminal shock environments. Source candidates were applied, such as Active Galactic Nuclei with mild relativistic shocks of boost factors of $\Gamma \approx 10$ and Gamma Ray Bursts with highly-relativistic shocks, $100 < \Gamma < 1000$.

The resulting particle spectra from those shocks have been investigated in the context of varying shock boost factor $\Gamma$ versus the shock obliquity, i.e. the inclination angle between the shock normal and the magnetic field, $\psi$. For three angles, $\psi = 23^\circ$, $33^\circ$, $43^\circ$, shocks were simulated with a series of different boost factors, $\Gamma = 10, 20, 30, 100, 300, 500, 700, 900, 1000$. While the result remained constant for the different inclination angles, a linear behavior between the spectral index $\alpha_p$ and the boost factor $\Gamma$ was found, leading to spectra of $\alpha_p \sim 2.2$ for mildly-relativistic shocks ($\Gamma \sim 10$), but producing much harder spectra for highly-relativistic shocks. Specifically, for boost factors of $\Gamma \sim 300 - 1000$, the spectra can become as flat as $\alpha_p \sim 1.5$.

Concerning the observed cosmic ray spectrum at the highest energies, our study shows that one cannot explain those high energies using solely a con-