



Estimation of the Energy Spectra of Primary Cosmic Ray Nuclei

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Abstract: Energy spectra and chemical composition of the primary cosmic ray nuclei for energies higher than 1 PeV are obtained mainly from measurements of intensities and various properties of extensive air showers. Additional and important data from the study of gamma ray families are available in addition. In bouth cases we have information from the range of very high fluctuations. An important fact that we are working with the very steep primary energy spectra has to be taking into account. In this research different probability distributions have been used as well as their convolutions with the power low primary spectrum. The role of the influence of different parameters on the measurements of primary spectra will be discussed in the paper.

Introduction

Since years in research of extensive air showers we are having difficulties with coherent interpretation of results, which came from different detectors. In the beginning the problem was to obtain the credible estimate of the vertical angle $\Delta Q \sim 5^0$ and total number of charged particles in $\Delta N_e/N_e \sim 30\%$. The estimates of the primary nucleus streams, which had been obtained on this base, were in 30% wrong.

Contemporary detectors KASCADE, HiRes, Auger allow calculating basic parameters with higher accuracy. Unfortunately, it does not mean that, we have automatically achieved several times better information about energetic spectrum and chemical composition of primary cosmic rays above 10^{15} eV. In author's opinion in the whole area form 10^{15} eV to 10^{20} eV the spectrum from different detectors are shifted concerning each over. It concerns also two structures: the "knee" region and the area of Greisen –Zatsepin – Kuzmin (GZK) cut-off.

Reconstruction of the primary energy spectrum

The general idea of this paper is discussion of reconstruction of the primary energy spectrum only from the mathematical point of view.

The spectrum with the constant slope

For the analyzes of the spectra without any structures we assumed the spectra to be:

$$(1). \quad j_o(E_0)dE_0 = A \cdot E_0^{\gamma-1} dE_0$$

We will consider two cases for reconstruction error function the log-normal distribution with density function:

$$(2). \quad f(E_0) = \frac{1}{\sqrt{2\pi}\sigma E_0} e^{-\frac{(\ln E_0 - m)^2}{2\sigma^2}}$$

and the generalized Gamma distribution with

$$(3). \quad f(E_0) = \frac{\alpha}{\lambda^{\frac{p}{\alpha}} \Gamma\left(\frac{p}{\alpha}\right)} E_0^{p-1} e^{-\frac{E_0}{\lambda}}$$

We look for such probability distribution of reconstructed energy E_0 to energy E with error func-

tion given by $g(E, E_0)$, which satisfied the following normalization condition that expected value μ of E_0 is equal to E . That is,

$$\mu = \int_{-\infty}^{+\infty} E_0 g(E, E_0) dE_0 = E \quad (4).$$

Consider the log-normal distribution given by (2) with the expected value $\mu = e^{\frac{m+\sigma^2}{2}}$. From assumption (4) we obtain :

$$m = \ln E - \frac{\sigma^2}{2} \quad (5).$$

As a result we can write (2) in the form:

$$g(E, E_0) = \frac{1}{\sqrt{2\pi}\sigma E_0} e^{-\frac{(\ln E_0 - \ln E + \frac{\sigma^2}{2})^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma E_0} e^{-\frac{(\ln \frac{E_0}{E} + \frac{\sigma^2}{2})^2}{2\sigma^2}} \quad (6).$$

For generalized Gamma distribution (3) we have the expected value:

$$\mu = \lambda^{\frac{1}{\alpha}} \frac{\Gamma\left(\frac{p+1}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}\right)}.$$

Hence the condition (4) is satisfied if

$$\lambda = \left(\frac{\Gamma\left(\frac{p}{\alpha}\right)}{\Gamma\left(\frac{p+1}{\alpha}\right)} \right)^{\alpha} \cdot E^{\alpha}.$$

Substitute the above λ into (3) we obtain the following generalized Gamma error function

$$g(E, E_0) = \frac{\alpha}{\Gamma\left(\frac{p}{\alpha}\right) \left(\frac{\Gamma\left(\frac{p+1}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}\right)} \right)^{\alpha}} \cdot \left(\frac{E_0}{E} \right)^{p-1} \times \frac{1}{E} e^{-\left(\frac{\Gamma\left(\frac{p+1}{\alpha}\right) E_0}{\Gamma\left(\frac{p}{\alpha}\right) E} \right)^{\alpha}} \quad (7).$$

