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Analytical description of the Day-Night neutrino asymmetry

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Abstract: We present a new treatment of the Earth matter effects on the neutrino oscillations that is valid for an arbitrary density profile. When applied to the the study of the day-night effect on the solar neutrino flux it renders us a simple analytical expression, which is more accurate than those derived by using the perturbation theory and can be extended to higher energies.

Introduction

Different types of experiments have provided compelling evidence for neutrino oscillations [1]. In the case of solar neutrinos the leading effects can be accounted by oscillations between two neutrino flavors, parameterized in terms of the mass square difference $\delta m^2 = m_2^2 - m_1^2$ and the mixing angle θ. A global fit of all the existing data gives $\delta m^2 = (7.9 - 8) \times 10^{-5}$ eV^2 and $\sin^2(\theta) = 0.310 - 0.315$ [1], which is in good agreement with the results of other groups. These values belong to the region in the parameter space referred as to the Large Mixing Angle Solution (LMA). According to the LMA, the ^{8}B electron neutrinos produced in the Sun undergo a highly adiabatic conversion and are almost totally converted into the mass eigenstate ν_2. Then, the electron neutrino survival probability is $P(\nu_e \to \nu_e) \cong \sin^2\theta$. However, during the night solar neutrinos arriving to terrestrial detectors travel a certain distance through the Earth's matter, which affects the oscillations pattern. This leads to a partial regeneration of the electron neutrino flux, a phenomenon known as the day-night effect.

Matter effects on the neutrino oscillations inside the Earth are conveniently accounted in terms of the parameter

$$\varepsilon(t) \equiv \frac{2EV(t)}{\delta m^2} \cong 0.019 \left[\frac{E}{10\text{ MeV}}\right]\left[\frac{n_e(t)}{N_A\text{ cm}^{-3}}\right] \times \left[\frac{8\times 10^{-5}\text{ eV}^2}{\delta m^2}\right], \tag{1}$$

where $V(t) = \sqrt{2}G_F n_e(t)$ represents the potential energy for ν_e, which comes from the charged-current interaction with electrons. Here, G_F is the Fermi constant, E is the neutrino energy, N_A is the Avogadro number, and $n_e(t)$ is the number density of electrons along the neutrino path.

For the favored value of δm^2 and the energy range of solar neutrinos, Earth's density is such that $\varepsilon \ll 1$. Taking advantage of this fact, the perturbation theory has been applied to derive an analytical expression for the day-night rate asymmetry to first order in ε [2, 3]. The formulas obtained are valid for any density profile and simplify the numerical calculations. The accuracy of the method has been subsequently improved by means of a second order expansion in ε [4]. In this work we show that the Magnus expansion of the evolution operator [5] provides a convenient alternative to the perturbative approach and derive an expression for the regeneration probability which is better, in the sense that the relative error is smaller than the one corresponding to the second order obtained in Ref. [4].

Neutrino Oscillation in Matter

We consider a system consisting of two neutrino flavors, $\Psi_f = (\Psi_e, \Psi_\mu)$, which are related to the mass eigenstate, $\Psi_{mass} = (\Psi_1, \Psi_2)$, according to,

$$\Psi_f = U(\theta)\Psi_{mass}, \tag{2}$$

where,

$$U(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{3}$$

The evolution operator of the system satisfies the equation

$$i\frac{d\mathcal{U}}{dt}(t,t_0) = H(t)\,\mathcal{U}(t,t_0)\,, \tag{4}$$

with the initial condition $\mathcal{U}(t_0,t_0) = \mathbb{1}$. The hamiltonian in the mass base is given by

$$H(t) = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\delta m^2}{2E} \end{pmatrix} + V(t)\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}, \tag{5}$$

and its eigenvalues are

$$\lambda_\pm(t) = \frac{1}{2}(V(t) + \frac{\delta m^2}{2E} \pm \Delta_m(t))\,, \tag{6}$$

with

$$\Delta_m(t) = \frac{\delta m^2}{2E}\sqrt{(\varepsilon(t) - \cos 2\theta)^2 + \sin^2 2\theta}\,. \tag{7}$$

Let us now write

$$\mathcal{U}(t,t_0) = \mathcal{P}(t,t_0)\,\mathcal{U}_{\mathcal{P}}(t,t_0), \tag{8}$$
$$\mathcal{P}(t,t_0) = \begin{pmatrix} e^{-i\alpha_-(t,t_0)} & 0 \\ 0 & e^{-i\alpha_+(t,t_0)} \end{pmatrix},$$

with $\alpha_\pm(t,t_0) = \int_{t_0}^t dt'\lambda_\pm(t')$. The operator $\mathcal{U}_{\mathcal{P}}(t,t_0)$ obeys Eq. (4) but with the Hamiltonian $H_{\mathcal{P}} = \mathcal{P}^\dagger(t,t_0)(H(t) - H_D(t))\mathcal{P}(t,t_0)$, where $H_D(t) = diag(\lambda_-(t), \lambda_+(t))$. If we expand the eigenvalues to first order in $\varepsilon(t)$ we obtain an approximated expression for the transformed hamiltonian in which the diagonal elements are zero, that is

$$H_{\mathcal{P}}(t,t_0) \cong V(t)\frac{\sin 2\theta}{2}\begin{pmatrix} 0 & e^{-i\phi_{t_0\to t}} \\ e^{i\phi_{t_0\to t}} & 0 \end{pmatrix}, \tag{9}$$

where, $\phi_{t_0\to t} = \int_{t_0}^t dt'\Delta_m(t')$.

