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Calculation of the Underground Muon Intensity Crouch Curve from a Parameterization of the Flux at Surface

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Abstract: Utilizing only the vertical muon intensity of the Gaisser parameterization of the muon flux at the surface and propagating this energy spectrum underground according to statistical ionization and radiative energy losses, it is possible to calculate the underground muon intensity Crouch curve. In addition, the primary spectral index of the Gaisser parameterization can be adjusted from $E^{-2.7}$ to $E^{-2.643}$ simply by minimizing the deviation from the Crouch curve. For chemical compositions other than standard rock, the propagation of the spectrum underground can be repeated with a different muon energy loss in the material. The resulting underground muon intensity curve represents a consistent conversion of the Crouch curve to the local rock, fully accounting for the energy dependence of the muon dE/dx.

Gaisser Parameterization of Muon Flux at Surface

The Gaisser parameterization of the cosmic induced muon flux at surface is an approximate extrapolation formula valid when muon decay is negligible ($E_\mu > 100/\cos\theta$ GeV) and the underground detector zenith angle θ can be assumed identical to the production angle in the upper atmosphere ($\theta < 70°$). Not including small contributions from charm and heavier flavors, which are negligible for energies below $10\,$TeV, the Gaisser parameterization is [1][2],

$$\frac{dN_\mu}{dE_\mu} \approx \frac{0.14 \cdot E_\mu^{-2.7}}{cm^2\, s\, sr\, GeV} \times \left(\frac{1}{1 + \frac{1.1\, E_\mu \cos\theta}{115\, GeV}} + \frac{0.054}{1 + \frac{1.1\, E_\mu \cos\theta}{850\, GeV}} \right) \qquad (1)$$

where the two terms give the contribution of pions and charged kaons. The energy spectrum steepens gradually to reflect the primary spectrum in the $10 - 100$ GeV range, and steepens further at higher energies because pions with $E_\pi >$ critical energy $\epsilon_\pi \approx 115\,$GeV tend to interact in the atmosphere before they decay. Asymptotically ($E_\mu \gg 1\,$TeV), the energy spectrum of atmospheric muons is one power steeper ($E^{-3.7}$) than the primary spectrum ($E^{-2.7}$). For the following calculations we only consider the vertical muon intensity for which $\cos\theta = 1$.

Muon Range Underground

The statistical energy loss of muons, traversing an amount X of matter, with energies far above the Bethe-Bloch minimum is given as

$$-\frac{dE_\mu}{dX} = a(E_\mu) + \sum_{n=1}^{3} b_n(E_\mu) \cdot E_\mu, \qquad (2)$$

where a is the collisional term (i.e. ionization mostly due to delta-ray production) and the second term accounts for the three radiative muon energy loss processes: 1. Bremsstrahlung, 2. pair production and 3. photonuclear interactions. In *Table* 1 these energy loss parameters are listed for standard rock. The critical energy where ionization losses equal radiative losses in standard rock is approximately $0.6\,$TeV. As the energy dependencies of the a and b parameters are relative mild, they are often assumed to be constant and the then resulting simplified differential equation can be easily solved by using an exponential function [2]. However, for our purpose of precisely determining the average muon range underground for each value of surface energy, the a and Σb values are parameterized with the following functions

$$a(E_\mu) = A_0 + (A_1 \cdot \log_{10} E_\mu[GeV]) \quad (3)$$

$$\Sigma b(E_\mu) = B_0 + (B_1 \cdot \log_{10} E_\mu[GeV]) + \{B_2 \cdot (\log_{10} E_\mu[GeV])^2\}. \quad (4)$$

Table 2 lists the fitted coefficients for standard rock. In addition, a Geant4 [3] based simulation is performed, tracking muons through a block of standard rock, defined as a mixture of $CaCO_3$ and $MgCO_3$ (with a density of $2.65\,g/cm^3$ and mass fractions of 52% O, 27% Ca, 12% C and 9% Mg). The comparison for the simulated Σb parameter as a function of energy and the parameterization in Eq. 4 validates that the fractional difference is less than 3 % over the entire energy range.

We used Eq. 2 to numerically compute the propagation of the muon by stepping ($\Delta X = 1\,g/cm^2$) through standard rock. Thus, for each initial value of muon energy, we determined, at what value of depth in meter-water-equivalent the muon ranges out. *Fig.* 1 shows the computed muon range in mwe of standard rock as a function of muon surface energy from 10 GeV to 10 TeV. Finally, the muon range X in standard rock can be simply parameterized as

$$X[mwe] = p_0 \cdot \log_e\{(p_1 \cdot E_\mu[GeV]) + p_2\}, \quad (5)$$

with the fitted parameters listed in *Table* 3.

