



Influence of the scattered Cherenkov light on the width of shower images as measured in the EAS fluorescence experiments

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Abstract: The EAS induced by ultra high energy cosmic rays excite the atmosphere which emits fluorescence light. Images of showers in this light can be registered from large distances, as narrow tracks, the intensity at a given level being proportional to the shower energy deposited there. However, there is also Cherenkov light accompanying the shower which, when scattered sideways, adds to the fluorescence light. We show that the width of the shower image is determined not only by the lateral spread of the shower electrons, but by the lateral distribution of the Cherenkov light, scattered to the observer. We analyse how this effect scales with the shower parameters of its development in the atmosphere. In particular, the importance of this effect grows with the distance (in meters) of the observed shower level to the shower maximum.

Introduction

The EAS induced by ultra high energy cosmic rays excite the atmosphere which emits fluorescence light. Images of showers in this light can be registered from large distances as narrow tracks, the intensity at a given level being proportional to the shower energy deposited there. However, there is also the Cherenkov light, emitted by shower electrons. When scattered sideways it adds to the fluorescence light.

We show that the width of the shower light image is determined not only by the lateral spread of the shower electrons, as claimed in [1], but also by the lateral distribution of the Cherenkov light, scattered to the observer. We analyse how this effect depends on shower parameters of its development in the atmosphere.

Dependence of the lateral distribution of Cherenkov light (LDCh) on shower parameters

We are going to calculate LDCh numerically, on different levels of a shower, using the results of our earlier [2, 3] and recent [4] analyses of various distributions (energy, angle, distance) of electrons in a shower.

The number of electrons, $\Delta N_{ch}(\theta, s, h)$, producing Ch light (called Cherenkov electrons) at a height h (above some fixed level on earth), at shower age s , at an angle $(\theta, \theta + \Delta\theta)$ to the axis equals

$$\Delta N_{ch}(\theta, s, h) = N_{tot} f(\theta; s, h) 2\pi \sin \theta \Delta\theta \quad (1)$$

where N_{tot} is the total number of electrons at this level, $f(\theta; s, h)$ is the angular distribution of Ch electrons (per unit solid angle, normalised to the fraction of Ch electrons at a given level, see [3]). It is important to note that this distribution depends on shower age s (as the electron energy spectrum depends on s only) and on (absolute) height h in

the atmosphere (because the threshold energy for Ch emission, E_{th} , depends on h only.)

Here we shall assume that Ch light is emitted along the direction of the electron. This assumption is well justified because Ch emission angles are $\sim 1^\circ$ or smaller, whereas electron angles are typically $\sim 10^\circ$. So, in such a case the actual angular distribution of Ch light would actually be broader by a factor $[1 + (1/10)^2]^{(1/2)}$ (which is 0.5%) than that calculated here.

Thus, the number of Ch photons, $d(\Delta Ch)$, emitted along path dX (in $g\text{ cm}^{-2}$) at angles $(\theta, \theta + \Delta\theta)$ equals

$$d(\Delta Ch_{em}) = k_{ch} \Delta N_{ch}(\theta, s, h) dX \quad (2)$$

where $k_{ch} = 172 \frac{\text{Ch photons}}{g\text{ cm}^{-2}}$, corresponding to the Ch emission (for $\lambda = 300 - 400\text{ nm}$) by one electron with energy $E \gg E_{th}$. The total Ch light arriving in a ring $(r, r + \Delta r)$ around shower axis is obtained by integration of the contributions from all levels above observation:

$$\Delta Ch_{obs}(r) = k_{ch} \int_0^{X_{obs}} T \left(\frac{X_{obs} - X}{\cos \theta} \right) \cdot N_{tot}(X) f(\theta; s, h) 2\pi \sin \theta \Delta\theta dX \quad (3)$$

where $T \left(\frac{X_{obs} - X}{\cos \theta} \right)$ is the attenuation factor, depending on the path (in $g\text{ cm}^{-2}$) traversed by the emitted photons at depth X to the observation depth X_{obs} . We have also the obvious relations: $\tan \theta = \frac{r}{y}$ and $\Delta\theta = \frac{y}{y^2 + r^2} \Delta r$, where y is the distance in meters (along shower axis) between the two levels X_{obs} and X . Taking this into account we have

$$\Delta Ch_{obs}(r) = 2\pi r \Delta r k_{ch} \cdot \int_0^{X_{obs}} T \left(\frac{X_{obs} - X}{\cos \theta} \right) \cdot N_{tot}(X) f(\theta; s, h) \frac{y}{(y^2 + r^2)^{3/2}} dX \quad (4)$$

This formula would be strictly correct if all electrons were exactly on the shower axis (no lateral distribution). However, we do take this effect (although small) into account (see later).