The relevant quantity is the regeneration probability defined as the difference between the day and night probability, $F_{reg}(E) \equiv P_{2\to e}(E) - \sin^2\theta$, where $P_{2\to e}(E) = |\langle\nu_e|\hat{\mathcal{U}}(t,t_0)|\nu_2\rangle|^2$. Here, we determine the evolution operator in the mass base from Eq. (8) by evaluating $\mathcal{U}_{\mathcal{P}}$ in terms of the lowest-order Magnus approximation, $\mathcal{U}_{\mathcal{P}}(t,t_0) \cong \exp(-i\int_{t_0}^t dt' H_{\mathcal{P}}(t',t_0))$. Proceeding in such a way we get

$$F_{reg}(E) = \frac{1}{2}\sin(2I)\sin 2\theta\sin(\phi_{\bar{t}\to t}) + \sin^2(I)\cos 2\theta, \tag{10}$$

where,

$$I = \sin 2\theta\int_{\bar{t}}^t dt' V(t')\cos(\phi_{\bar{t}\to t'}). \tag{11}$$

In writing Eq. (10), we assumed that the potential is symmetric with respect to middle point of the trajectory $\bar{t} = (t+t_0)/2$, which is the case for a neutrino that propagates through a medium, like the Earth, with a spherically symmetric density profile. By doing an expansion up to first order in I, our result for $F_{reg}(E)$ reduces to the one calculated to first order in ε [4]. To make a numerical comparison of both formulas, we examine the case of a neutrino that crosses the Earth passing trough the center. For the density profile of the Earth we use a simplified model called mantle-core-mantle [6]. According to it, $n_e(r)$ is approximated by a step function and the radius of the core and the thickness of the mantle are assumed to be half of the Earth radius. Then, we put

$$n_e(r) = N_A\begin{cases} 5.953\ \text{cm}^{-3}, & r \le R_\oplus/2 \\ 2.48\ \text{cm}^{-3}, & R_\oplus/2 < r \le R_\oplus \end{cases} \tag{12}$$

where $R_\oplus$ is the radius of the Earth.

As in Ref. [4] we consider the function,

$$\delta(E) = \frac{1}{\bar{F}_{reg}(E)}(F_{reg}^{(appr)}(E) - F_{reg}^{(exact)}(E)), \tag{13}$$

where $F_{reg}^{(appr)}$ is given by the approximated analytical expressions, $F_{reg}^{(exact)}$ is obtained from the exact (numerical) solution, and

$$\bar{F}_{reg}(E) = \frac{1}{2}\varepsilon(t_s)\sin^2\theta, \tag{14}$$

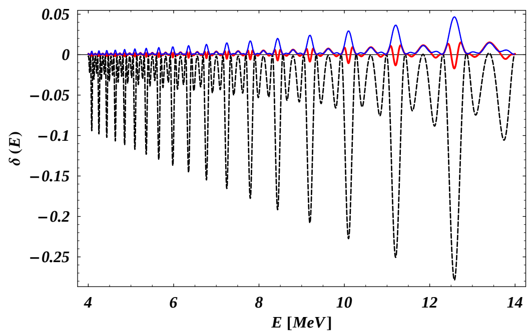

Figure 1: The relative error $\delta(E)$ as a function of the energy corresponding to a neutrino that propagates inside the Earth and pass through the Earth center at first (dashed line) and second (solid blue line) order in ε compared with the result obtained by using the equation (10) (solid red line) for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2(\theta) = 0.4$.

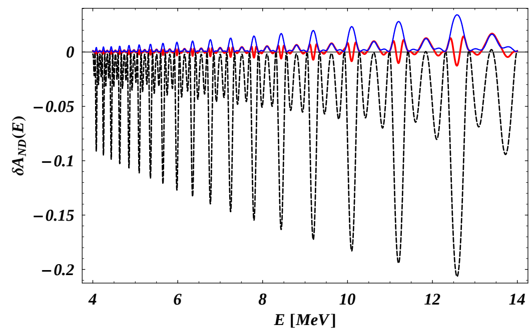

Figure 2: Relative error of the Day-Night asymmetry $\delta A_{ND}(E)$ as a function of the energy corresponding to a neutrino that propagates inside the Earth and pass through the Earth center ($\eta_1 = 0°$) at first (dashed line) and second (solid blue line) order in ε compared with the our result (solid red line) for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2(\theta) = 0.4$.

is the average regeneration factor corresponding to the density of the surface layer. $\delta(E)$ corresponds essentially to the relative error of the approximated expression. Figure 1 shows $\delta(E)$ as a function of the energy for $F_{reg}^{(appr)}$ computed at first (dashed line) and second (solid blue line) order in V compared with the result obtained by using our approximation (solid red line), for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2(\theta) = 0.4$.