The reconstructed spectrum $j(E)$ will be given by the convolution j_0 with the error function g

$$j(E) = \int_0^{+\infty} g(E, E_0) j_0(E_0) dE_0 \quad (8).$$

Let us define the ratio of reconstructed spectrum to the primary spectrum by

$$K = \frac{j(E)}{j_0(E)} \quad (9).$$

We can evaluate K for log normal distribution

$$K = e^{\frac{\sigma^2}{2}(\gamma+1)(\gamma+2)} \quad (10).$$

and for generalized Gamma distribution

$$K = \frac{\Gamma\left(\frac{p-\gamma-1}{\alpha}\right) \left(\frac{\Gamma\left(\frac{p+1}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}\right)} \right)^{\gamma+2}}{\Gamma\left(\frac{p+1}{\alpha}\right) \left(\frac{\Gamma\left(\frac{p}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}\right)} \right)}, \quad \text{for } p > \gamma + 1 \quad (11).$$

For $\alpha=1$ Gamma distribution we have

$$K = \frac{\Gamma(p-\gamma-1)}{\Gamma(p)} p^{\gamma+1}, \quad \text{for } p > \gamma + 1 \quad (12).$$

If $\alpha=p$ we get known Weibull's distribution, the value of K is

$$K = \Gamma\left(\frac{p-\gamma-1}{p}\right) \left(\frac{\Gamma\left(\frac{p+1}{p}\right)}{\Gamma\left(\frac{p}{p}\right)} \right)^{\gamma+1}, \quad \text{for } p > \gamma + 1 \quad (13).$$

In Tab.1, the value of parameter K we evaluate for $\gamma=1,7$ and different standard deviations σ .

σ	Log	Gamma	
	normal	($\alpha=1$)	Weibull ($p=\alpha$)
0,30	1,56	1,65	2,51
0,20	1,22	1,23	1,35
0,10	1,05	1,05	1,06

Table 1: Comparison of the parameter K for different deviations σ and various distributions.

From the above considerations we obtain the following plots of the reconstructed spectra:

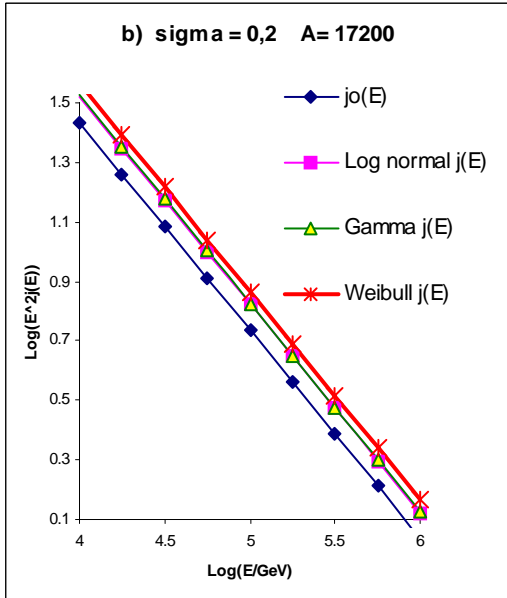
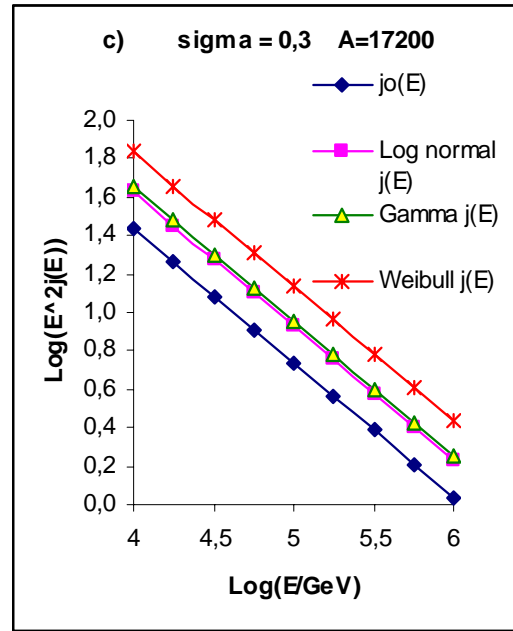
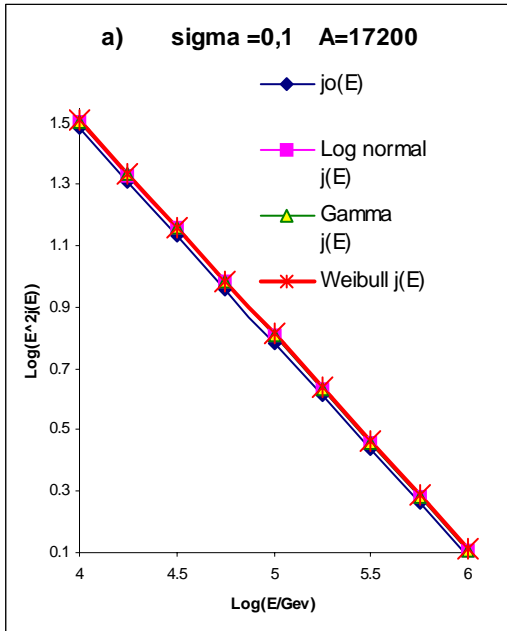


Figure 1: Primary spectra reconstructed for different values of σ : 0.1 fig. a), 0.2 fig b) and for 0.3 fig. c).

