Effect of muon-nuclear inelastic scattering on high-energy atmospheric muon spectrum at large depth underwater

S. I. Sinegovsky[1], A. Misaki[2], K. S. Lokhtin[1] and N. Takahashi[3]

[1] Irkutsk State University, Irkutsk, Russia

[2] Advanced Research Institute for Science and Technology, Waseda University, Tokyo, Japan

[3] Department of Advanced Physics, Hirosaki University, Hirosaki, Japan

Introduction

The muon inelastic scattering on nuclei contributes noticeably to the total energy loss of cosmic-ray muons. The influence of this interaction on the shape of ultra-high energy muon spectra at the large depth of water or rock is still unknown in detail. Of interest is also to estimate the number of cascade showers produced by very high-energy muons in inelastic interactions with nuclei and to study the influence of this process on the energy spectra of cosmic-ray muons in water at the depth of the underwater/ice neutrino telescopes, operating and projected – NT200+, AMANDA, IceCube, ANTARES, NESTOR, NEMO and KM3NeT.

In this work, the energy spectra of hadron cascade showers produced by atmospheric muons in water by inelastic scattering on nuclei, as well as the integral energy spectra of atmospheric muons in water at depths up to 4 km are computed using two models – the hybrid model of inelastic scattering of leptons on nuclei ($2C$ or $3C$) [1,2] and, for a comparison, the known generalized vector-meson-dominance (GVMD) model of photonuclear muon interactions by Bezrukov and Bugaev [3].

Hybrid models, $2C$ & $3C$

A two-component ($2C$) model and three-component ($3C$) one are constructed to describe inelastic high-energy scattering of muons and taus on nucleus in standard rock or water. The $3C$ model involves photonuclear interactions at $Q^2 < 0.1$ GeV^2, as well as moderate Q^2 processes and the deep inelastic scattering. For low Q^2 there was applied the electromagnetic structure function parametrization by Bezrukov and Bugaev [3] based on the generalized vector meson dominance (GVMD) model. The Regge approach by Kaidalov, Merino and Pertermann (KMP) [4] for moderate values of the Q^2 was employed as the component. In the region $Q^2 > 5$ GeV^2 the global fit of parton distributions, CTEQ [5] or MRST [6], was used to compute electroweak structure functions of the nucleon.

$$2C\,\mathrm{model}: \quad \begin{cases} 0 < Q^2 \le 5\,\mathrm{GeV}^2 & \Leftarrow \mathrm{KMP}, \\ Q^2 > 5\,\mathrm{GeV}^2 & \Leftarrow \mathrm{CTEQ}\;/\;\mathrm{MRST} \end{cases}$$

$$3C\,\mathrm{model}: \quad \begin{cases} 0 < Q^2 < 0.1\,\mathrm{GeV}^2 & \Leftarrow \mathrm{GVMD}, \\ 0.1 < Q^2 \le 5\,\mathrm{GeV}^2 & \Leftarrow \mathrm{KMP}, \\ Q^2 > 5\,\mathrm{GeV}^2 & \Leftarrow \mathrm{CTEQ}\;/\;\mathrm{MRST} \end{cases}$$

Soft processes $\ell^\pm + A \to \ell^\pm + X$

Photonuclear interactions at $Q^2 < 0.1$ GeV^2:

$$\frac{d^2\sigma^{\ell A}}{dQ^2 dy} = \frac{\alpha E}{\pi}\left[\Gamma_T \sigma_T^A(\nu, Q^2) + \Gamma_L \sigma_L^A(\nu, Q^2)\right]$$

$$\Gamma_T = \frac{K}{Q^2(Q^2+\nu^2)\,E^2}\left[E(E-\nu) + \frac{\nu^2}{2}\left(1 - \frac{2m_\ell^2}{Q^2}\right) + \frac{Q^2}{4} - m_\ell^2\right],$$

$$\Gamma_L = \frac{K}{Q^2(Q^2+\nu^2)\,E^2}\left[E(E-\nu) - \frac{Q^2}{4}\right], \quad K = \nu - \frac{Q^2}{2M_p}.$$