To see the dependence on the zenith angle z more clearly let us consider an inclined shower with the same $N(X)$ as a vertical one. It is easy to show that all distances y are now larger by factor $1/\cos z$ than those for a vertical shower. Thus, the LDCh should be broader by $1/\cos z$. As the Molière radius at a fixed slant depth X grows as $1/\cos z$, using r/r_M instead of r eliminates the geometrical effects. Introducing new variables $y_0 = y \cos z$, $r_0 = r \cos z$ and $x = r/r_M$ we obtain

$$\Delta Ch_{obs}(x) = 2\pi x \Delta x r_M^2(0) k_{ch} \cdot \int_0^{X_{obs}} T \left(\frac{X_{obs} - X}{\cos \theta} \right) \cdot N_{tot}(X) f(\theta; s, h) \frac{y_0}{(y_0^2 + r_0^2)^{3/2}} dX \quad (5)$$

where $r_M(0)$ is the Molière radius at the depth X_{obs} for all vertical shower ($z = 0$).

Thus, we see that LDCh at a given X_{obs} (slant) and for a given $N_{tot}(X)$ depends on the zenith angle only because $f(\theta; s, h)$ does. It is so because an inclined shower with the same $N_{tot}(X)$ is higher in the atmosphere by $\Delta h = -H \cdot \ln(\cos z)$ (for a purely exponential decrease of the air density, with H being the scale height) with respect to the vertical one. As the threshold energy for Ch emission increases with height, so $f(\theta; s, h)$ decreases (as it is normalised to the fraction of Ch electrons at the level considered).

If the shapes of shower curves $N_{tot}(X)/N_{max}$ were the same (actually they do not differ much from shower to shower as we measure X from shower initiation point) the shape of LDCh would depend only on the shower age s at the observation level and on the absolute height h of the shower (say, of its maximum) in the atmosphere.

To take into account the lateral distribution (LD) of electrons we use the fact that it affects LDCh rather weakly, particularly at larger distances. Thus, any contribution to LDCh from a given path element at age s has been broadened by factor $\left[1 + \left(\frac{\langle r_{Ch}(s) \rangle}{r} \right)^2 \right]^{1/2}$, where r_{Ch} is the mean radius of Ch electrons at s (when expressed in units of r_M it depends on s only [3]). To calculate it we have used the results of an analysis from [4], where the LD of electrons with a fixed energy E have

been calculated and parametrised, as functions of s and E . Calculating the means $\langle r^2(E, s) \rangle$ (in units proportional to the air density) we can find $\langle r_{Ch}^2(s, h) \rangle$ for Ch electrons by integrating the former (weighted with electron energy distribution) over energy above Ch threshold, depending on height.

Having LDCh we calculate next what is the lateral distribution of the *scattered* Ch light at a given level and we want to compare it with the fluorescence light, in order to see what determines the lateral width of shower light images.

Here we take into account Rayleigh scattering only which is roughly isotropic as fluorescence. Thus, comparing both lights emitted in *all* directions (as we do) reflects the real situation when a detector sees light emitted at a particular angle only. Adding highly anisotropic Mie scattered light would make sense only if we were calculating shower images for various observation angles.

Results of calculations of LDCh

We have assumed a typical shape (Gaisser-Hillas i.e. a gamma function) of the shower curve, $N(X)$, with $X_0 = 0$, $\lambda = 70$ and $X_{max} = 787 \text{ g cm}^{-2}$, corresponding to an average proton shower and $X_{max} = 698 \text{ g cm}^{-2}$ for iron shower with $E_0 = 10^{19} \text{ eV}$. Figure 1,2,3 show LDCh for ages $s = 0.9, 1.1$ and 1.3 (corresponding to $X_{obs}/X_{max} = 0.86, 1.16$ and 1.53 respectively). On the vertical axis there is the number of Cherenkov photons at the core distance larger than indicated on the horizontal axis, scattered in a 1 m path in all directions, divided by $N_{tot}(X_{obs})$. Also shown is the total fluorescence light (Fl) produced along 1 m of shower path by electrons at lateral distances larger than r , divided by $N_{tot}(X_{obs})$. As shown in [5] the fluorescence curve depends on s only if r/r_M are used rather than r . To calculate it we took the fitted formula from [1].

