The effect of a modified Parker field on the modulation of the galactic cosmic rays

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Abstract: The effects on the galactic cosmic ray modulation of a Parker field modified by the motion of footpoints of the magnetic field lines and the latitudinal dependent solar wind velocity are numerically investigated. The calculation of the propagation of the galactic cosmic ray is made by solving a coupled set of stochastic differential equations (SDE) which is equivalent to the so-called diffusion convection partial differential equation. The three-dimensional code based on the SDE method has been developed into the modified Parker field. We present the details of the effects of latitudinal footpoints motion on the solar modulation of the galactic cosmic rays.

Introduction

The heliospheric magnetic field (HMF) plays an important part in the model for the solar modulation of the galactic cosmic rays. The model for HMF by Parker [1] has been successfully used in modulation studies, however, this model may be an oversimplification to describe the field in the polar region of the heliosphere. Recently some models of HMF have been proposed to take account of the footpoint motion of open magnetic fields (e.g. [2], [3]). When we adopt the footpoint velocity $u_\perp = R_{ss} \omega_\theta \hat{e}_\theta + R_{ss} \omega_\phi \hat{e}_\phi$ on the source surface in a frame rotating with the Sun, the HMF takes the following form

$$B = B_{ss} \left( \frac{R_{ss}}{r} \right)^2 \times \left( \hat{e}_r - \frac{\omega_\theta}{V} \hat{e}_\theta - \frac{r (\Omega_\odot - \omega_\phi) \sin \theta}{V} \hat{e}_\phi \right) \times \left[ 1 - 2H (\theta - \theta') \right],$$

(1)

where $r$ is the heliocentric distance, $\theta$ is the co-latitude, $R_{ss}$ is the radius of the source surface, $B_{ss}$ is the magnetic field strength at the source surface, and $\Omega_\odot$ is the rotational angular velocity of the Sun, $V$ is the solar wind velocity, $H$ is the Heaviside function, and $\theta'$ is the co-latitude of the heliospheric current sheet. Note that this equation can be applicable only for a constant solar wind velocity dependence, especially near at the solar minimum phase. Schwadron and McComas [4] studied the effect of the latitudinal dependent solar wind on the HMF (eq.1) and the cosmic ray acceleration at the termination shock. Burger and Sello [5] calculated the latitudinal gradient of the galactic cosmic rays by adopting a simple two-dimensional HMF which takes account of the latitudinal dependent solar wind velocity. The direction of the latitudinal footpoint motion affects the structure of the HMF in the region where the solar wind velocity changes significantly with latitude. We have numerically investigated the solar modulation of the galactic cosmic rays by the same HMF assumed by Burger and Sello [5]. We present details of the effects of the direction of the latitudinal footpoint motion on the solar modulation of the galactic cosmic rays.

Numerical models

Transport of the galactic cosmic rays in the heliosphere is described by the so-called diffusion convection partial differential equation. This diffusion convection equation is equivalent to a coupled set of stochastic differential equations (SDE) [6, 7]. This SDE method allows us to get some information about the solar modulation phenomena of the galactic cosmic ray not obtained by the
other numerical methods, such as the distributions of arrival time, energy lost, and trajectory. The stochastic numerical code adapted for the wavy heliospheric current sheet in the standard Parker field has been developed by Miyake and Yanagita [8]. We have developed this three-dimensional code based on the SDE method into a Parker field modified by the latitudinal footpoint motion and the latitudinal dependent solar wind velocity. The SDE equivalent to the diffusion convection partial differential equation is written as

\[ d\mathbf{X} = \left( \nabla \cdot \mathbf{k} - \mathbf{V}(\theta) - \mathbf{V}_d \right) dt + \sum_s \sigma_s dW_s(t), \]  
\[ dP = \frac{1}{3} P \left( \nabla \cdot \mathbf{V}(\theta) \right) dt, \]  

where \( \mathbf{X} \) and \( P \) are the position and the momentum of the particle, \( \mathbf{V}(\theta) \) is the solar wind velocity which has a latitudinal dependence, \( \mathbf{V}_d \) is the gradient-curvature drift velocity, \( \mathbf{k} \) is the diffusion coefficient tensor, \( \sum_s \sigma_s^p \sigma_s^q = 2\kappa_{\mu\nu} \), and \( dW_s \) is a Wiener process given by the Gaussian distribution. We adopted \( \kappa_{\mu\nu} = 1.5 \times 10^{21} (\mu_0/(1 GeV/c)) (Be/B) \text{ cm}^2/\text{s} \) and \( \kappa_{\perp} = 0.05 \kappa_{\parallel} \). We have used the “drift velocity field method” [9] for the calculation of drift in the heliospheric current sheet. We investigated a case of the tilt angle of the heliospheric current sheet \( \alpha \) fixed at 10°.

