Abstract: Young rotating neutron stars (pulsars) are considered as strong sources of TeV muon neutrinos, which are produced through the delta resonance in interactions of pulsar accelerated ions with its thermal radiation field. Here we have argued that the observed upper limit of gamma ray fluxes from nebula of potential neutrino pulsars severely constrain the fluxes of muon neutrinos from the sources.

Introduction

Pulsars are potential sites for acceleration of charged particles. Observation of pulsed and/or steady flux of electromagnetic radiations from radio to gamma ray energies provides direct evidence that some pulsars are site of energetic particles of at least several TeV. According to the Hillas condition [1], the maximum energy of a particle of charge Ze that can be contained near the light cylinder of a pulsar of angular speed $\Omega$ rad s$^{-1}$, radius $R$ and with the surface magnetic field $B_s = B_{12} \times 10^{12}$ Gauss is

$$E_{\text{max}} = 3.4 \times 10^{17} Z B_{12} \left( \frac{\Omega}{10^3 \text{ s}^{-1}} \right)^2 \left( \frac{R}{10^6 \text{ cm}} \right)^3 \text{ eV}$$ (1)

which shows that the maximum possible energy can be very large, for fast rotating pulsars $E_{\text{max}}$ even could reach around 100 EeV, the highest energy cosmic ray particles observed so far.

In a recent work [2], Link and Burgio have shown that pulsars could be strong sources of TeV muon neutrinos with fluxes observable by the operating or planed large area neutrino observatories. As a conjecture to be verified by observations they considered that protons or heavier ions are accelerated near the surface of the pulsar (by the polar caps) to PeV energies. When these accelerated ions interact with the thermal radiation field of pulsar the $\Delta$ resonance state may occur provided their energies exceed the threshold energy for the process. Though radiation losses limit the maximum energy that can be attained by a nuclei in the acceleration process but it appears that such an energy condition should be satisfied for several pulsars. Muon neutrinos are subsequently produced from the decay of $\Delta$ particles.

A young neutron star is generally encircled by pulsar wind nebula (PWN). Positive ions, after gaining energy from polar gaps will move away from the pulsar practically along the open field lines and will finally inject into the nebula. It is very likely that these energetic ions would be trapped by the magnetic field of the nebula for a long period and consequently they should produce an appreciable of high energy gamma rays by interacting with the matter of the nebula.

In this work we would estimate the expected fluxes of gamma rays resulting from interaction of pulsar injected energetic ions with matter of nebula for potential neutrino pulsars. The results will be compared with the observations.

TeV muon neutrinos from pulsars

Since the moment of inertia of a neutron star is around $10^{45}$ erg s$^{-2}$, a millisecond pulsar has a rotational energy $E = \frac{1}{2} I \Omega^2 \sim 10^{52}$ ergs. A fraction of such a huge rotational energy of a pulsar may be converted to the kinetic energy of the particles those present in the magnetosphere. The pulsar magnetosphere is usually considered to be
composed of electron-positron pairs but hadronic component also may exist in magnetosphere [3]. These nuclei can be accelerated by pulsars through large potential drop associated with strong electric field parallel to the pulsar magnetic field. Several detailed mechanisms have so far been suggested for accelerating particles by pulsars that include the popular polar gap [4, 5, 6], and the outer gap [7] models. In the former model, acceleration of particles takes place in the open field line region above the magnetic pole of the neutron star whereas in the case of outer gap model it occurs in the vacuum gaps between the neutral line and the last open line in the magnetosphere. Thus, the region of acceleration in the polar gap model is close to the pulsar surface, while the same in the outer gap model is close to the light cylinder.

