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Differences in dE/dX for $\mu+$ and $\mu-$ and its Effect on the Underground Charge Ratio

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Abstract: Theoretical calculations predict a small fractional difference in energy loss for μ^+ and μ^- of the order of $0.15\,\%$ at high energies. This is predominantly due to a z^3 term in an extended ionization dE/dX relation, in analogy to the Barkas effect at low energies around the Bethe-Bloch maximum. The atmospheric muon energy spectrum is steeply falling off with approximately $E^{-3.7}$ and thus the small difference in dE/dX between μ^+ and μ^- at high energies results in an amplified charge asymmetry of about $0.6\,\%$ a few thousand meters water equivalent deep underground.

Introduction

The MINOS far detector is the first large underground experiment with a magnet that can measure the ratio of μ^+ to μ^- with high precision.[1, 2]. A precise measurement of the charge ratio can then be used to ascertain special properties of the cosmic ray showers, such as the π^+/π^- ratio and the K^+/K^- ratio[3]. The survival probability for muons to reach an underground detector depends on the energy loss, so if there is any difference in energy loss between μ^+ and μ^-, that would affect the measured charge ratio. The statistical error on the MINOS measurement is remarkably small, and MINOS reports:

$$r = \frac{N(\mu^+)}{N(\mu^-)} = 1.374 \pm 0.003(stat)^{+0.012}_{-0.010}(sys) \quad (1)$$

for surface muons with an energy near 1 TeV or higher. Thus, even very small differences in the energy loss between μ^+ and μ^- could be important in the interpretation of these measurements.

The statistical energy loss of muons, traversing an amount X of matter in g/cm^2, with energies far above the Bethe-Bloch minimum is usually parameterized as

$$-\frac{dE_\mu}{dX} = a(E_\mu) + \sum_{n=1}^{3} b_n(E_\mu) \cdot E_\mu, \quad (2)$$

where a is the collisional term (i.e. ionization, mostly due to delta-ray production) and b in the second term accounts for the three radiative muon energy loss processes: 1. Bremsstrahlung and 2. pair production, as well as 3. photonuclear interactions. In *Table* 1 these energy loss parameters are listed for standard rock. The critical energy where ionization losses equal radiative losses in standard rock is approximately 0.6 TeV. The average muon energy for a muon which reaches the depth of MINOS is greater than 1 TeV, so the b term and its energy dependence are important in calculating the energy loss. This paper focuses on the (small) differences in the a and b terms for μ^+ and μ^-.

E_μ [GeV]	a_{ion} [MeV cm²/g]	b_{brems}	b_{pair}	b_{DIS}	Σb
		$[10^{-6}\ cm^2/g]$			
10	2.17	0.70	0.70	0.50	1.90
10^2	2.44	1.10	1.53	0.41	3.04
10^3	2.68	1.44	2.07	0.41	3.92
10^4	2.93	1.62	2.27	0.46	4.35

Table 1: Average muon energy loss parameters calculated for standard rock [4][5]

Calculated Difference in Ionization dE/dX for μ^+ and μ^-

At low energies, around the Bethe-Bloch maximum, the difference in ionization energy loss is known as the Barkas effect[6], and there have been efforts to both measure and calculate those differences[7]. Calculations show that negative particles lose energy at a slower rate, with the difference dropping from tens of percent at MeV energies to about $0.3\,\%$ in the GeV range. Such differences were experimentally verified both at MeV energies[8, 9, 10] and in the GeV range [11]. At higher energies, this difference in ionization energy loss has usually been neglected, and we are not aware of any measurements. As described in Reference [12], the usual ionization energy loss term for muons (of either sign) depends on z^2, and the difference between μ^+ and μ^- arises from a small additional z^3 correction term. This correction term in dE/dx is:

$$\left(\frac{dE}{dX}\right)^{corr}_{ion} = \frac{\pi\alpha z^3 0.307 Z}{2\beta A} \quad [MeV\, cm^2\, g^{-1}]$$

(3)

where α is the fine structure constant, z is the charge, β is the relativistic velocity, and Z and A are the nuclear properties of the material through which the muon is passing. The absolute value of the difference in ionization energy loss between positive and negative muons in standard rock[5] is plotted in Figure 1. It is fairly constant above $10\,$GeV, at a value corresponding to approximately $0.15\,\%$ of the mean energy loss in the ionization dominated energy regime (*c.f.* Table 1).

