Abstract: The AMANDA-II neutrino telescope detects upward-going atmospheric muon neutrinos penetrating the Earth from the Northern Hemisphere via the Cherenkov light of neutrino-induced muons, allowing the reconstruction of the original neutrino direction. Due to the high energy threshold of about 50 GeV, the declination spectrum is minimally affected by standard neutrino oscillations; however, alternative oscillation models predicting subdominant effects can be tested and constrained. Of particular interest are models that allow one to test Lorentz invariance and the equivalence principle. Using the AMANDA-II data from the years 2000 to 2003, a sample of 3401 candidate neutrino-induced events was selected. No indication for alternative oscillation effects was found. For maximal mixing angles, an upper limit is set on both the Lorentz violation parameter $\delta c/c$ and the equivalence principle violation parameter $2|\phi| \delta \gamma$ of $5.3 \times 10^{-27}$ at the 90% confidence level.

Introduction and detector description

Cosmic ray particles entering the Earth’s atmosphere generate a steady flux of secondary particles, including muons and neutrinos. High energy muons pass through the atmosphere and can penetrate several kilometers of ice and rock, while atmospheric neutrinos of energies only above roughly 40 TeV start to be absorbed in the Earth. Lower energy muon neutrinos penetrating the diameter of the Earth can oscillate into tau neutrinos. However, the oscillation maxima at 30 GeV [1] and below are beneath the AMANDA-II threshold. Departures from conventional mass-induced oscillations could emerge at higher neutrino energies due to relativity-violating effects (see below). Such mechanisms would distort the expected angular distribution and energy spectrum of atmospheric neutrinos and could be detectable by AMANDA-II.

The AMANDA-II neutrino telescope is embedded 1500 – 2000 m deep in the transparent and inert ice of the Antarctic ice sheet, close to the geographic South Pole. AMANDA-II consists of 677 optical modules (OMs) on 19 vertical strings, which are arranged in three approximately concentric circles of 60 m, 120 m and 200 m diameter. Muons produced in $\nu_\mu$-nucleon interactions can be directionally reconstructed by observing the Cherenkov radiation that propagates through the ice to the array of photosensors. To ensure that the observed muon is due to a neutrino interaction, the Earth is used as a filter against atmospheric muons, and only tracks from the Northern Hemisphere (declination $\delta > 0^\circ$) are selected.

Phenomenology of standard and alternative neutrino oscillations

It is commonly accepted that standard (mass-induced) $\nu_\mu \rightarrow \nu_\tau$ oscillations\(^1\) are responsible for the measured deficit of atmospheric muon neutrinos (see e.g. [2]). Atmospheric neutrino data can also be used to test non-standard oscillation mechanisms that lead to observable differences at higher

\(^1\) In the regime of atmospheric neutrino oscillations, it suffices to consider a two-flavor system of eigenstates ($\nu_\mu$, $\nu_\tau$).
neutrino energies. Various new physics scenarios can result in neutrino flavor mixing. Two of these scenarios, which can be described in a mathematically analogous way, have been tested in this analysis. The underlying theories assume small deviations from the principles of the theory of relativity and lead to measurable neutrino oscillations:

- In theories predicting violation of Lorentz invariance (VLI), a set of additional neutrino eigenstates with different maximal attainable velocities (MAV) $c_n/c$ is introduced, violating special relativity [2].
- In theories predicting violation of the weak equivalence principle (VEP), gravitational neutrino eigenstates are introduced which couple with distinct strengths $\gamma_n$ to a gravitational potential $\phi$, conflicting with the universal coupling assumed in general relativity [3, 4].

The main difference between these oscillation scenarios and standard oscillations is the linear energy dependence of the oscillation frequency, shifting observable oscillation effects into the energy range of AMANDA-II. For the sake of simplicity, we will focus on the VLI scenario. As both theories are mathematically equivalent, the results can be transferred to the VEP case by simply exchanging the relativity-violating oscillation parameters $\delta c/c \rightarrow 2|\phi|\delta\gamma$ and mixing angles $\Theta_c \rightarrow \Theta_\gamma$.