The spectra with the cut-off

Let us consider the following energy primary spectrum of the form given by:

$$j_o(E_0)dE_0 = A \cdot E_0^{-\gamma-1} e^{-\frac{E_0}{R}} dE_0 \quad (14),$$

where $R=3 \cdot 10^5$ GeV for the 'knee' region and 50 EeV in the GZK region.

The ratio K (see (6)) for log normal distribution is

$$K = \frac{e^{\frac{\sigma^2}{2}(\gamma+1)(\gamma+2)}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} e^{\frac{E}{R}(1-e^{\sqrt{2}\alpha x - (\gamma+1.5)\sigma^2})} dx \quad (15),$$

and for generalized distribution Gamma is:

$$K = \frac{1}{\Gamma\left(\frac{p+1}{\alpha}\right)} \cdot \left(\frac{\Gamma\left(\frac{p+1}{\alpha}\right)}{\Gamma\left(\frac{p}{\alpha}\right)} \right)^{\gamma+2} \times \int_0^{\infty} x^{\frac{p-\gamma-1}{\alpha}} e^{-x} e^{-\frac{E}{R} \left(\frac{\Gamma\left(\frac{p}{\alpha}\right)}{\Gamma\left(\frac{p+1}{\alpha}\right)} x^{\frac{1}{\alpha}-1} \right)} dx$$

The results for the ‘knee’ region are presented on the Figures 2 for different errors distributions.

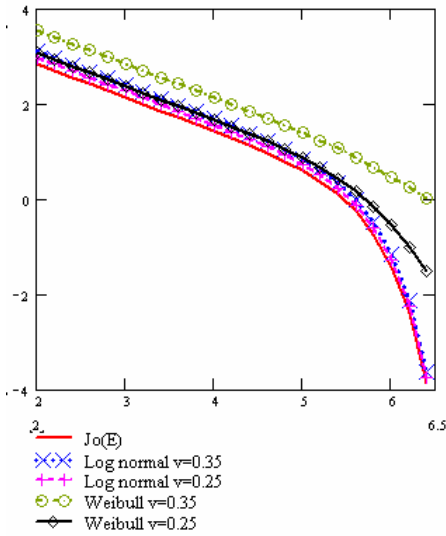


Figure 2: The reconstructed spectra around the knee’ region for different distributions. The $J_0(E)$ is the true spectrum.

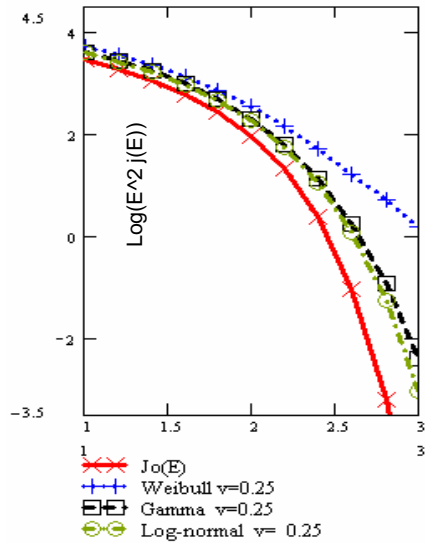


Figure 3. The reconstructed spectra around the GZK region for different distributions. The $j_0(E)$ is assumed as the true spectrum.

For the fluctuations problem considered in this paper the region of GZK has the same mathematical view.

We took the same shape of the primary spectrum but for higher energies. For example the used value of R was equal to 50 EeV. The results are presented in the Figure 3.

Conclusion

The extensive air showers researches demands understanding the role of fluctuation during the work with primary beam types $E^{-2.7}$ or E^{-3} . These are researches, in which the fluctuation of the measured parameters, which are being used for calculating primary energies and chemical composition are strongly asymmetrical with positive skewness. This situation causes that, even the assumption, that we measure properly average the primary energy dose not automatically mean good reconstruction of primary spectrums. In our publication [1] we have proved that depending on standard deviation of distributions and even assuming the independency of σ energy, we should expect the higher shift of spectrum, the bigger is value of the standard deviation.

We have very specific situation in the region of knee and of the GZK cut-off in spectrum. Showers, which come from primary nucleus of higher A have less electrons and worse symmetry of density distribution of charged particles in showers. It causes bigger problems with their detection near the ‘knee’ region, which simply can be transforming to the higher dispersion of the reconstructed energy. This fact can have fundamental importance for artificial evaluating of the spectrum shape in this region of higher energy around the ‘knee’ or near the GZK cut-off and average land showers detected. We should do not forget that we have natural tendency to over estimate the fluxes around the points with the structures in the primary spectrum.

References

[1] J.Kempa and R.Malecki (2007) Nuclear Physics B (Proc.Suppl.) in press.