Link and Burgio [2] conjectured that protons or heavier ions are accelerated near the surface of the pulsar by the polar caps to PeV energies. When pulsar accelerated ions interact with the thermal radiation field of pulsar the $\Delta$ resonance state may occur provided their energies exceed the threshold energy for the process. The threshold condition for production of $\Delta$ resonance state in $p\gamma$ interaction is given by

$$\epsilon_p\epsilon_\gamma (1 - \cos \theta_{p\gamma}) \geq 0.3 \ GeV^2$$

where $\epsilon_p$ and $\epsilon_\gamma$ are the proton and photon energies respectively and $\theta_{p\gamma}$ is the incident angle between the proton and photon in the laboratory frame. This implies that in a young pulsar atmosphere the condition for production of the $\Delta$ resonance is $B_{12} P_{\nu\mu}^{-2} T_{0.1 \ keV} \geq 3 \times 10^{-4}$ where $T_{0.1 \ keV} \equiv (kT_{\infty}/0.1 keV)$, $T_{\infty} \sim 0.1 keV$ being the typical surface temperature of young pulsars. Such a condition holds for many young pulsars and thus $\Delta$ resonance should be reached in pulsar atmosphere. Muon neutrinos are subsequently produced through the following channels:

$$p\gamma \rightarrow \Delta^+ \rightarrow n\pi^+ \rightarrow ne^+\nu_e\nu_\mu\bar{\nu}_\mu$$

The flux of muon neutrinos from pulsars can be estimated as follows. The charge density near the pulsar surface is $n_\nu \simeq eZn_o$, where $n_o(r) \equiv B_3 R^3 \Omega/(4\pi Z e c^3 v^3)$ is the Goldreich-Julian density of ions at radial distance $r$. For acceleration to take place there must be a charge depleted gap (here polar gap) and the density in the gap may be written as $f_d n_o$, where $f_d \leq 1$ is the depletion factor which is a model dependent quantity. Denoting the duty cycle of the neutrino beam as $f_b$ (typically $f_b \sim 0.1-0.3$), the phase averaged neutrino flux at Earth from a pulsar of distance $d$ is given by [2, 8]

$$\phi_\nu \simeq \eta f_b f_d (1 - f_d)n_o \left(\frac{R}{d}\right)^2 P_{\text{conv}}$$

where $P_{\text{conv}}$ is the probability that a PeV energy proton starting from the pulsar surface will produce $\Delta$ particle by interacting with thermal field (typically $P_{\text{conv}} \approx 0.02 T_{0.1 \ keV}^3$). Here it is assumed that $\eta$ fraction of the unit sphere is available for acceleration of ions ($\eta$ is taken as 1 in the Link and Burgio work [2, 8]).

### Magnetic trapping of pulsar accelerated PeV ions in nebulae

Conservation of magnetic flux across the light cylinder entails that outside the light cylinder $B \sim r^{-1}$ whereas far from the light cylinder radial component of magnetic field varies as $B_r \sim r^{-2}$. Thus (far) outside the light cylinder the azimuthal component of the magnetic field dominates over the radial field. Therefore, accelerated protons while moving away from the pulsar have to cross the field lines (for instance magnetic field lines at wind shock). The Larmor radius of particles (even for proton) of energy about 1 PeV is expected to be smaller than the radius of nebula during most of the time of the evolution of nebula. Thus it is very likely that energetic particles of PeV energies would be trapped by the magnetic field of the nebula. The energetic particles propagate diffusively in the envelope and they escape from from the nebula when the mean radial distance traveled by the particles becomes comparable with the radius of nebulae at the time of escaping. This time is somewhat uncertain due to uncertainty of the value of diffusion coefficient but is estimated as atleast few thousand years [9].

If still one insists that somehow accelerated protons manage to escape from the nebula within a short period, then resultant cosmic ray anisotropy due to nearby pulsars such as the Vela pulsar will
become inconsistent with the observations. To elaborate the point, let us consider that pulsar accelerated particles start to escape from the nebula after a short period of time and thus the pulsar continuously emitting cosmic rays of energy $E$ at a constant rate $c\eta f_d(1 - f_d) n_o R^2$ (as considered in the Link-Burgio model with $\eta = 1$) from time say $t_{on}$ until the present. Then the density of cosmic rays at time $t$ at a distance $d$ from the pulsar is [10]

