



## Theoretical Uncertainty in the Tau Energy Loss

N. ARMESTO<sup>1</sup>, C. MERINO<sup>1</sup>, G. PARENTE<sup>1</sup>, E. ZAS.<sup>1</sup>

<sup>1</sup> *Departamento de Física de Partículas & Instituto Galego de Física de Altas Enerxías  
Universidade de Santiago de Compostela, 15786 Santiago de Compostela, Galicia, Spain  
gonzalo@fpaxpl.usc.es*

**Abstract:** We evaluate the tau lepton energy loss produced by photonuclear interactions at high energies by using different theoretical and phenomenological models. The theoretical uncertainty is estimated by taking different extrapolations of the DIS structure function  $F_2$  in the low and moderate  $Q^2$  range, at extremely low values of  $x$  where nuclear shadowing and parton saturation effects could be stronger than usually considered. For tau leptons of energy above  $E = 10^7$  GeV photonuclear interactions are the dominant energy loss mechanism which controls the effective volume for converting Earth-skimming tau neutrinos into tau leptons that exit the Earth.

## Introduction

The possibility to search for tau neutrinos by looking for tau leptons that exit the Earth, Earth-skimming neutrinos, has been shown to be particularly advantageous to detect neutrinos of energies in the EeV range [1, 2]. The sensitivity to tau neutrinos through this channel depends directly both on the charged current (CC) neutrino cross section and on the tau range, or equivalently the tau energy loss, which determine the amount of matter in which the neutrino can interact and still produce an emerging tau [3, 4].

While the energy loss for muons is shared by roughly equivalent contributions from pair production, bremsstrahlung, and photonuclear interactions, for tau leptons of energies  $E > 10^7$  GeV, photonuclear interactions are responsible for the largest and most uncertain contribution.

Both the neutrino cross section and the photonuclear energy loss are calculated from theory using structure functions which carry the information of the nuclear structure.

In order to study the uncertainties in the calculation of Earth-skimming neutrinos the same structure functions should be used for both processes. Unfortunately this is not possible since at EeV energies the kinematical  $Q^2$  (minus the momentum

transfer squared) and Bjorken- $x$  ranges that contribute for both processes are quite different and available parameterizations are not entirely adequate to describe both ranges simultaneously.

The  $Q^2$  scale that contributes to the tau energy loss, dominated by photon exchange, is low and moderate  $Q^2$  at very low  $x$ , where perturbative and non perturbative QCD effects are mixed. The CC neutrino-nucleon cross section is produced by  $W$ -boson exchange that sets the relevant scale of  $Q^2$  to values up to  $M_W^2$  at low  $x$ , a region where perturbative QCD is expected to work. In both cases the relevant  $x$  range lie well outside the regions where structure functions are measured (see Fig. 5) and rely on extrapolations.

In this article we study the tau energy loss in the frame of the more relevant models describing photonuclear processes, the Generalized Vector Dominance (GVD) Model and the Regge-like approach, and we cover the range of possible scenarios for the extrapolation of nuclear structure functions at low  $x$  and low  $Q^2$ .

Two important effects to be taken into account in this extrapolation of the structure functions are nuclear shadowing corrections and saturation due to partonic screening. Nuclear corrections are deviations from the naive picture in which the nucleus is treated as an incoherent sum of nucleons. Saturation accounts for the fact that the structure func-

tions cannot rise indefinitely as  $x$  goes to zero. A new calculation of the energy loss based on saturation physics [5] is also presented.

## Photonuclear energy loss

The contribution of photonuclear interactions to the average energy loss rate per unit depth  $X$  of muons or tau leptons,  $b(E)$ , is obtained by integration of the lepton nucleus differential cross section:

$$b(E) = -\frac{1}{E} \left\langle \frac{dE}{dX} \right\rangle = \frac{N_A}{A} \int dy y \int dQ^2 \frac{d\sigma^{lA}}{dQ^2 dy}, \quad (1)$$

where  $N_A$  is Avogadro's number,  $A$  the mass number, and  $y$  the fraction of energy lost by the lepton in the interaction. For the lepton-nucleus differential cross section we consider the expression for virtual photon exchange in terms of structure functions. The limits in the double integral of Eq. (1) are standard and well established and we have neglected the longitudinal structure function in the calculations.

The calculation of the photonuclear interaction cross section in the GVD Model [6] (BB) and in its extension to higher energies by including a perturbative component based on the color dipole model [7] (BS), has been widely used to explore muon and tau lepton propagation in matter (see for instance [8, 9, 10] and references therein).