Bugaev and Bezrukov (BB) parametrization [3] :

$$\sigma_T^A(\nu, Q^2) = A\,\sigma_{\gamma N}(\nu)\left[\frac{0.75\,m_1^4}{(m_1^2+Q^2)^2}G(z) + \frac{0.25\,m_2^2}{m_2^2+Q^2}\right]$$

$$\sigma_L^A(\nu, Q^2) = 0.25\,A\,\sigma_{\gamma N}(\nu)\left[\frac{0.75\,m_1^2 Q^2}{(m_1^2+Q^2)^2}G(z) + \frac{0.25 m_2^2}{Q^2}\ln\left(1 + \frac{Q^2}{m_2^2}\right) - \frac{0.25\,m_2^2}{m_2^2+Q^2}\right]$$

Shadowing of nucleons:

$$r^A = \frac{\sigma_{\gamma A}}{A\sigma_{\gamma N}} = 0.75 G(z) + 0.25, \quad G(z) = \frac{3}{z^3}\left[\frac{z^2}{2} - 1 + e^{-z}(1+z)\right],$$

$$\sigma_{\gamma N}(\nu) = 114.3 + 1.647\ln^2(0.0213\,\nu), \qquad z = 0.00282 A^{1/3}\sigma_{\gamma N}(\nu)$$

Semihard scattering $\ell^{\pm} + N \to \ell^{\pm} + X$

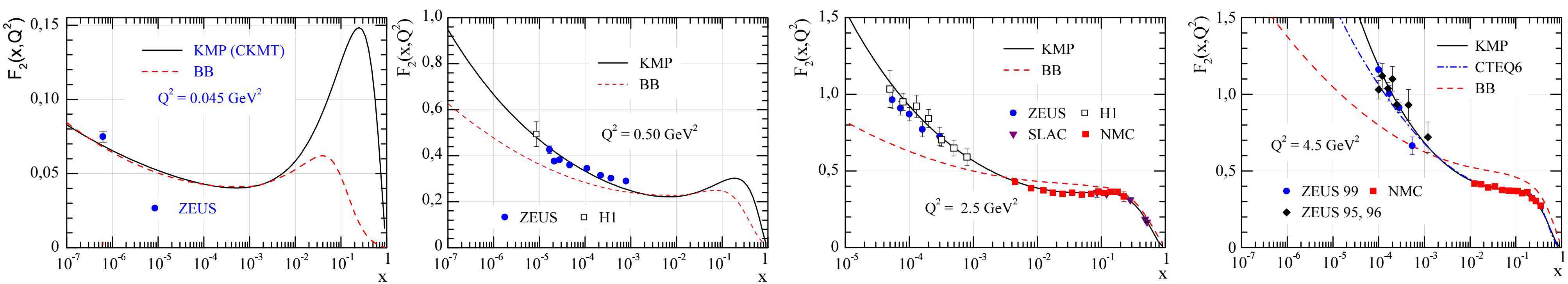
$$\frac{d^2\sigma}{dQ^2dy} = \frac{4\pi\alpha^2}{yQ^4}\left[1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2(1+R)}\left(1 - \frac{2m_\ell^2}{Q^2}\right)\left(1 + \frac{Q^2}{E^2y^2}\right)\right]F_2$$

$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \left(1 + \frac{Q^2}{\nu^2}\right)\frac{F_2}{2xF_1} - 1,$$

CKMT (KMP) parametrization ($0 < Q^2 < 5$ GeV^2) [4]:

$$F_2(x,Q^2) = A\,x^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} + B\,x^{1-\alpha_R}\,(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}$$

GVMD (BB) and KMP proton structure function $F_2(x,Q^2)$:



Deep inelastic scattering $\ell^{\pm} + N \to \ell^{\pm} + X$

$$\frac{d^2\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{yQ^4}\left\{\left[1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2}\left(1 - \frac{2m_l^2}{Q^2}\right)\right] F_2 \pm \left(\frac{y^2}{2} - y\right) xF_3\right\}$$

$$F_2(x,Q^2) = F_2^{\gamma} - g_V^{\ell}\eta_{\gamma Z}F_2^{\gamma Z} + ({g_V^{\ell}}^2 + {g_A^{\ell}}^2)\eta_{\gamma Z}^2 F_2^Z,$$

$$F_3(x,Q^2) = -g_A^{\ell}\eta_{\gamma Z}F_3^{\gamma Z} + 2g_V^{\ell}g_A^{\ell}\eta_{\gamma Z}^2 F_3^Z,$$

$$\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\frac{Q^2}{M_Z^2 + Q^2}, \quad g_V^{\ell} = -\frac{1}{2} + 2\sin^2\theta_W, \quad g_A^{\ell} = -\frac{1}{2}.$$