$$I_{CR}(E) = c\eta f_d(1 - f_d) n_o R^2 \frac{I(x)}{4\pi D(E) d}$$

where $D(E)$ stands for the diffusion coefficient, $x = 4D(t - t_{on})/r^2$ and

$$I(x) = \frac{1}{\sqrt{\pi x}} \int_{1/x}^{\infty} \frac{du}{\sqrt{u}} e^{-u}$$

The amplitude of cosmic ray anisotropy due to a point source is given by [10]

$$\delta = h(E) \frac{3d}{2ct_{on}}$$

Here $h(E)$ denotes the ratio of the cosmic rays of energy $E$ from the source to the total observed flux of cosmic rays at the same energy from all sources. Now estimating the amplitude of anisotropy due to the Vela pulsar, for $\eta = .1$ we find that $\delta \sim 0.01$. The observed upper limit of cosmic ray anisotropy at 1 PeV is smaller than 0.001 [11, 12, 13]. If one also considers phase analysis of the first harmonic, the discrepancy will be greatly enhanced. Similarly for Geminga pulsar also the estimated cosmic ray anisotropy is higher than the observed limit.

Thus observed upper limit of cosmic ray anisotropy straightway suggests that either pulsar accelerated ions remain trapped for a long period before escape, which is a commonly accepted view or polar caps can not accelerate ions to PeV energies or sufficient number of ions are not available for acceleration near the surface.

**Gamma rays from neutrino pulsars**

As pointed out in the preceding section the pulsar injected ions of PeV energies should be trapped by the magnetic field of the nebula for a long period and consequently there would be an accumulation of energetic ions in nebula. These energetic ions will interact with the matter of the nebula. The rate of interactions ($\xi$) would be $n o \sigma_{pA}$, where $n$ is the number density of protons in nebula and $\sigma_{pA}$ is interaction cross-section. In each such interaction charged and neutral pions will be produced copiously. Subsequently the decays of neutral pions will produce gamma rays whereas charged pions and their muon daughters will give rise to neutrinos. If $m$ is the mean multiplicity of charged particles in proton-ion interaction, then the flux of gamma rays at a distance $d$ from the source roughly would be

$$\phi_{\gamma} = c\eta f_d n_o \left( \frac{R}{d} \right)^2 \xi mt$$

where the parameters have their usual meaning. Note that there should not be any reduction of flux due to pulsar duty cycle in the case of emission to nebula. Though $n_o$ is taken as constant but actually at the early stages of pulsar $n_o$ should be larger owing to the smaller pulsar period. So the above expression gives only a lower limit of flux. The flux could be even higher if ions are efficiently trapped by the dense filaments in nebula. Typical energy of these resultant gamma rays would be

$\sim 10^{15}/(6 * m) \text{ eV}$.

Assuming pulsar accelerated ions are protons, numerical values of the TeV gamma ray fluxes from nearby PWNe have been estimated from Eq.(1) and are shown in Table 1. The observational results [14, 15] are also given there for comparison.

**Conclusion**

It is clear that observed fluxes are substantially lower than those obtained from Eq.(1) unless $\eta$ is very small. This suggests that the polar cap can not accelerate ions to PeV energies and/or ions with sufficient numbers are not available in pulsar magnetosphere for acceleration. Consequently the fluxes of muon neutrinos from the sources should be much less than those given in [2].
Table 1: TeV gamma ray fluxes from some nearby PWNe

<table>
<thead>
<tr>
<th>PWNe</th>
<th>$n_{cm^{-3}}$</th>
<th>Expected flux $f_d \times 10^{-12} cm^{-2}s^{-1}$</th>
<th>Observed flux $10^{-12} cm^{-2}s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crab nebula</td>
<td>150 [16]</td>
<td>6000 $\eta$</td>
<td>10</td>
</tr>
<tr>
<td>Vela PWN</td>
<td>1</td>
<td>800 $\eta$</td>
<td>2.5</td>
</tr>
<tr>
<td>PSR1706 – 44</td>
<td>1</td>
<td>300 $\eta$</td>
<td>1.5</td>
</tr>
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References