The same computation can be done for overburdens made up of other rock compositions like e.g. for more dense rock ($\rho \approx 2.85\,g/cm^3$) at the Soudan iron mine in northern Minnesota, US (*c.f.* again *Fig.* 1 and *Table* 3). In order to easily transform the muon energy loss parameters a_s and b_s in standard rock (average nuclear properties $\overline{Z_s} = 11$, $\overline{A_s} = 22$ [2]) to a' and b' in other rock (e.g. Soudan $\overline{Z'} = 12$, $\overline{A'} = 24$ [4]), the a and total b parameter at a given muon energy can be scaled as

$$a'(E_\mu) = \frac{\overline{Z'/A'}}{\overline{Z_s/A_s}} \cdot a_s(E_\mu) \quad (6)$$

$$\Sigma b'(E_\mu) = \left(\frac{\overline{Z'^2/A'}}{\overline{{Z_s}^2/A_s}} \cdot 0.9 + 0.1 \right) \cdot \Sigma b_s(E_\mu) \quad (7)$$

The muon energy loss from Bremsstrahlung and pair production can be scaled with $\overline{Z^2/A}$ to the first order [4], whereas the simulated contribution from photonuclear interactions to dE/dX has only a very weak dependence on the nuclear properties in rock and is herein assumed constant per mwe. Furthermore, the MC simulation shows that in the region of interest from 250 GeV to 10 TeV the photonuclear interactions account for a constant fraction (10 %) of all radiative muon energy losses in standard rock. The scaled a and Σb values can be parameterized again with the functions in Eq. 3, 4 and the average muon range underground for each value of surface energy can be computed again in the same procedure as described above (*c.f.* again *Table* 2 and 3).

E_μ $[GeV]$	a_{ion} $[MeV\,cm^2/g]$	b_{brems}	b_{pair}	b_{DIS}	Σb
		$[10^{-6}\,cm^2/g]$			
10	2.17	0.70	0.70	0.50	1.90
10^2	2.44	1.10	1.53	0.41	3.04
10^3	2.68	1.44	2.07	0.41	3.92
10^4	2.93	1.62	2.27	0.46	4.35

Table 1: Average muon energy loss parameters calculated for standard rock [5][2]

	A_0	A_1	B_0	B_1	B_2
	$[MeV\,cm^2/g]$		$[10^{-6}\,cm^2/g]$		
standard rock	1.925	0.252	0.358	1.711	-0.178
Soudan rock	1.925	0.252	0.393	1.878	-0.195

Table 2: Fitted coefficients for the parameterizations of the ionization (Eq. 3) and the total radiative (Eq. 4) average muon energy loss in standard and Soudan rock.

	p_0 $[mwe]$	p_1 $[GeV^{-1}]$	p_2 [ ]
standard rock	2298.2	0.001920	0.99809
Soudan rock	2098.9	0.002119	0.99789

Table 3: Fitted coefficients for the parameterization (Eq. 5) of the average muon range in standard and Soudan rock.

Calculating the Muon Intensity Underground

Together with Eq. 1, which is the Gaisser parameterization of the differential intensity of vertical muons at the surface ($\cos\theta = 1$), and Eq.

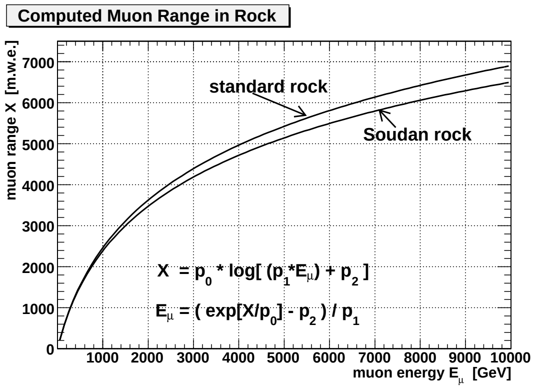

Figure 1: Calculated average muon range in standard and Soudan rock as a function of initial muon energy at the surface.

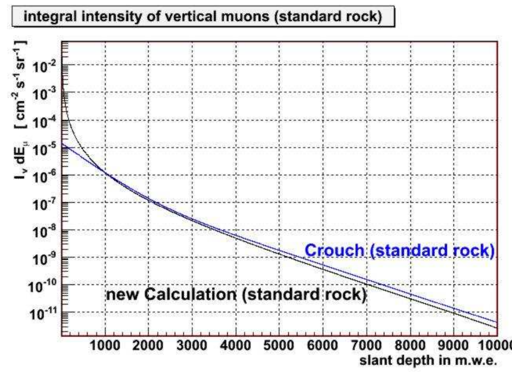

Figure 2: Calculated integral vertical muon intensity underground as a function of slant depth in standard rock in comparison with the Crouch function [6].