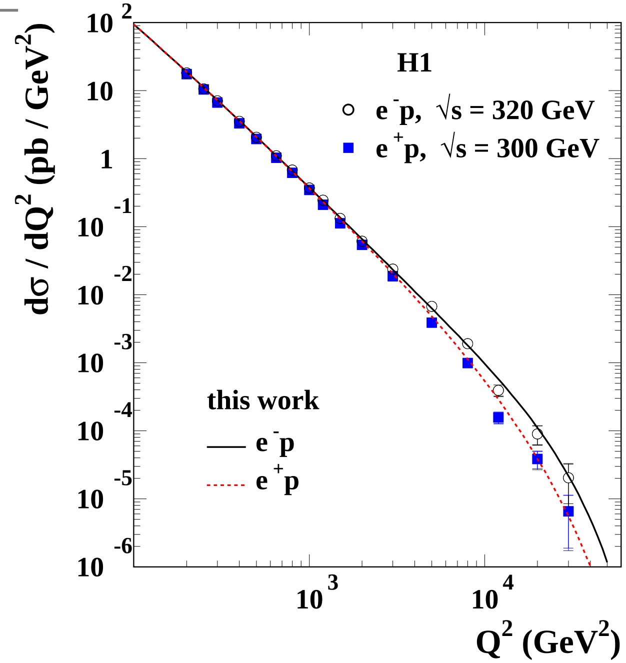
Nucleon structure functions F_2^{γ}, $F_2^{\gamma Z}$, F_2^Z, $F_3^{\gamma Z}$, F_3^Z**:**

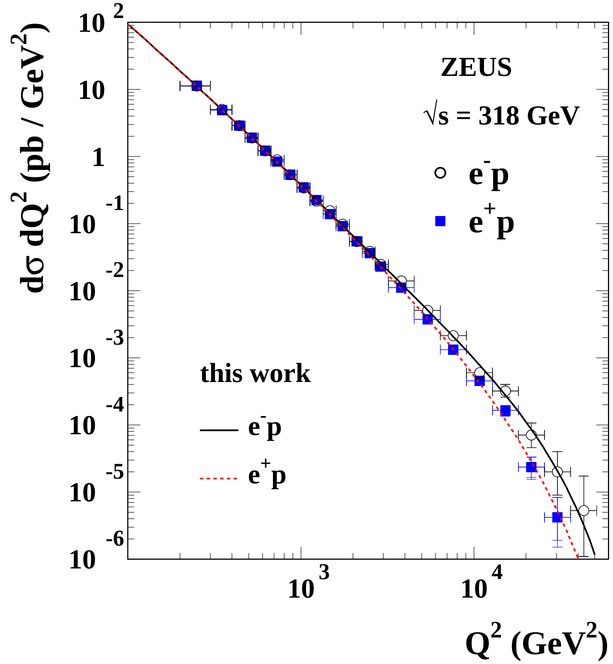
$$\left[F_2^{\gamma}, F_2^{\gamma Z}, F_2^Z\right] = x\sum_q \left[e_q^2, 2e_q g_V^q, {g_V^q}^2 + {g_A^q}^2\right](q + \bar{q}),$$

$$\left[F_3^{\gamma Z}, F_3^Z\right] = \sum_q \left[2e_q g_A^q, 2g_V^q g_A^q\right](q - \bar{q}).$$

