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Likelihood deconvolution of diffuse prompt and extra-terrestrial neutrino fluxes in the AMANDA-II detector

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Abstract: The AMANDA-II detector at the South Pole station, Antarctica, has been used in several searches for a flux of extra-terrestrial neutrinos from the sum of all sources in the universe. These searches are complicated by uncertainties in the expected fluxes of background neutrinos, both those from cosmic-ray pion and kaon meson production (conventional atmospheric neutrinos) and those from charm-containing mesons (prompt atmospheric neutrinos). In this work, we explore the use of a full likelihood analysis on flux sensitive distributions in order to account for the uncertainties and place simultaneous constraints on the fluxes of interest. The method is illustrated using simulated data sets, with application to the real AMANDA-II data to come.

Introduction

The search for an extra-terrestrial diffuse flux is one of the most challenging tasks of a neutrino detector. In contrast to a point source search, where backgrounds are measured from off-source data, a diffuse search requires a good understanding and prediction of the expected backgrounds. In the case of a diffuse neutrino search, the backgrounds are atmospheric neutrinos. There are two components to this flux, one thought to be well understood, and another less certain. The conventional atmospheric neutrinos[1, 2] are due to decay of pions and kaons produced by cosmic radiation interacting with the earth's atmosphere. Prompt atmospheric neutrinos[3, 4, 5, 6, 7], from the production and decay of mesons containing charm quarks, have never been identified and predictions of this flux span orders of magnitude. The prompt component should follow the spectral index of the primary cosmic rays, whilst the conventional component has a spectrum about one power steeper. The expected flux of extra-terrestrial neutrinos from, for example, the sum of all active galaxies in the universe, is expected to have a harder spectrum ($\sim E^{-2}$) than either of the atmospheric neutrino components. The low expected event rates and similarity of the spectra of prompt and extra-terrestrial neutrinos will make their independent identification difficult[8]. The AMANDA-II detector data from the years 2000-03 have been searched for prompt and extra-terrestrial components[9, 10]. Spectral differences in the neutrino fluxes would manifest themselves in different expected energy distributions of detected events in the AMANDA-II neutrino detector. The number of optical modules ($N_{\rm ch}$) registering at least one photon was used as an energy estimator. A diffuse extra-terrestrial signal would appear as an excess of events at higher values of the $N_{\rm ch}$ parameter. In order not to bias the analysis, a blind analysis, and a simulation based unbiased optimum limit setting technique were used to choose the best cut appropriate for each signal spectrum. The atmospheric neutrino background simulation was normalised to observed data below the cut in order to constrain some of the uncertainties. The prompt neutrinos were treated in two ways, firstly, they were included as a background for the extra-terrestrial searches, and secondly, they were treated as an unknown signal, to be constrained by the observed data. The final limit on an E^{-2} flux was set at a level of $\rm E_\nu^2 \times dN_\nu/dE_\nu = \ 7.4 \times 10^{-8}$ $\rm GeV\ cm^{-2}\ s^{-1}\ sr^{-1}$, valid over an energy range 16-2500 TeV. This is the best limit to date on extra-terrestrial neutrino fluxes. Despite this success, the cut and count method does suffer from some drawbacks. Primarily, the shape of the $N_{\rm ch}$ distribution is not used in the analysis, only the integrated number of events above the cut value. A likelihood analysis can be used to take advantage of the full shape of the $N_{\rm ch}$ distribution. In addition, such an analysis can simultaneously constrain all the parameters, both those of direct interest (the numbers of prompt and extraterrestrial neutrinos) and those of indirect interest - known as "nuisance parameters" (normalisation and shape of the conventional atmospheric neutrinos). Another key point is that if an entire distribution is used in a likelihood analysis, then there is no need to optimise a selection cut on that parameter, removing discussion of what is the optimal cut criterion. These likelihood methods with nuisance parameters are standard for neutrino oscillation analyses[11], and for "unbinned" astrophysical point source searches[12, 13, 14].

Methodology

The likelihood function in this analysis is the product over a binned version of the $N_{\rm ch}$ distribution of the bin-by-bin Poisson probabilities of events observed given events expected.

$$P(\{n_i\} \mid \{\mu_i\}) = \Pi_i \frac{(\mu_i)^{n_i}}{n_i!} \exp(-\mu_i) + \frac{\Delta\epsilon^2}{\sigma_\epsilon} \quad (1)$$