From figure 1 we see that the relative error for our result remains always smaller than the corresponding to the second order in V. Although it increases with the neutrino energy it is smaller than $\sim 2\%$ at energies of order of 14 MeV.

Day-Night asymmetry

Following Ref. [2] the day-night asymmetry as a function of the energy is given by,

$$A_{DN}(E) = \frac{2\,\langle\cos 2\hat{\theta}\rangle(E) F_{reg}(E)}{1 - \langle\cos 2\hat{\theta}\rangle(E)(F_{reg}(E) - \cos 2\theta)}, \tag{15}$$

where,

$$\langle\cos 2\hat{\theta}\rangle(E) = \int_0^{R_\odot} dr\, f(r) \times \frac{\cos 2\theta - \varepsilon(E,r)}{\sqrt{(\varepsilon(E,r) - \cos 2\theta)^2 + \sin^2 2\theta}}. \tag{16}$$

Here $f(r)$ is the spatial distribution function of the neutrino sources in the Sun [7] and $\varepsilon(E,r)$ is determined by Eq. (1) with the electron density of the Sun [7]. Figures 2 and 3 show the relative error of A_{ND} as a function of the energy for the three approximations examined here and for a neutrino trajectory of nadir angle $\eta_1 = 0°$ (neutrino passing through the Earth center) and $\eta_2 = 30°$ (neutrino passing tangent to the core region), respectively. We used the spatial distribution function $f(r)$ corresponding to ^{8}B. In both cases, the smallest relative error is obtained for our expression of the regeneration probability. We can also see that the relative error is smaller for η_2 in the three cases, which is due to the fact that the electron density, and therefore ε, is smaller in the mantle region.

We also calculate the integrated day-night asymmetry,

$$A_{DN} = 2\int_{E_{th}}^{\infty} dE\phi_\nu(E)\langle\cos 2\hat{\theta}\rangle(E)F_{reg}(E) \times \left[1 - \int_{E_{th}}^{\infty} dE\phi_\nu(E)\langle\cos 2\hat{\theta}\rangle(E) \times (F_{reg}(E) - \cos 2\theta)\right]^{-1}, \tag{17}$$

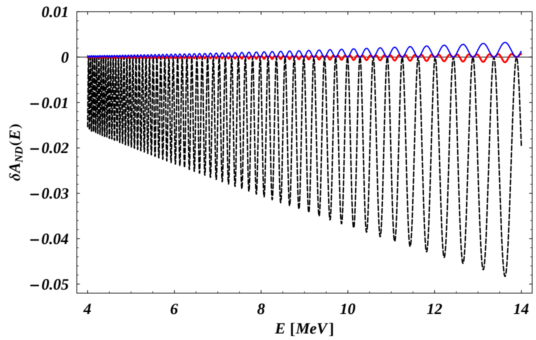

Figure 3: Relative error of the Day-Night asymmetry $\delta A_{ND}(E)$ as a function of the energy corresponding to a neutrino that propagates inside the Earth and pass tangent to the core region ($\eta_2 = 30°$) at first (dashed line) and second (solid blue line) order in ε compared with our result (solid red line) for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2(\theta) = 0.4$.

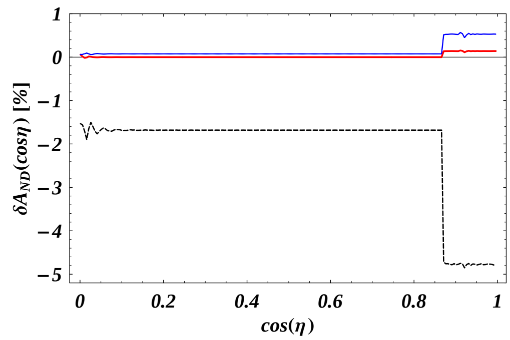

Figure 4: Relative error of the integrated Day-Night asymmetry $\delta A_{ND}(\cos\eta)$ as a function of the Nadir angle at first (dashed line) and second (solid blue line) order in ε compared with our result (solid red line) for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2(\theta) = 0.4$.

where $\phi_\nu(E)$ is the normalized flux of solar ^{8}B neutrinos and $E_{th} = 5$ MeV is the detection energy threshold for Super-Kamiokande and SNO. Figure 4 shows the relative error of A_{DN} as a function of the cosine of the nadir angle for the three approximated formulas. As we see, there are two regions; one of them corresponds to the propagation in the mantle, $0 < \cos\eta < \sqrt{3}/2$, and the other to the propagation in the mantle and the core, $\cos\eta > \sqrt{3}/2$. The relative error in this regions is practically constant. In the mantle region it takes the values -1.7%, 0.07%, and -0.001% for the first order in ε, the second order in ε, and formula (10), respectively. In the core-mantle the corresponding values are -4.7%, 0.51%, and 0.13%.

Conclusions

In this work we have applied the Magnus expansion of the time evolution operator to find approximated analytical solutions of the system of two neutrino flavors coupled very weakly with matter. From this result we derived new expressions for the regeneration probability and the Day-Night asymmetry which give better approximations to the exact numerical results than those obtained by using a perturbative approach.

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