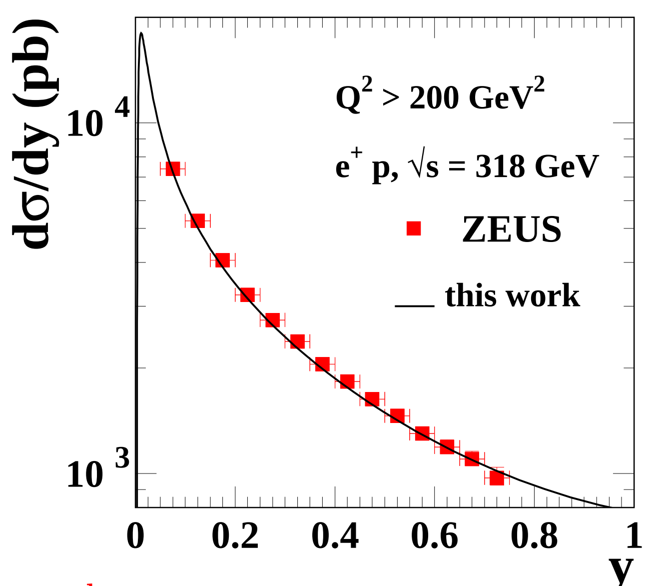
Here $g_V^q = \pm\frac{1}{2} - 2e_q\sin^2\theta_W$ **– vector coupling of quarks,** $g_A^q = \pm\frac{1}{2}$ **– axial coupling. The +(-) sign corresponds to** $q = u, c, t\ (d, s, b)$**.**

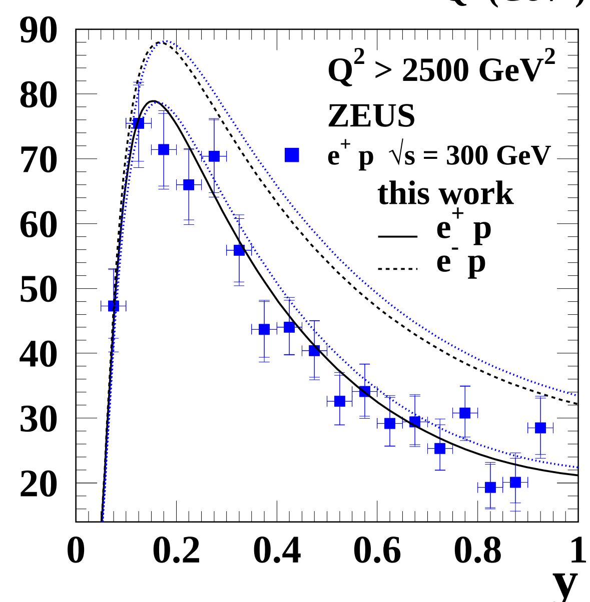
Deep inelastic $e^{\pm}p$-scattering: $d\sigma/dQ^2, d\sigma/dy$

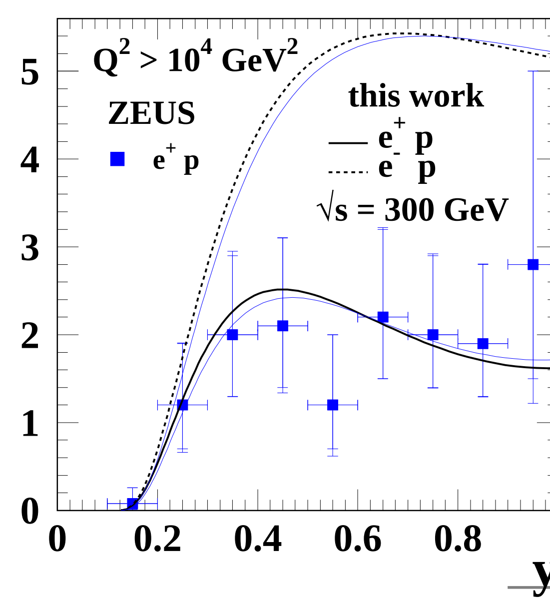




Comparison of computations in which we use the CTEQ6 PDF set and MRST one (thin lines in bottom Figs.) with the data of HERA experiments.







