# Schwarzschild-Couder two-mirror telescope for ground-based $\gamma$-ray astronomy 

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#### Abstract

Schwarzschild-type aplanatic telescopes with two aspheric mirrors, configured to correct spherical and coma aberrations, are considered for application in $\gamma$-ray astronomy utilizing the groundbased atmospheric Cherenkov technique. We use analytical descriptions for the figures of primary and secondary mirrors and, by means of numerical ray-tracing, we find telescope configurations which minimize astigmatism and maximize effective light collecting area. It is shown that unlike the traditional prime-focus Davies-Cotton design, such telescopes provide a solution for wide field of view $\gamma$-ray observations. The designs are isochronous, can be optimized to have no vignetting across the field, and allow for significant reduction of the plate scale, making them compatible with finely-pixilated cameras, which can be constructed from modern, cost-effective image sensors such as multi-anode PMTs, SiPMs, or image intensifiers.


## Introduction

All present-day atmospheric Cherenkov telescopes (ACTs) have prime-focus optical systems (OS) with Davies-Cotton or segmented parabolic primary mirror surfaces. Although these designs have many benefits and have proved to be reliable in ACT applications, they may be incompatible with the demanding requirements of nextgeneration large-area arrays of ACTs, which are being planned in Europe (Cherenkov Telescope Array, CTA), and in the U.S. (Advanced Gammaray Imaging System, AGIS). The desire to significantly improve the angular resolution, simultaneously increase the field of view, and reduce the focal plane scale of the telescopes, for compatibility with highly integrated, multi-pixel photon detectors, motivates research of alternative designs for ACTs.
Since comatic aberration is the factor limiting the imaging quality of existing ACTs, we consider two-mirror, aplanatic optical systems which are free from both coma and spherical aberrations. A subset of these two-mirror systems with fast optics, those with considerably decreased focal length, are well-suited to the goals of designing wide-field optics and reducing plate scale. Aplanatic op-
tical systems were first systematically studied in the classical 1905 paper by Karl Schwarzschild, in which he described a telescope (shown in figure 1) with two concave mirrors in which the secondary is placed between the primary and its focus, so that it de-magnifies the image at the focal plane [1]. This original telescope design suffered from significant astigmatism, due in part to the additional requirement of having a flat focal plane that Schwarzschild imposed. In 1926 Andre Couder derived the mirror surfaces to the third order in the square of the running radius and showed that astigmatism could be drastically reduced if the primary and secondary mirrors are separated by a distance equal to twice the equivalent focal length of the telescope, and a curved, convex focal plane is introduced [2]. The Schwarzschild-Couder (S-C) telescope has been only infrequently studied since [3, 4, 5], and has never been built (although construction near Paris was initiated by Couder during the 1930s) due to the sensitivity of its optical performance on the accurate figuring of the aspheric mirrors and on the exact values of the optical system parameters, $\alpha$ and $q$ (see definitions on figure 1).
Recently, we attempted to adapt the S-C configuration for wide-field or small plate scale ACT application [6]. In that study the figures of both aspheric


Figure 1: Schematic of Schwarzschild-Couder two-mirror optical system.
mirrors were completely constrained by the relative positioning of the mirrors and the focal plane, and by the requirements to eliminate on-axis spherical aberrations and to satisfy the intrinsic symmetry of aplanatic systems given by the "Abbe sine" condition. An analytic solution for both aspheric mirrors, in parametric form, surprisingly exists for all arbitrary two-mirror optical systems characterized by $(q, \alpha)$, and was first discovered and described by Linden-Bell in an elegant paper [7].

## Configurations

Figure 2 shows the PSF as a function of parameters $\alpha$ and $q$ for systems with $1,2,6,8,10$, and 14 degrees un-vignetted fields of view. The lines show OSs with minimal astigmatism for a given value of $\alpha$. The dashed line indicates configurations with unvignetted FoV less than $2^{\circ}$; the Couder telescope is shown with symbol "C". Optimal OSs for FoV in the range $2^{\circ}$ to $\sim 12^{\circ}$ are shown with a solid line; symbol " $X$ " denotes our choice for ACT applications. The dot-dashed line illustrates OSs with minimal astigmatism for which the unvignetted FoV exceeds $12^{\circ}$; the original solution proposed by Schwarzschild is denoted as " S ".
Figures 3 and 4 show the effective telescope light collecting area and curvature of the focal plane as a function of parameters $\alpha$ and $q$ for 6,8 , and 10 degrees unvignetted FoV. The solutions with minimal astigmatism are indicated as lines. The focal length of all OSs was chosen to be 500 cm . The Schwarzschild solution, ("S"), was originally proposed as the design with minimum curvature of the
focal plane. It is evident that both the telescope effective light collecting area, and the radius of curvature of the focal plane are effectively one dimensional functions of the combination of $\alpha$ and $q$ parameters, since the contour isolines are almost parallel. The light collecting area and maximal incident angle of photons onto the focal plane are determined predominately by the value of $\alpha$.
The characteristics of OSs optimized for use in ACTs, and which have no vignetting within the central $6^{\circ}$ of their FoV, are shown in figure 5. All systems have focal length of $\mathrm{F}=500 \mathrm{~cm}$, which can be rescaled as required, for example to give a certain light collecting area, camera plate scale, or maximal angle of incidence of rays onto the focal plane. The PSF at a field angle of $3^{\circ}$ and curvature of the focal plane are shown on the right figures, as a function of $\alpha$.

The effective collecting area and PSF for our preferred OS (with $\alpha=0.7, q=0.606$, which is marked on all figures with the symbol " X ", are shown in figure 6 as a function of field angle.

## Conclusions

Schwarzschild-Couder aplanatic optical system provides a viable solution for ACTs designed to have small plate scale or to conduct wide field of view $\gamma$-ray observations. The designs are isochronous and can be optimized to have no vignetting across the field. The significantly reduced plate scale makes them compatible with finelypixilated cameras, which can be constructed from modern, potentially cost-effective image sensors


Figure 2: $\mathrm{PSF}=2 \times \max \left\{\mathrm{RMS}_{\text {sagittal }}, \mathrm{RMS}_{\text {tangential }}\right\}$, as a function of parameters $\alpha$ and $q$ for systems with different fields of view.




Figure 3: Effective light collection area as a function of parameters $\alpha$ and $q$ for systems with different fields of view.
such as multi-anode PMTs, SiPMs, or image intensifiers.

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Figure 4: Focal plane curvature as a function of parameters $\alpha$ and $q$ for systems with different fields of view.


Figure 5: The characteristics of OSs optimized for use in ACTs.


Figure 6: Effective collecting area and PSF for our preferred OS.