In our simulation, particles start at 1 AU on the equatorial plane and run backward in time until they exit the heliospheric boundary, 90 AU. The momentum spectrum \( f_{\mathbf{X}}(p) \) at arbitrary position \( \mathbf{X} \) is written as a convolution of the spectrum \( f_{\mathbf{X}_0}(p_0) \) at the heliospheric boundary with the normalized transition probability \( F(p_0, \mathbf{X}_0\|p, \mathbf{X}) \) obtained by our SDE method as

\[ f_{\mathbf{X}}(p) = \int f_{\mathbf{X}_0}(p_0) F(p_0, \mathbf{X}_0\|p, \mathbf{X}) dp_0. \]  

Here the spectrum at the boundary is \( f_{\mathbf{X}_0}(p_0) \propto (m^2 c^4 + p_0^2 c^2)^{-1.85} / p_0 \) which is assumed to be uniform at the heliospheric boundary.

We adopted the simplified model of HMF which is assumed by Burger and Sello [5] and takes account of the footpoint motion of open magnetic fields and the latitudinal dependent solar wind velocity,

\[ \mathbf{B} = B_{ss} \left( \frac{R_{ss}}{r} \right)^2 \left( \begin{array}{c} \frac{r_p}{4V(\theta)} \frac{dV(\theta)}{d\theta} \hat{e}_r \\ -\frac{r_{\Omega_p}\sin\theta}{V(\theta)} \hat{e}_\phi \end{array} \right), \]  

where \( r_p \) is a dimensionless constant.

Figure 1 shows the solar wind velocity profile used in this study. We assumed the solar wind velocity depends only on \( \theta \). Figure 2.a shows the effect of the solar wind velocity profile shown in Figure 1 on the HMF strength at 1 AU where \( r_p \times dV/d\theta \leq 0 \); (b) Same as (a) where \( r_p \times dV/d\theta \geq 0 \).
for both cases is localized in the region where the solar wind velocity changes significantly with latitude, the feature of the difference is quite different qualitatively between Fig 2.a and b. These differences would affect the solar modulation of the galactic cosmic rays. We have numerically investigated the effect of these differences on the modulation of the galactic cosmic rays.

**Results and Conclusions**

Figure 3 shows the latitudinal dependence of the 1 GeV proton intensities at 1 AU where \( r_p \times dV/d\theta \leq 0 \). The intensity is normalized to the intensity at the heliospheric boundary. The solid line indicates the intensities where the absolute value of \( r_p \) is 4. The dotted line indicates the intensities where \( r_p = 0 \).

Figure 4: Same as Figure 3 but \( r_p \times dV/d\theta \geq 0 \).

In Figure 3, the intensities for the modified Parker field where \(|r_p| = 4\) is lower than the intensities for the standard Parker field, especially in the region where the solar wind velocity changes significantly with latitude. This tendency comes from the dependence of the diffusion mean free path and the drift scale on the HMF strength. In our simulation, the diffusion coefficient and the drift scale are inversely proportional to the HMF strength. Accordingly once the galactic cosmic rays get into the region where the HMF strength is relatively high, i.e. into the region where the solar wind velocity changes significantly with latitude, the galactic cosmic rays stay there much longer time compared with the case of the standard Parker model. Figure 5 shows the region where the difference in the HMF strength from the standard Parker field exceeds 0.1 \( \mu G \) for \( r_p \times dV/d\theta \leq 0 \). The difference shown in Figure 3 between the result for the modified Parker model and that for the standard Parker model comes from the time spent by the cosmic rays during their journey in this region (“modified region”). Figure 6 shows the distribution of the time spent in the “modified region” and the momentum lost there by 1 GeV protons at 1 AU and \( \theta = 60^\circ \) where \( r_p \times dV/d\theta \leq 0 \). The time spent in the “modified region” is longer than the time spent for the standard Parker model. Accordingly the momentum loss has increased as seen in Figure 6.b.

As seen in Figure 3, this tendency is opposite to the results by Burger and Sello [5]. This contradiction in the results between Burger and Sello [5] and ours may come from the difference in the assumption of the dependence of the diffusion coefficient on the HMF strength. Burger and Sello [5] adopted a model the parallel mean free path of the diffusion is proportional to \( B^{5/3} \) at low rigidities and is independent of \( B \) at high rigidities. Therefore the different latitudinal dependence may result in for the case of \( r_p \times dV/d\theta \leq 0 \).
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References