Nuclear stucture functions

Lepton-nucleus inelastic scattering $\ell^\pm + A \to \ell^\pm + X$:

$$\frac{d^2\sigma^{\ell A}}{dQ^2 dy} = A r_A(x, Q^2) \frac{d^2\sigma^{\ell N}}{dQ^2 dy}, \qquad (1)$$

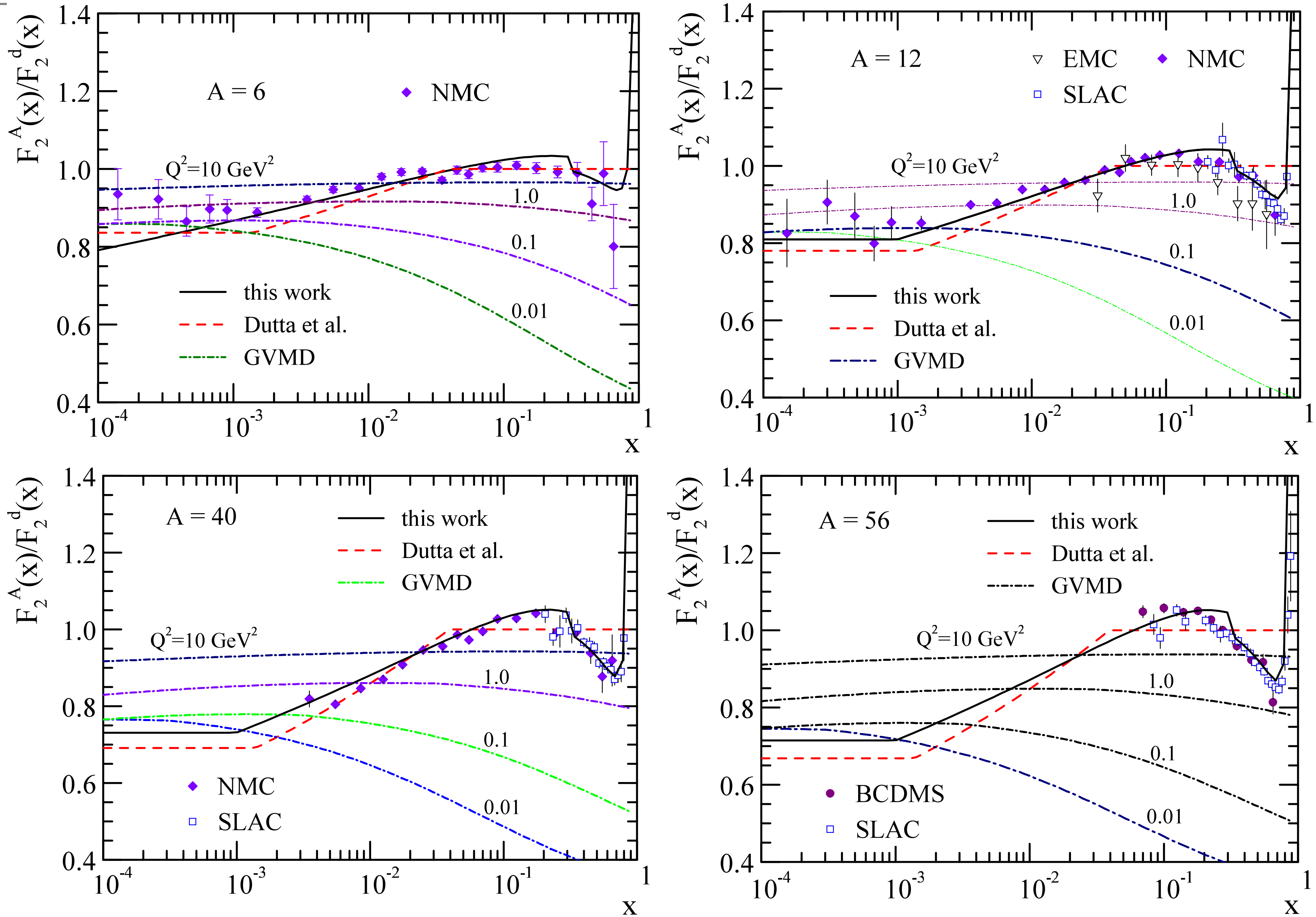
$r_A(x, Q^2) = F_2^A(x, Q^2)/F_2^N(x, Q^2)$ is the ratio of the intranuclear structure function of the nucleon (F_2^A) to that of free isoscalar nucleon (isospin averaged nucleon structure function), $F_2^N \equiv (F_2^p + F_2^n)/2$.