Combining standard and VLI oscillations, one obtains three systems of neutrino eigenstates (flavor, mass, and MAV eigenstates), resulting in a total of 5 oscillation parameters: the mass-squared difference $\Delta m^2$, two mixing angles $\Theta_m$ and $\Theta_c$, the VLI parameter $\delta c/c$, and a complex phase $\eta$. Fixing $\Delta m^2 = 2.3 \times 10^{-3}$ eV$^2$ and $\Theta_m = 45^\circ$, the survival probability may then be written as:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\Theta \sin^2 (\Omega L) \quad (1)$$

$$2\Theta = \arctan (s/t) \quad \Omega = \sqrt{s^2 + t^2} \quad (2)$$

$$s = 2.92 \times 10^{-3} \left| 1/E_\nu + 8.70 \times 10^{20} \delta c/c \sin 2\Theta_c E_\nu e^{i\eta} \right|,$$

$$t = 2.54 \times 10^{18} \delta c/c \cos 2\Theta_c E_\nu \ . \quad (3)$$

Here the the muon neutrino path length $L$ is expressed in km and the neutrino energy $E_\nu$ in GeV. For the given set of parameters, one can observe a significant effect within the analyzed energy range (100 GeV – 10 TeV) and declination range ($\delta \geq 20^\circ$), for certain values of $\Theta_c$ and $\delta c/c$.

**Data selection**

The data analyzed in this analysis are selected from 7.9 $\times$ 10$^9$ events recorded from 2000 to 2003. Detector signals are recorded when 24 or more OMs report signals within a sliding window of 2.5 $\mu$s. Signals from unstable OMs, electronic and OM noise or cross-talk, as well as hits due to uncorrelated muons coincident within the trigger time, are rejected. Also, data periods with reduced data quality are discarded, corresponding to 87.8 days. The 17.3% deadtime of the data acquisition system results in a total livetime of 807.2 days used for the analysis.

The events are processed with a fast pattern recognition algorithm (A) to select tracks that are likely to be upgoing ($\delta_A > -20^\circ$). The calculated track direction serves as a first guess for 16-fold iterative maximum likelihood reconstruction algorithms (B), restricted to upgoing tracks with $\delta > 0^\circ$. The alternative hypothesis of a downgoing track is tested with a two-fold iterative fit requiring $\delta < -10^\circ$. In order to reduce the probability of wrongly reconstructed tracks due to spurious hits, both fits are repeated after rejecting hits with timing residuals larger than two standard deviations. Background rejection and angular resolution are further improved by a 10-fold iterative fit (C) incorporating the probabilities that modules registered hits for the given track. From an examination of the likelihood contours in declination and right ascension [5], an estimate of the median space angle resolution $\sigma_\phi$ is obtained for individual tracks. The following selection criteria are applied, with $L_{\text{diff}} \equiv \Delta \ln L$ being the difference of up- and downgoing likelihood minima: (1) declinations $\delta_A > -20^\circ$, $\delta_B > 0^\circ$ and $\delta c > 20^\circ$; (2) space angle differences $\Psi(A,B) < 30^\circ$, $\Psi(B,C) < 7.5^\circ$; (3) space angle resolutions $\sigma_\phi(B) < 6^\circ$ and $\sigma_\phi(C) < 3.0^\circ$; (4) likelihood difference $L_{\text{diff}}(B,C) > 32.5$. 

1296
The oscillation probability depends on the neutrino flight length (i.e. declination) and the neutrino energy. As an energy estimator, we use a correlated observable, the number of OMs triggered in an event ($N_{\text{ch}}$). Using Monte Carlo simulations, declination- and $N_{\text{ch}}$-dependent selection criteria have been developed by dividing the distribution of the angular resolution into equal declination and $N_{\text{ch}}$ bins. For each of these bins, a fixed, optimized percentage (8%) of the events with poor angular resolution is rejected. The same was done for the likelihood difference distributions. These criteria improved the efficiency of the data selection by 30% compared to simple selections in the angular resolution and the likelihood difference. The resulting number of selected neutrino candidate events is 3401. From a study of the distribution of space angle difference, the background of wrongly reconstructed atmospheric muons is estimated to be 4%.

A full simulation chain, including neutrino absorption in the Earth, neutral current regeneration, muon propagation, and detector response is used to simulate the response of AMANDA-II to atmospheric neutrinos [6, 7]. The expected atmospheric neutrino flux before oscillations is taken from Lipari [8].