In the calculations of the muon and/or the tau energy loss by photonuclear interactions in Ref. [11] (DRSS), in Ref. [12] (BM), and in Ref. [13] (KLS), the  $F_2$  structure function is given by a phenomenological parameterization of data based on Regge Theory. For the proton structure function,  $F_2^p$ , DRSS (see also Ref. [14]) uses the ALLM model [15] while BM and KLS both consider the CKMT model [16] at low  $Q^2$  matched at high  $Q^2$  to perturbative QCD predictions based on different parameterizations of parton distribution functions. The ALLM and CKMT  $F_2^p$  structure functions are shown in Fig.1 with the HERA data at the lowest measured  $x$  values at different  $Q^2$ .

In DRSS, BM, and KLA calculations the nuclear structure function is related to the proton structure function through  $F_2^A = f^A(A F_2^p)$ . At high energy only the low  $x$  behavior of the nuclear correction factor  $f^A$  is relevant to the calculation of

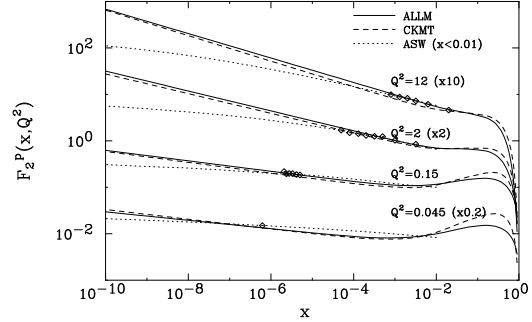


Figure 1: The proton structure function  $F_2$  vs  $x$  for different  $Q^2$  ( $\text{GeV}^2$ ). Data points are from HERA [17, 18].

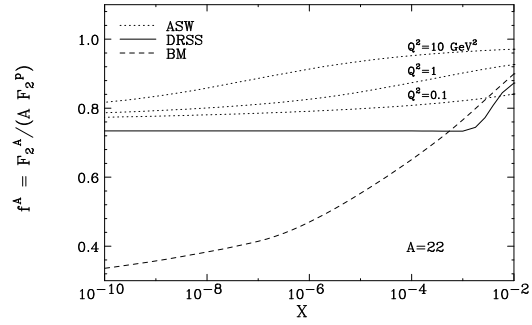


Figure 2: The nuclear correction factor  $f^A$

$b(E)$  (see Fig. 5 below). In the DRSS calculation the low  $x$  behavior of  $f^A$  freezes at the value  $f^A = A^{-0.1}$  for  $x < 0.0014$  ( $\sim 0.73$  for standard rock,  $A = 22$ ), while in the BM and KLS calculations  $f^A$  reaches a maximal asymptotic regime  $f^A = A^{-1/3}$  ( $\sim 0.36$  for  $A = 22$ ) at much lower  $x$  (see Fig. 2).

In addition to the existing calculations we present a new computation of the photonuclear tau energy loss using the results of Ref. [5] (ASW) which are based on the geometric scaling property [19] that all data on  $\sigma^{\gamma^*p}$  and on  $\sigma^{\gamma^*A}$  lie on a single universal curve in terms of the scaling variable  $\tau = Q^2/Q_{sat}^2$ . The functional shape of the curve is motivated in saturation physics and it is given by [5, 20]:

$$\sigma^{\gamma^*p}(x, Q^2) \equiv \Phi(\tau) = \bar{\sigma}_0 [\gamma_E + \Gamma(0, \xi) + \ln \xi], \quad (2)$$

with  $\gamma_E$  the Euler constant,  $\Gamma(0, \xi)$  the incomplete  $\Gamma$  function, and  $\xi = a/\tau^b$ , with  $a = 1.868$  and  $b = 0.746$  extracted from a fit to lepton-proton

data. The saturation scale  $Q_{\text{sat}}^2$  is parameterized as  $Q_{\text{sat}}^2(\text{GeV}^2) = (\bar{x}/x_0)^{-\lambda}$  [21], where  $x_0 = 3.04 \cdot 10^{-4}$ ,  $\lambda = 0.288$ , and  $\bar{x} = x(Q^2 + 4m_f^2)/Q^2$  with  $m_f = 0.14$  GeV. The normalization is fixed by  $\bar{\sigma}_0 = 40.56$  mb.