The lepton energy loss due to inelastic scattering on nuclei in matter is

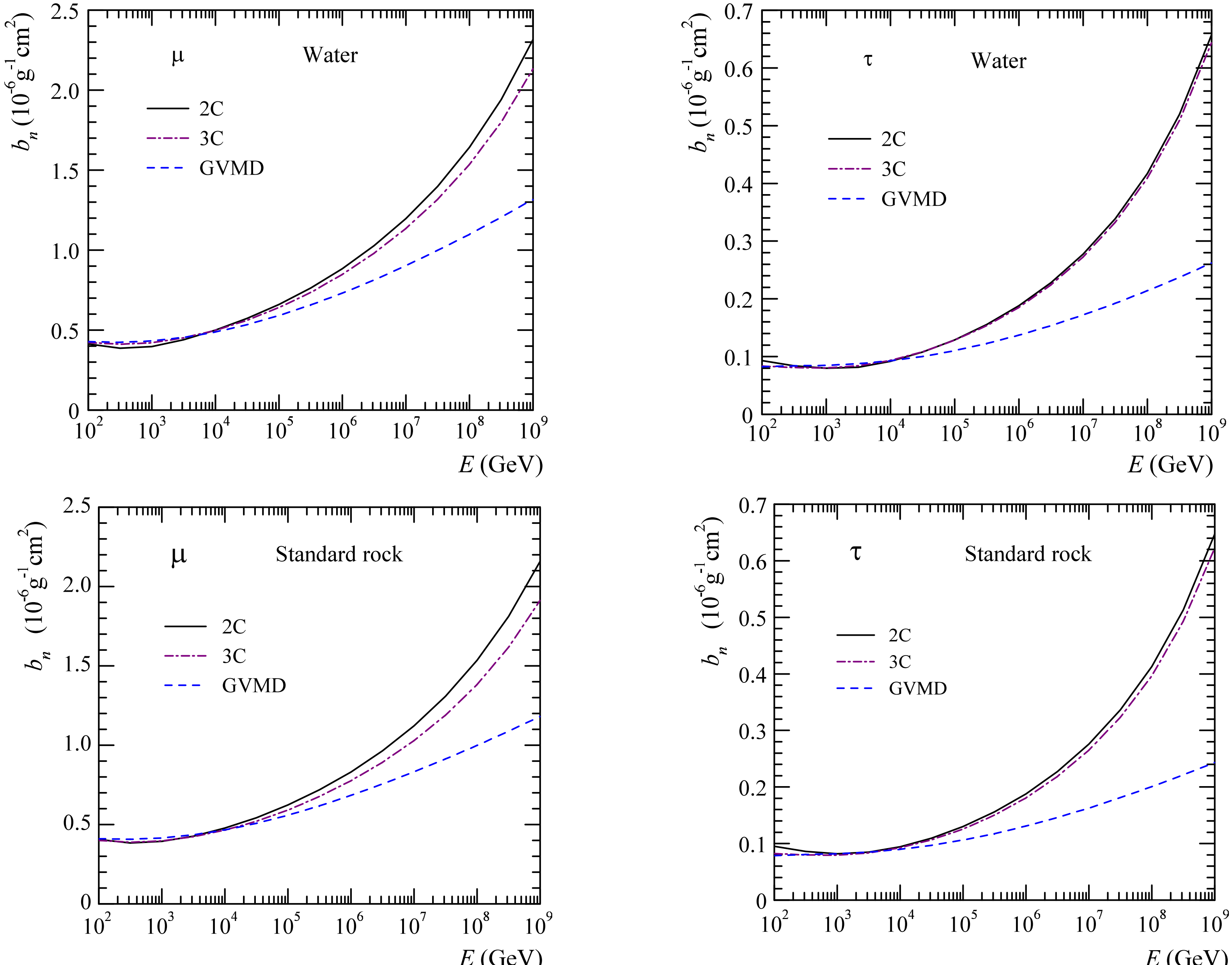
$$b_n^{(\ell)}(E) \equiv -\frac{1}{E}\frac{dE}{dh} = \frac{N_A}{A} \int_{y_{\rm min}}^{y_{\rm max}} y dy \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} dQ^2 \, \frac{d^2\sigma^{\ell A}}{dQ^2 dy} \qquad (2)$$

Nuclear effects (nucleon shadowing, anti-shadowing, Fermi motion) are taken into account according to Ref. [7] (see also [8]).