**Analysis method and systematic errors**

The analysis method uses a $\chi^2$-test to compare the declination and $N_{\text{ch}}$ distributions of data with Monte Carlo simulations including VLI oscillation effects. The systematic uncertainties affecting the Monte Carlo prediction are integrated into the $\chi^2$ expression:

$$
\chi^2 \left( \delta c/c, \Theta_c, \cos \eta \right) = \sum_{i=1}^{N_{\text{Bins}}} \frac{\left( N^D_i - N^\text{BG}_i - F_i N^\text{MC}(\delta c/c, \Theta_c, \cos \eta) \right)^2}{N^D_i + N^\text{BG}_i + (F_i)^2} 
$$

$$
+ \left( \frac{\sigma_{\epsilon}}{\epsilon} \right)^2 + \left( \frac{\sigma_{\kappa}}{\kappa} \right)^2 + \left( \frac{\sigma_{\sigma}}{\sigma} \right)^2 ,
$$

where $N^x_i$ denotes the number events in bin $i$ and $x$ denotes data (D), background (BG) and Monte Carlo (MC). The function $F$ represents the product of functions $f_\alpha \cdot f_\kappa \cdot f_\epsilon$ which are defined as:

$$
f_\alpha = 1 + \alpha, \quad f_\kappa = c_i \cdot \kappa + 1, \quad f_\epsilon = 1 + 2 \epsilon \left( 0.5 - \sin \delta_i \right).
$$

$\alpha$ parametrizes the systematic uncertainty in the overall normalization due to uncertainties in the detector response and theoretical uncertainties of the atmospheric neutrino flux ($\sigma_\alpha = 30\%$). The uncertainty due to the relative production rate between kaons and pions, which affects the shape of the declination distribution, is parametrized by $\epsilon$ and is estimated as $\sigma_\epsilon = 6\%$ in total [9]. The uncertainty in the sensitivity of the optical modules is parametrized by $\kappa$ ($\kappa = 0$ for 100% sensitivity) and was measured to be $\sigma_\kappa = 11.5\%$. The function $f^i_\kappa$ was derived from the changes in the declination distribution generated by Monte Carlo distributions with different OM sensitivities. In order to determine the optimal number of declination and $N_{\text{ch}}$ bins and their optimal range, toy Monte Carlo samples of 10000 events have been generated reflecting the simulated flux and systematic uncertainties as assumed above. The mathematical properties of expression (4) were checked, and belts for 90%, 95% and 99% confidence level were derived from a high statistics toy Monte Carlo sample.

The results of the toy Monte Carlo studies favor an analysis using the following 4 bins: ($N_{\text{ch}} \leq 49$, $f^i_\epsilon$).
The exclusion regions for alternative oscillation effects are obtained by scanning through the oscillation parameter space. For each point $[\delta c/c, \sin(2\Theta_c), \cos(\eta)]$ the $\chi^2$ expression is minimized in the error variables $\alpha, \epsilon$ and $\kappa$.

**Results and Outlook**

The analysis of the final atmospheric neutrino sample finds no evidence for alternative oscillations, and a preliminary upper limit on the VLI parameter $\delta c/c$ is set of $5.3 \times 10^{-27}$ at the 90\% confidence level, for nearly maximal mixing angles $\Theta_c \approx \pm \pi/4$. The dependence on the unconstrained phase $\eta$ is found to be small (see figure 2); the most conservative limit is obtained for $\cos \eta = 0$. The limit can also be interpreted in the context of VEP theories, leading to an upper limit of $2|\phi|\delta \gamma \leq 5.3 \times 10^{-27}$. This result improves the limits obtained using data from Super-Kamiokande [10] and MACRO [11]. However, AMANDA is not sensitive to small mixing angles due to the systematic errors and its higher energy threshold. A likelihood analysis of the 2000-2005 AMANDA-II data sample is in progress, with improved systematic error estimation and increased sensitivity [12]. This analysis will also extend the technique to search for evidence of quantum decoherence resulting from interaction of neutrinos with the background space-time foam [13]. The next-generation IceCube detector, when completed in 2010, will be able to extend the sensitivity to VLI effects by about one order of magnitude [14].

**Acknowledgments**

J. A. thanks the German Research Foundation (DFG) and the German Federal Ministry of Education and Research (BMBF) for financial support.

**References**