5, which relates the average muon range underground to a given value of surface energy, it is now very simple to compute the intensity underground as a function of slant depth in standard rock. For this, the differential intensity of vertical muons at the surface (Eq. 1 with $\cos\theta = 1$) is stepwise added ($\Delta E_\mu = 10\,MeV$) from 100 TeV down to 1 GeV. At each step the average muon range X in mwe is calculated with Eq. 5 and assigned to the interim value of the intensity sum. Thus, the integral vertical muon intensity underground as a function of slant depth in standard rock can be efficiently computed from $10000\,mwe$ up to the surface. *Fig.* 2 depicts the result in comparison with the Crouch curve, which is the parameterized 'world average' of deeply underground measured ($> 1000\,mwe$) vertical muon intensities; compiled by Crouch and referring to standard rock [6]. The result of our calculation for standard rock agrees already well with the underground measured Crouch curve. Both the intensity values and the spectral shape of the exponentially falling off Crouch function are well reproduced. At the small slant depth value of $1000\,mwe$, where underground measurements start to be considered in the Crouch fit, both curves match very well, whereas for larger values of slant depth the new calculation tends to fall off slightly faster.

Refining the Gaisser Parameterization by Comparison with Crouch Curve

We tried to further improve the agreement between the calculation and the Crouch curve by refining the primary spectral index (γ) and the absolute normalization (c) of the Gaisser parameterization of the muon flux at the surface (substitute $E^{-2.7}$ with $c \cdot E^{\gamma}$ in Eq. 1). In a Newtonian iteration procedure, the two variables γ and c were varied before each new calculation, such that the new values at $\sim\ 1500\,mwe$ and $\sim\ 9500\,mwe$ are in optimum agreement with Crouch and the amplitude of the fractional differences for slant depth values in between is minimal (*c.f. Fig.* 3). The best fit values obtained by this procedure are $\gamma_{fit} = -2.643$ for the primary spectral index and $c_{norm} = 80.5\,\%$ for the normalization factor (*c.f. Fig.* 4, 2). We found the maximum fractional difference between the optimized calculation and the Crouch curve to be $6\,\%$, which is of the order of the uncertainty on the measured Crouch curve.

Discussion

The fact that the parameterized muon flux at the surface in combination with the propagated muon energy loss in the rock reproduces the 'world average' Crouch curve implies that the underlying physics, as described in this paper, is well understood. Our optimized Gaisser parameteriza-

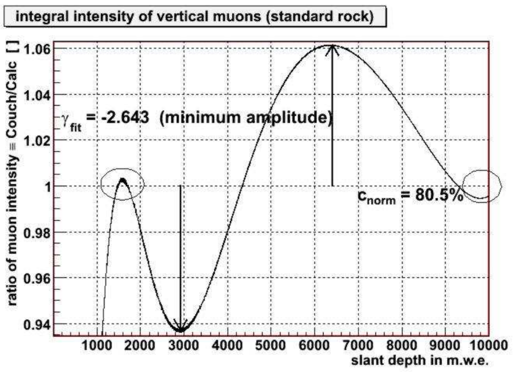

Figure 3: Fractional difference between the integral vertical muon intensity underground according to the Crouch curve and the optimized calculation for slant depth values in standard rock.

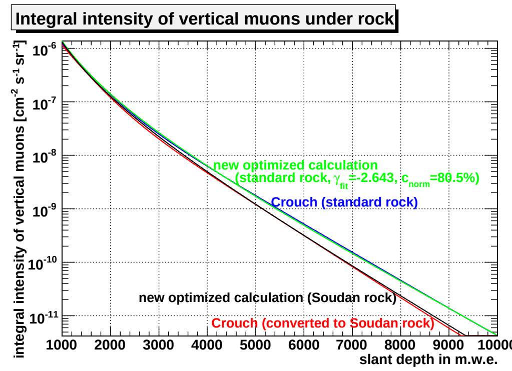

Figure 4: Integral vertical muon intensity underground according to the Crouch curve (blue) and the optimized calculations for slant depth values in standard (green) and Soudan rock (black), as well as an approximate conversion (red) of the Crouch curve to Soudan rock (constant muon energy loss parameters a and b at 1 TeV).

tion, with an absolute value of the primary spectral index of slightly less than 2.7 and a $\sim 20\,\%$ lower muon intensity at surface, is in reasonable agreement with experimental data [2]. Crouch [6] converted the underground muon intensity data to depth values in standard rock utilizing an approximate mathematical solution of the differential Eq. 2 assuming the a and b parameters to be constant [2], which becomes less valid at increasing depths. It is possible now to perform the herein described calculation in other rock than standard rock, yielding new underground muon intensity curves for the local rock composition.

Summary

Taking the vertical muon intensity of the refined Gaisser parameterization of the muon flux at the surface ($80.5\% \cdot E^{-2.643}$ instead of $E^{-2.7}$) and propagating this energy spectrum underground according to statistical ionization and radiative energy losses yields a good fit to the underground muon intensity Crouch curve. The obtained agreement is for most part better than the uncertainty of $\sim 5\,\%$ associated with the Crouch function. For chemical compositions other than standard rock, a consistent computation of the underground muon intensity curve can be repeated, fully accounting for the energy dependence of the muon dE/dx in the local rock. This can yield a better determination of a map of the overburden of an underground muon detector, by normalizing the measured intensity to the computed distribution as a function of slant depth in the local rock. Furthermore, the methods described herein also allow for a more precise extrapolation of the underground muon energy back to the surface.

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