Nucleus effect on the structure function



Lepton energy loss for interactions with nuclei



Comparison of calculated $b_n^{(\mu,\tau)}$ values in rock

E, GeV	$b_n^{(\ell)}(E)$, 10^{-6} cm$^2\cdot$ g^{-1}				
	[1, 2] (2C)	[9]	[8]	[10]	[11]
muon					
10^5	0.62	0.60	0.68	0.70	0.7
10^6	0.82	0.80	0.88	1.08	1.0
10^8	1.53	1.60	–	2.25	2.5
10^9	2.16	2.18	–	3.10	4.0
tau					
10^5	0.13	0.12	–	0.14	0.12
10^6	0.19	0.18	–	0.21	0.20
10^8	0.41	0.40	–	0.50	0.60
10^9	0.65	0.60	–	0.72	1.30

Approximating formula for the energy loss $b_n^{(\ell)}(E)$

$$\ell^{\pm} + A \to \ell^{\pm} + X \qquad (\ell = \mu, \tau)$$

$$b_n^{(\ell)}(E) = N_0 \int_{y_{\min}}^{y_{\max}} y \frac{d\sigma_{\ell A}}{dy} dy$$

Approximation for energy loss of muons and taus in water or standard rock fits the numerical results in the energy range $10^2 - 10^9$ with an accuracy of $\sim 1\%$:

$$b_n^{(\ell)}(E) = (c_0 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4) \cdot 10^{-6}\,\mathrm{cm}^2/\mathrm{g}\,,$$

$$\eta = \lg(E/1\,\mathrm{GeV}); \quad E \in 10^2 - 10^9\,\mathrm{GeV}$$

Water:

μ : $c_0 = 1.06416,\ c_1 = -0.64629,\ c_2 = 0.20394,\ c_3 = -0.02465,\ c_4 = 0.00113;$

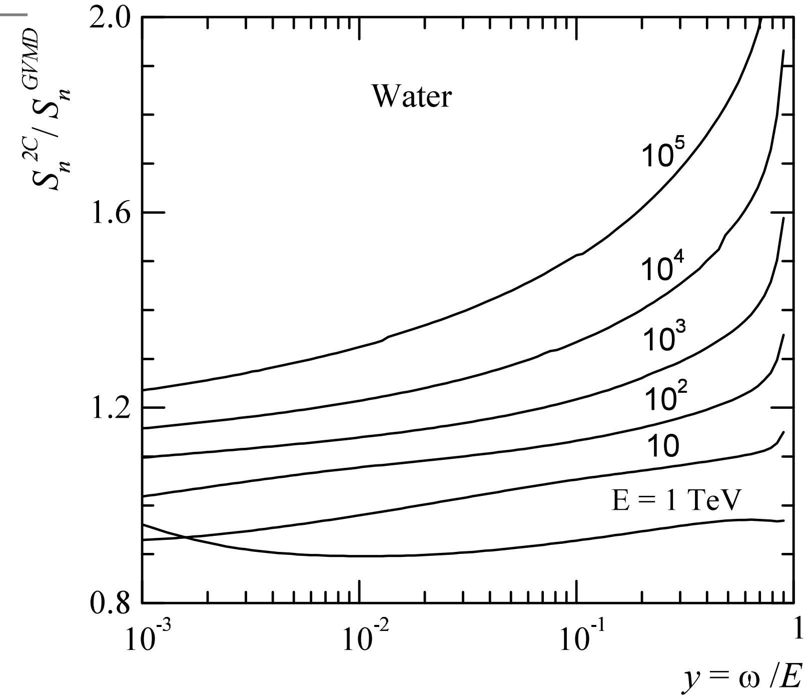
τ : $c_0 = 0.35697,\ c_1 = -0.24437,\ c_2 = 0.07403,\ c_3 = -0.00940,\ c_4 = 0.00051.$

Rock:

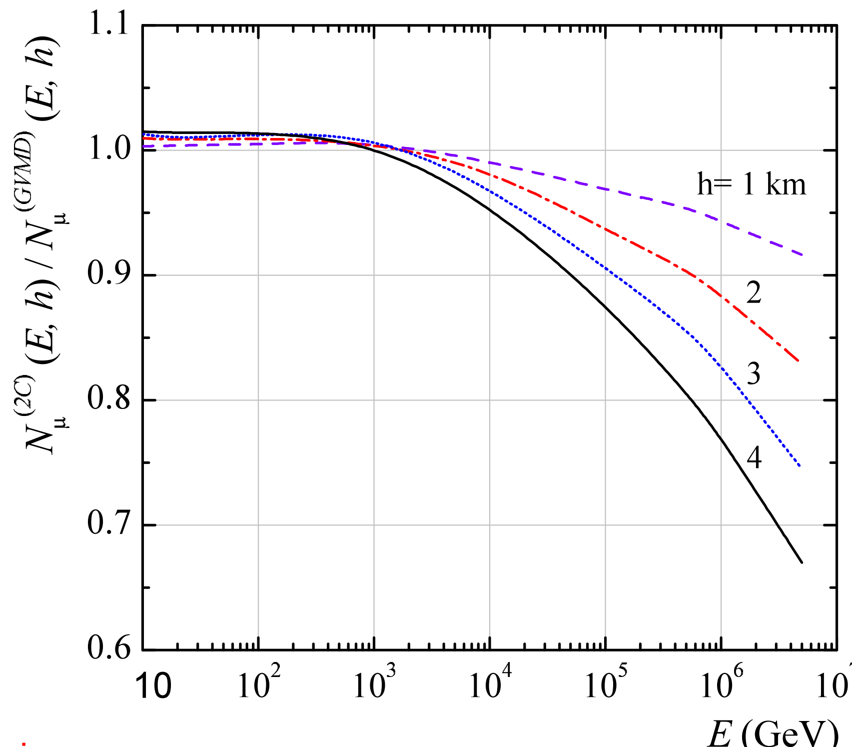
μ : $c_0 = 0.98711,\ c_1 = -0.56840,\ c_2 = 0.17677,\ c_3 = -0.02114,\ c_4 = 0.00112;$

τ : $c_0 = 0.33247,\ c_1 = -0.22283,\ c_2 = 0.06811,\ c_3 = -0.00873,\ c_4 = 0.00048.$

Hadron showers & Muon flux underwater



For muon energy $E > 10^3$ TeV, the number of muon-induced hadron showers which computed with the $2C$ model exceeds that of obtained with use of the GVMD model by 20–50 % even for small energy loss ($y \sim 0.1$). For the catastrophic energy loss ($y > 0.5$), the number of showers, obtained with the $2C$ model, exceeds that of the GVMD prediction by factor about 2.



Right figure shows the ratio of integral muon spectra underwater (close to vertical) at depth 1–4 km, computed for $2C$ model, to those for GVMD one. At large depth the effect is a noticeable at the $E > 10$ TeV. For $h = 4$ km this ratio decreases to 0.75 at $E = 10^3$ TeV . Thus a sizeable increase of the muon inelastic scattering cross section results in an appreciable decrease of the deep underwater muon flux as compared to that obtained [13-14] with the GVMD model.

Summary

A sizeable increase of the muon inelastic scattering cross section results in an appreciable decrease of the deep underwater muon flux: cosmic ray muon fluxes $N_{\mu}^{2C}(E,h)$ underwater at depth 3–4 km computed with the 2C model of muon-nuclear scattering is less by one fourth as compared to that computed using the GVMD model, $N_{\mu}^{GVMD}(E,h)$, for the energy $E=10^3$ TeV. It should be noted that this result refers only to the atmospheric conventional (π,K) muons. As concerns muons produced in charmed particle decays (prompt muons), which become presumably dominant at $E>100$ TeV (see e. g. [13, 14]), the role of the muon-nucleus inelastic scattering needs further study.

Evidently the increase of the cross section of inelastic muon scattering in matter, while leading to diminished cosmic-ray muon flux deep underwater, results in growing efficiency of muon or tau detection. This last factor is positive for neutrino astronomy since neutrino-induced muons may yield the signal from astrophysical high-energy muon neutrinos.

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