The extension to the nuclear case is done through  $\sigma^{\gamma^*A} = \pi R_A^2 / (\pi R_p^2) \sigma^{\gamma^*p}(\tau_A)$  where  $\tau_A$  is related to  $\tau$  through  $\tau_A = \tau [\pi R_A^2 / (A\pi R_p^2)]^{\frac{1}{2}}$ . The nuclear radius is given by the usual parametrization  $R_A = (1.12A^{1/3} - 0.86A^{-1/3})$  fm, and  $\delta = 0.79 \pm 0.02$  and  $\pi R_p^2 = 1.55 \pm 0.02$  fm<sup>2</sup> are extracted from a fit to lepton-nucleus data. The nuclear structure function  $F_2^A$  is obtained through the standard relation  $F_2^A(x, Q^2) = Q^2 \sigma^{\gamma^*A} / (4\pi^2 \alpha)$ . The ASW structure function for the proton case is recovered by taking  $A = 1$  in the expressions above (see Fig. 1).

## Analysis of the results

The photonuclear contribution to  $b(E)$  computed (for standard rock  $A = 22$  throughout all this paper) with ALLM and with CKMT structure functions, and the same nuclear corrections [11], give very close results (see Fig. 3).

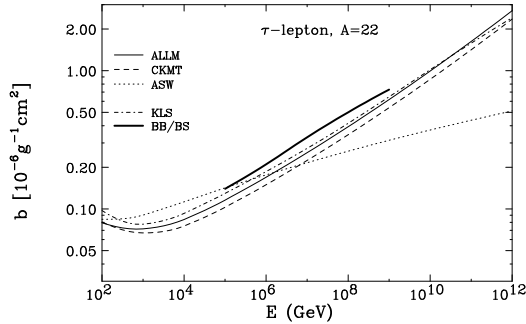


Figure 3: The photonuclear energy loss rate,  $b(E)$ , computed in different models

Although ALLM and CKMT parameterizations share a common theoretical base, with a reggeon and pomeron component, and they are fitted to the same data sets, ALLM systematically lies above CKMT at low  $x$  (see Fig. 1), which accounts for the difference in  $b(E)$  observed in Fig. 3.

The lowest of the  $b(E)$  values at high energies is obtained with the ASW structure functions. The ASW structure functions contain mild nuclear cor-

rections at low  $x$  when compared with DRSS and BM nuclear corrections (see Fig. 2), but their saturation effects at the nucleon level are rather strong and limit the rise of  $b(E)$  with energy as observed in Fig.3. For energies below  $E = 10^6$  GeV the result from the ASW structure function is higher than those from ALLM or CKMT (see Fig.3). This is because the ASW structure function at low  $Q^2$  is significantly higher for the  $x$  range  $10^{-6}$ - $10^{-3}$  (see Fig.1) which is relevant for energies below  $E = 10^6$  GeV (see Fig.5). Thus the saturation based ASW prediction lowers the energy loss rate  $b(E)$  with respect to the already existing predictions by a factor 2 at  $E = 10^9$  GeV, and by a factor even larger at higher energies.

The BB/BS calculation gives the largest of the predicted energy loss rates, while the KLS result is slightly above the calculations in which we use ALLM and CKMT proton structure functions with the nuclear corrections in DRSS (see Fig. 3).

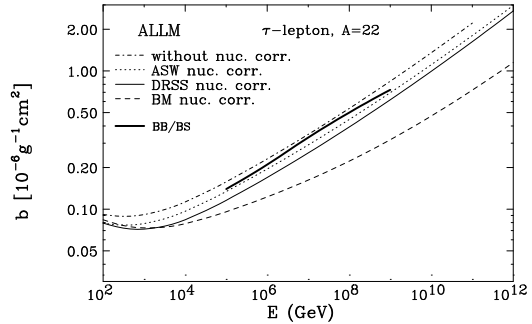


Figure 4: The effect of nuclear corrections on the photonuclear energy loss,  $b(E)$ .

Much of the uncertainty in the calculation of the photonuclear tau energy loss is actually due to nuclear effects. The choice of the nuclear corrections in Ref. [11], Ref. [12], or in Ref. [5], translates into differences in the calculated value of  $b(E)$  (in this case using the ALLM structure function) by a factor rising from 1.5 to 2.5 as energy increases in the range  $E = 10^6$ - $10^9$  GeV (see Fig. 4), which corresponds to the region of very low  $x$  (see Fig. 5) where the differences in the predictions are large (see Fig. 2).

Finally, it is also interesting to see how much different regions of  $x$  contribute to  $b(E)$ . The dependence of  $b(E)$  on the maximum value of  $x$  consid-

ered in the photonuclear process is shown in Fig. 5 both for ALLM and ASW.