Calculated Difference in Bremsstrahlung dE/dX for μ^+ and μ^-

Above an energy near $0.6\,$TeV in standard rock, radiative energy loss becomes comparable to ionization energy loss, and continues to grow at higher muon energies. From Reference [13], the fractional difference in Bremsstrahlung energy loss between positive and negative muons is

$$\frac{[\frac{dE}{dX}]^{\mu^+}_{brems} - [\frac{dE}{dX}]^{\mu^-}_{brems}}{[\frac{dE}{dX}]_{brems}} = \frac{8Z\alpha}{\gamma}$$
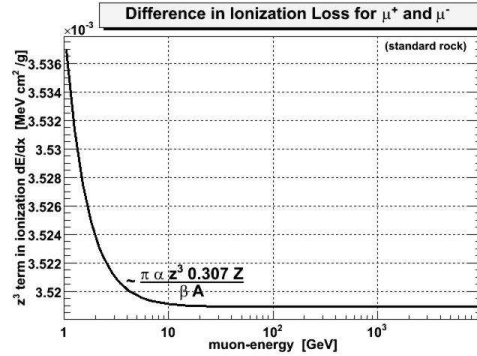
(4)

Figure 1: Calculated difference in ionization energy loss between positive and negative muons in standard rock (average nuclear properties: $\overline{Z} = 11$, $\overline{A} = 22$ [5]).
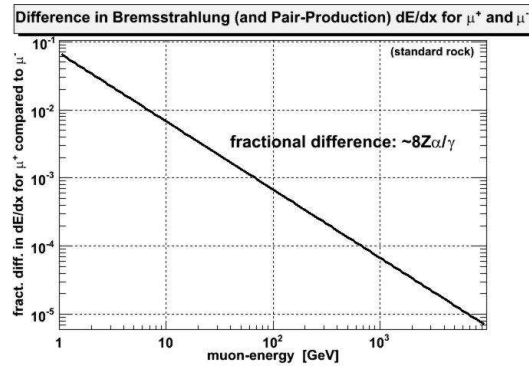

Figure 2: Calculated fractional difference in Bremsstrahlung energy loss between positive and negative muons.

where γ is the Lorentz factor of the muon. Again, the μ^+ has a slightly higher energy loss. This fractional difference *decreases* with energy and is already negligible where radiative energy losses become important. This fractional difference is plotted in Figure 2 for muons in standard rock. Presumably, the same fractional difference can also be assigned for pair-production, as the underlying process is a two-photon exchange between the muon and the constituents of the nucleus, and thus the cross sections for μ^+ and μ^- should scale in the same way as for Bremsstrahlung.

Muon Range Underground for μ^+ and μ^-

Taking the vertical muon intensity from an optimized Gaisser parameterization of the muon flux at the surface and propagating this energy spectrum underground according to statistical ionization and radiative energy losses, it is possible to precisely calculate the underground muon intensity. This procedure is described in detail in [14] for overburdens of standard and Soudan rock (MINOS). First, the average muon range underground, for each value of surface energy, has to be precisely computed. For this, the energy dependent a and Σb values are parameterized for standard and Soudan rock as in [14]. The additional ionization loss according to Eq. 3 is then added for μ^+ to the value of the function for a (subtracted for μ^-). Conservatively, $90\,\%$ of the value of the function for Σb (the total radiative losses), are scaled with the energy dependent fractional difference $R = 8Z\alpha/\gamma$ according to Eq. 4 as $\Sigma b^{\pm} = 0.9 \cdot \Sigma b \cdot (1 \pm R/2)$ for μ^+ and μ^-, respectively. The contribution from photonuclear production (DIS), which accounts for a constant fraction of $10\,\%$ of all radiative muon energy losses in standard rock (in the region of interest from $250\,\mathrm{GeV}$ to $10\,\mathrm{TeV}$) is not scaled with the fractional difference. According to differential Eq. 2 the propagation of the muon energy is then numerically computed, separately for μ^+ and μ^- (in standard and Soudan rock, respectively). Thus, for each initial value of muon energy, the slant depth in meter-water-equivalent where the muons of different charge range out is determined.