For each bin, the expectation μ_i is the sum of conventional and prompt atmospheric neutrinos, and extra-terrestrial neutrinos

$$\mu_i = \epsilon(A_c\mu_{ci}(\Delta\gamma) + A_p\mu_{pi} + A_e\mu_{ei}) \quad (2)$$

where subscripts c, p and e stand for conventional, prompt and extra-terrestrial neutrino fluxes respectively. As an example, the term $A_e\mu_{ei}$ is the number of events expected in bin i after convolving an extra-terrestrial flux, normalised to a total of A_e events, with the effective area of the detector (which includes absorption effects in the earth). The parameter $\Delta\gamma$ of the conventional atmospheric flux allows for changes in the spectral shape relative to the prediction. Full calculations[1, 2] of the angular and spectral dependence of the flux have been made, here we allow for deviations away from the exact form $\Phi_0(E,\theta)$ by using $\Phi(E,\theta) = \Phi_0(E,\theta)E^{\Delta\gamma}$. Since the spectrum only approximately follows a power-law (and this varies with angle) we choose to fit for deviations away from the actual spectrum, rather than fit for a simple power law γ. Fitting for $\Delta\gamma$ would allow statements to made such as "the data favour a similar/harder/steeper spectral form than that calculated theoretically," rather than simply fitting for a single value of γ. The parameter ϵ is an efficiency term reflecting uncertainties in the effective area of the detector. While this is strictly energy- and thus bin-dependent, with strong bin-to-bin correlations, here we simplify to a constant form for this initial illustration of the method. Epsilon is constrained to a Gaussian form with width σ_ϵ by the penalty term in the likelihood function, with $\Delta\epsilon$ being the difference between the tested value of the efficiency, ϵ, and the notional best fit value for the efficiency, $\epsilon_0 = 1$. To test a given hypothesis, e.g. that $A_p = 20.0$ and $A_e = 10.0$, the likelihood is maximised, fixing A_p and A_e to the desired values and allowing ϵ, A_c and $\Delta\gamma$ to float. This likelihood, denoted $\mathcal{L}$, is then compared to the likelihood $\hat{\mathcal{L}}$ where all parameters are free to float in the fit. The tested hypothesis is then rejected at a confidence level set by the probability of observing a greater likelihood ratio, given the truth of the null hypothesis A_p and A_e, than the specific one that was observed. The distribution of the likelihood ratio statistic under the null hypothesis is known approximately from Wilks' theorem. Asymptotically, the likelihood ratio defined by $-2\log\mathcal{L}/\hat{\mathcal{L}}$ follows a chi-square distribution with degrees of freedom equal to the number of fixed parameters in the $\mathcal{L}$ fit. The confidence level at which the hypothesis is then rejected is found from checking the ratio $-2\log\mathcal{L}/\hat{\mathcal{L}}$ against the appropriate chi-square value (e.g. a 90% c.l. corresponds to a chi-square of 4.6 for two degrees of freedom). In order to compute the exact confidence level for each null hypothesis, the likelihood ratio may be compared to its expected distribution, generated from many random event distributions drawn from the null hypothesis[15]. In this paper, we use the chi-square approximation for simplicity, leaving the full interval constuction for final analysis.

Having written down the form of the likelihood function, the details of the components must be determined. Here, we take the shape of the conventional atmospheric neutrino detector response, $\mu_c(\Delta\gamma)$ as the convolution of the Bartol flux[1], with the detector effective area, multiplied by the factor $E^{\Delta\gamma}$. There are two primary sources of uncertainty in the prediction of the atmospheric neutrino flux - the cosmic ray primary spectrum and the interaction model. Together, these manifest themselves as overall uncertainties in the normalisation (fitted by A_c), and as an increasing uncertainty in the flux as a function of energy (see figure 12 of [9]). This energy dependent uncertainty can be approximately parameterised as a change of slope in the neutrino spectrum. The prompt flux is the "Charm D" model[7], an older prediction, but with a spectral shape similar to more recent preditions. The extra-terrestrial flux follows an E^{-2} power law. The value of the effective area uncertainty, σ_ϵ, is taken as 10%, effectively bounding (95% region) it to extrema of plus/minus 20%.