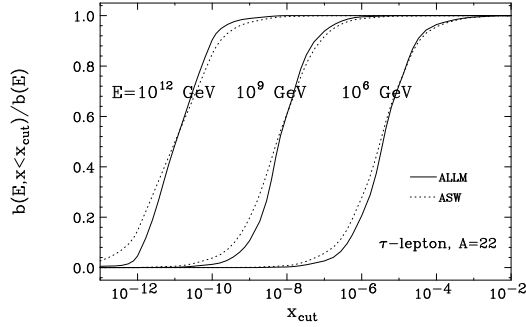


Figure 5: The relative contribution of  $x < x_{cut}$  to the photonuclear energy loss,  $b(E)$ .

## Conclusions

The highest prediction for  $b(E)$  in the case of the tau energy loss by photonuclear interactions is provided by the BB/BS calculation, while the lowest estimation at high energy is obtained with the ASW structure functions. The difference between these two limiting predictions is a factor 2.5 at  $E = 10^9$  GeV and it rises with energy. The BB/BS, ALLM, and CKMT calculations agree within a 30% and go parallel for all energies, what is an indication of the different normalization of the structure function in each model.

The application of a much stronger than usually considered nuclear shadowing at low  $x$  lowers the prediction of  $b(E)$  with respect to the already existing calculations by a factor up to 2 at  $E = 10^9$  GeV.

The obtained uncertainty for the tau energy loss is to be used, together with the corresponding one for the CC neutrino-nucleon cross section, in analytical calculations and implemented into the Monte Carlos for the setting of a tau neutrino bound in cosmic ray physics, though this task is out of the scope of this paper.

## Acknowledgements

Work supported by Ministerio de Educacion y Ciencia, Spain under grant FPA2005-01963 and Consellería de Educación, Xunta de Galicia.

## References

- [1] D. Fargion, *Astrophys. J.* 570 (2002) 909.
- [2] X. Bertou, P. Billoir, O. Deligny, C. Lachaud, and A. Letessier-Selvon, *Astropart. Phys.* 17 (2002) 183.
- [3] O. Blanch Bigas (Pierre Auger Collaboration), these proceedings.
- [4] J. Alvarez-Muñiz (Pierre Auger Collaboration), these proceedings.
- [5] N. Armesto, C. Salgado, and U.A. Wiedemann, *Phys. Rev. Lett.* 94 (2005) 022002.
- [6] L.B. Bezrukov and E.V. Bugaev, *Yad. Fiz.* 33 (1981) 1195.
- [7] E.V. Bugaev and Yu.V. Shlepin, *Phys. Rev. D* 67 (2003) 934027.
- [8] P. Lipari and T. Stanev, *Phys. Rev. D* 44 (1991) 3543.
- [9] E.V. Bugaev, T. Montaruli, Yu.V. Shlepin, and I. Solkalski, *Astropart. Phys.* 21 (2004) 491.
- [10] C. Aramo et al, *Astropart. Phys.* 23 (2005) 65.
- [11] S.I. Dutta, M.H. Reno, I. Sarcevic, and D. Seckel, *Phys. Rev. D* 63, 094020 (2001).
- [12] A.V. Butkevich and S.P. Mikheyev, *Zh. Eksp. Teor. Fiz.* 122 (2002) 17.
- [13] K.S. Kuzmin, K.S. Lokhtin, and S.I. Sinegovsky, hep-ph/0412377; A.A. Kochanov, K.S. Lokhtin, and S.I. Sinegovsky, Proc. 29th ICRC (2005)00,101-106.
- [14] S.I. Dutta, Y. Huang, and M.H. Reno, *Phys. Rev. D* 72, 013005 (2005).
- [15] H. Abramowicz and A. Levy, hep-ph/9712415.
- [16] A. Capella, A. Kaidalov, C. Merino, and J. Tran Thanh Van, *Phys. Lett.* B337 (1994) 358.
- [17] C. Adloff et al. (H1 Collaboration), *Eur. Phys. J.* C21 (2001) 33.
- [18] J. Breitweg et al., (ZEUS Collaboration), *Phys. Lett.* B487 (2000) 53.
- [19] A.M. Stasto, K. Golec-Biernat, and J. Kwiecinski, *Phys. Rev. Lett.* 86, 596 (2001).
- [20] J.L. Albacete, N. Armesto, J.G. Milhano, C. A. Salgado, and U. A. Wiedemann, *Eur. Phys. J.* C 43, 353 (2005).
- [21] K. Golec-Biernat and M. Wüsthoff, *Phys. Rev. D* 59, 014017 (1999).