Muon Charge Asymmetry from Ratio of Intensities Underground

Using the average muon range underground, calculated for positive and negative muons in rock as described in the last section, and an optimized Gaisser parameterization of the differential intensity of vertical muons at the surface, we have computed the corresponding underground intensities of positive and negative muons as a function of slant depth for a given rock composition. The resulting ratio of the μ^+ and μ^- intensity curves is shown in Figure 3 for Soudan rock. The upper curve corre-

sponds to the fractional difference in integral intensities of μ^+ and μ^- at a given slant depth. For slant depth values above about $1000\,mwe$ the underground ratio $N(\mu^+)/N(\mu^-)$ is lowered by roughly $0.4\,\%$. However, since the charge of only the lower energy muons can be identified in a magnetic detector, owing to its maximum detectable momentum [15], the detected intensity corresponds to the charge ratio of the muons at depth below some momentum. The lower curve in Figure 3 depicts the fractional difference in intensity for underground muon momenta below $250\,\mathrm{GeV/c}$, corresponding to the approximate maximum detectable momentum of MINOS. For increasing slant depth values the measured underground ratio $N(\mu^+)/N(\mu^-)$ is further reduced and saturates at about $0.6\,\%$ below its surface value for slant depths larger than roughly $5000\,mwe$.

The dominant 0.15% difference in ionization energy loss between μ^+ and μ^- gets amplified by a factor of about 3.7, due to the approximate $E^{-3.7}$ dependence of the differential muon spectrum. The impact of the rock composition is almost negligible, as the induced muon charge asymmetry under Soudan rock lowers the surface value of the ratio by an additional amount less than $0.02\,\%$ compared to standard rock.

Summary

There is a small fractional difference in energy loss for μ^+ and μ^- of the order of $0.15\,\%$ predicted by theoretical calculations at high energies, predominantly due to a z^3 correction term in the ionization energy loss. This causes that measurements of the atmospheric muon charge ratio $N(\mu^+)/N(\mu^-)$ deep underground (e.g. with the MINOS detector), to observe a slightly lower ratio than at the surface. Moreover, as the atmospheric muon energy spectrum is steeply falling off with approximately $E^{-3.7}$, the small difference in energy loss between μ^+ and μ^- at high energies results in an amplified charge asymmetry of about $0.6\,\%$ several thousand meters water equivalent deep underground. The calculations presented herein allow for a correction of the underground measured muon charge ratio to its surface value.
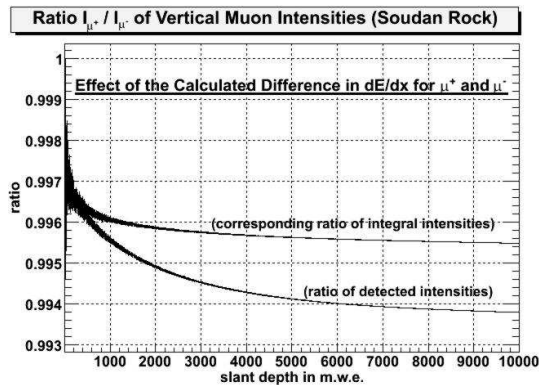
Figure 3: Calculated ratio of positive to negative vertical muon intensities in Soudan rock as a function of slant depth. The upper curve is for all muons, the lower curve is for muons with a remnant momentum of less than 250 GeV/c ($\approx$ the maximum detectable momentum in the MINOS far detector).

Acknowledgments

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