Example fitting of a test data set

To demonstrate the power of the likelihood method, we derive a random test data set by sampling 450 events from the Bartol $N_{\rm ch}$ distribution. These event are then treated as though they are the real data set. Figure 1 shows the result of the fitting procedure, where the data set is best fit by 446.5 atmospheric neutrinos and 3.6 extra-terrestrial neutrinos. The normalisation and $\Delta\gamma$ of the atmospheric neutrinos, and the effective area parameter ϵ, were allowed to float during this fit. The potential to constrain the atmospheric neutrino parameters is shown in figure 2, where an acceptance region was found while allowing the effective area uncertainty to float. The size of this experimentally determined allowed region is similar to the theoretical uncertainties of flux. This simple $N_{\rm ch}$ fitting procedure is not powerful enough to constrain the theory with only AMANDA-II. However, with increased exposure (more AMANDA-II data and the larger IceCube detector) the experimental observations will begin to constrain the theory, allowing for proper measurements of the flux. In figure 3 the allowed regions for prompt and extra-terrestrial fluxes are shown. Since there is only background in the test data set, the allowed region includes the background only corner of the plane. The upper bounds of the allowed regions define combinations of allowed amounts of the two components. The 90% confidence level count on the extra-terrestrial axis (25 events) corresponds to a flux level of $\mathrm{E}_\nu^2 \times \mathrm{dN}_\nu/\mathrm{dE}_\nu = 1.2 \times 10^{-7}$ $\mathrm{GeV\ cm^{-2}\ s^{-1}\ sr^{-1}}$. Since this result is just for one specific test data set, a meaningful comparison to the standard analysis[9] cannot be made, without determining a sensitivity over many repeated random experiments. It is expected that the likelihood method will lead to an improvement in the sensitivity. The actual predicted level of the CharmD prompt flux corresponds to 8 events in this acceptance region.

Future work

To properly estimate the sensitivity and discovery potential, many test sets, drawn from mixtures of backgrounds and signals must be processed and the acceptance regions combined. This will be done using the median likelihood ratios at each point in the plane. Required signal combinations for definite discovery of either or both of the signal fluxes could also be determined.

The nature of the parameterisations of the fluxes can be further developed and improved. In principle, the atmospheric neutrinos could be parameterised in ways more directly connected to the physics of the cosmic ray fluxes and interaction models, for instance to fit for the charm production cross-sections, and to allow for the charm spectral index to float. Instead of using an E^{-2} extra-terrestrial spectrum, the spectral index of this additional component flux could be a fit parameter. The uncertainties on the detector response could be treated in a proper bin-to-bin correlated manner.

Conclusions

A likelihood ratio fitting method, incorporating nuisance parameters, has been developed for application to a neutrino search with the AMANDA-II detector. This method allows for the simultaneous constraint of background and signal flux parameters. The use of an entire distribution in the analysis removes the need for optimisation of a selection cut, and allows all the available information to be

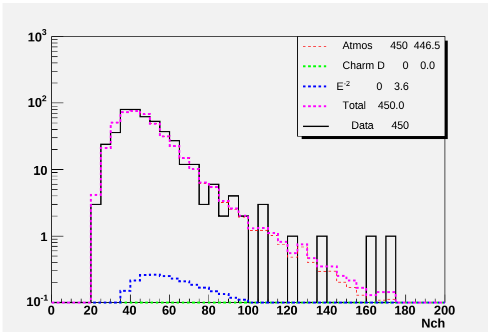

Figure 1: Fitting of a test data set with the likelihood procedure. The data set, drawn from the Bartol atmospheric neutrino distribution, is best fit by a near pure atmospheric neutrino contribution, plus 3.6 extra-terrestrial events. The allowed regions for the additional components are shown in figure 3.

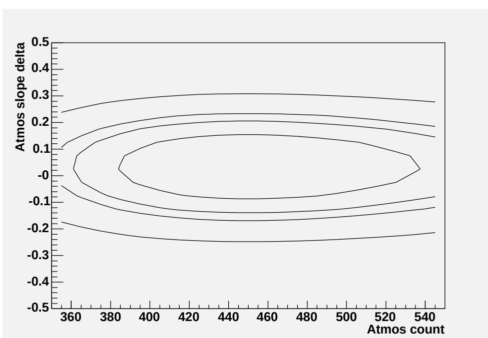

Figure 2: Test data set allowed regions of the atmospheric neutrino total event count, and spectral slope difference $\Delta\gamma$. The confidence level contours correspond to one-sigma, 90%, two and three sigma, moving outward from the best fit point.

incorporated into the confidence interval construction.

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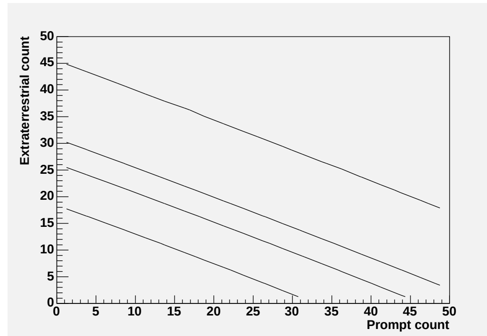

Figure 3: Allowed regions of the prompt and extra-terrestrial neutrino contributions for the test data set, allowing the atmospheric neutrino and detector effective area parameters to float. The 90% confidence level count on the extra-terrestrial axis (25 events) corresponds to a flux level of $\mathrm{E}_\nu^2 \times \mathrm{dN}_\nu/\mathrm{dE}_\nu = 1.2 \times 10^{-7}$ $\mathrm{GeV\ cm^{-2}\ s^{-1}\ sr^{-1}}$.

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