Inspecting the three figures one can see that, whereas the Fl curve becomes only a little flatter with s , the absolute values of Ch light increase rather strongly with age. At $s = 0.9$ the scattered Ch constitutes only 10% of Fl for a proton shower at $z = 45^\circ$, almost independently on r . But at $s = 1.1$ it becomes equal to Fl at $r = 2r_M$ dom-

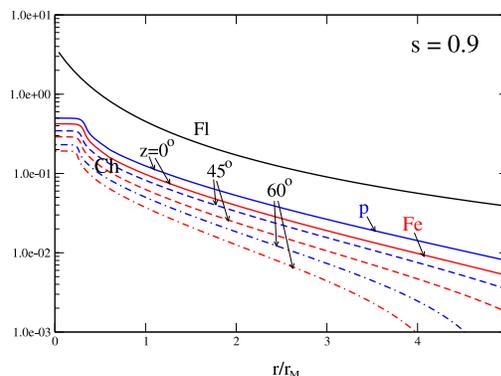


Figure 1: Total number of fluorescence (Fl) and scattered Cherenkov photons (Ch) emitted from core distances larger than r/r_M , divided by N_{tot} at the observation level ($s = 0.9$); zenith angles z and primary particles are indicated.

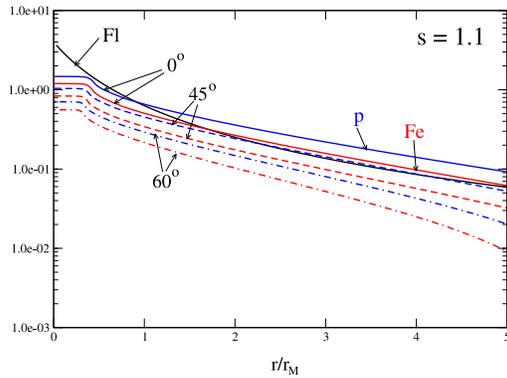
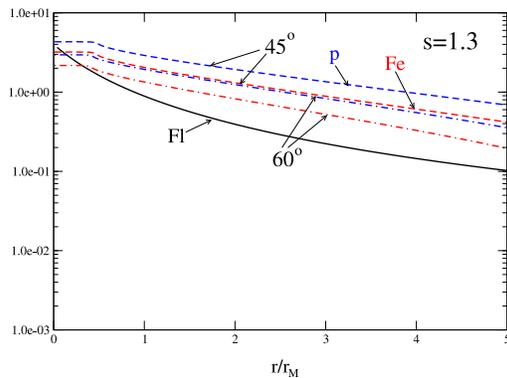
inating at larger distances. At $s = 1.3$ the scattered Ch becomes dominating at practically all distances.

In the figures there are Ch curves for proton and iron primaries and for different zenith angles z . The only reason why they differ is different height of the showers in the atmosphere.

Application to the fluorescence detectors of EAS

In the experiments measuring large EAS by observing their fluorescence tracks it is necessary to know what light one is actually collecting. It is well known that the scattered Cherenkov contribution is not negligible, particularly from lower parts of a shower. As it is the Fl light that is proportional to the shower energy deposit in the atmosphere (from what one calculates the primary energy) the exact knowledge of the width of the Ch track has to be known.

Our calculations allow to predict LDCh once a shower curve $N(X - X_{max})$ and the height of X_{max} are given. For example, let us consider a proton shower with $E_0 = 10^{19} \text{ eV}$ at $z =$


 Figure 2: The same as in Fig.1, but for different s .

 Figure 3: The same as in Fig.1, but for different s .

45° . From a distance of 25 km one pixel in the Auger experiment (full field of view = 1.5°) sees $\pm 328 \text{ m}$ perpendicularly to the shower track. At $s = 1.1$ (see Figure 2) the Molière radius $r_M = 113 \text{ m}$, so the pixel collects all light emitted within $\frac{r}{r_M} = 2.9$ (if the axis goes through the centre of it). The neighbouring pixels will collect as much Fl as Ch light, so to recover all Fl light one has to take them into account.

Conclusions

We have shown that the lateral distribution of Cherenkov light below shower maximum is broad enough to contribute to the shower light image by its scattered component.

The effect depends rather strongly on the shower age s (increases with s) and its absolute height h in the atmosphere (decreases with h). As the shapes of shower curves $N(X - X_{max})/N_{max}$ do not fluctuate much from shower to shower, it is practically these two parameters s and h which determine the width of the Ch image. As the fluorescence width depends also on these two parameters only, it will be possible to determine their relative contributions on s and h only.

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