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A Macroscopic Description of Coherent Geo-Magnetic Radiation from Cosmic Rays

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Abstract: We have developed a macroscopic description of geo-magnetic radiation from air showers initiated by ultra-high-energy cosmic rays. This description offers a simple and direct insight in the relation between the properties of the air shower and the time structure of the electromagnetic pulse.

The Formalism

When an ultra-high-energy (UHE) cosmic-ray particle enters the upper layers of the atmosphere, a cascade of high-energy particles – called a cosmic-ray air shower – develops. Due to the relativistic velocities, most of the particles are concentrated in the relatively thin shower front, which is called the 'pancake'. The pancake is assumed to be charge neutral for simplicity of the present discussion and contains large numbers of electrons and positrons. Near the core of the shower this plasma has a typical thickness of a few meters and is moving to the surface of the Earth with (almost) the velocity of light, $\beta_s c$ with $\beta_s \approx 1$. This pancake is moving through the magnetic field of the Earth. The Lorentz force on the charged particles induces a drift velocity and thus an electric current in the $\hat{x}$ direction, which is perpendicular to the magnetic field and the shower axis. At the surface of the Earth the electromagnetic radiation can be detected, emitted by this relatively constant electric current moving with high velocity towards the Earth. The shape of the electromagnetic pulse is determined by the (relatively slow) variation in time of the magnitude of the current, combined with time retardation effects.

We confine ourselves to a rather simple geometry to emphasize the basic principles. The cosmic shower moves straight towards the Earths surface (the $\hat{z}$ direction) and the magnetic field (with magnitude B_E) is parallel to the surface (the $\hat{y}$ direction), $\vec{B} = B_E \hat{y}$. The strength of the induced electric current depends on the distance h from the front of the shower and on the time t in the shower development. The direction of the current is in the $\hat{x}$ direction. The position of the shower front above the Earths surface is given by $z = -\beta_s ct$, reaching the surface at time $t = 0$. All quantities are measured in the rest system of the observer which coincides with the rest frame of the Earth.

In the calculation of the current density we will initially assume a finite extent in the horizontal directions (x and y), which soon will be integrated out, knowing that the charged-particle density is strongly peaked near the center of the shower. The electron/positron density is written as

$$\rho_e(x, y, z, t) = \int \tilde{\rho}_e(x, y, z, t, h) dh \qquad (1)$$

where we assume a simple factorized form,

$$\begin{aligned} \tilde{\rho}_e(x, y, z, t, h) &= N_e \delta(z + \beta_s ct + h) \\ &\times f_t(t) \rho_{\rm NKG}(x, y) \rho_p(h) \end{aligned} \qquad (2)$$

where N_e equals the total number of charged particles at the time of maximum shower development and the velocity of the shower front is given by $-\beta_s c\hat{z}$. The lateral distribution function [1] is normalized according to $\int \rho_{\rm NKG}(x, y)\, dx\, dy = 1$ [2]. The pancake distribution, parameterized following Ref. [3],

$$\rho_p(h) = h^\beta e^{-2h/L} \times (4/L^2) \, , \qquad (3)$$

obeys a similar normalization $\int \rho_p(h)\,dh = 1$. The temporal (or longitudinal) distribution $f_t(t)$ is parameterized in terms of the shower age as

$$N_e f_t(t) = N_e e^{(X - X_{\text{mx}} - 1.5X \ln s)/X_0} \ . \quad (4)$$

The parameter X_{mx} is chosen such as to reproduce the positions of the shower maxima as have been determined from shower simulations [4]. The maximum of the temporal distribution $f_t(t)$ is normalized to unity.

The magnitude of the induced current in the $\hat{x}$-direction is calculated as the number of electrons (and positrons) multiplied by an average drift velocity,

$$j(x,y,z,t) = \int \langle v_d \rangle \, e \, \tilde{\rho}_e(x,y,z,t,h) dh \ . \quad (5)$$

The drift velocity, v_d, of an electron with vertical velocity βc, depends critically on the electron path length L,

$$v_d = \frac{cLeB_E}{2\beta\gamma m_e c}. \quad (6)$$

At high energies, $\epsilon_e > 10$ MeV, the electron cross section is dominated by pair creation and the path length is given by $L_R = X_0/\rho_{air}$ where the electronic radiation length is $X_0 = 36.7\ \text{g}\,\text{cm}^{-2}$ and the density of air is $\rho_{air} = 10^{-3}\ \text{g}\,\text{cm}^{-3}$ at sea level. For smaller energies Coulomb scattering becomes the dominant scattering mechanism with a path length between successive collisions of $1\ \text{g}\,\text{cm}^{-2}$. Since Coulomb scattering is strongly forward peaked, several collisions are necessary to randomize the momentum and the scattering angle due to multiple Coulomb scattering after traversing a distance d, is $\delta\langle\theta^2\rangle = d(20/\gamma)^2\rho_{air}/X_0$. When $\delta\langle\theta^2\rangle \approx 1$, the original direction is lost, giving an effective path length $L_C = (\gamma/20)^2 X_0/\rho_{air}$. Over the whole energy range, the path length L may be written as $1/L = 1/L_C + 1/L_R$ or

$$L = \frac{\gamma^2}{\gamma^2 + 20^2} \frac{X_0}{\rho_{air}} \ . \quad (7)$$

The drift velocity can now be expressed as

$$v_d = \frac{c}{2\,\beta} \frac{\gamma}{\gamma^2 + 20^2} \frac{eB_E}{mc} \frac{X_0}{\rho_{air}} \quad (8)$$

The average drift velocity $\langle v_d \rangle$ is obtained by averaging over the energy distribution of electrons in a cosmic-ray air shower, see Ref. [5], giving $\langle v_d \rangle = 0.046$ c.

Given a current density, j^μ, the vector potential can be obtained using the Liénard-Wiechert fields,

$$A^\mu(x) = \frac{1}{4\pi\varepsilon_0} \int \left. \frac{j^\mu}{R(1 - \vec{\beta}\cdot\hat{n})} \right|_{\text{ret}} dh \ . \quad (9)$$

We use the common notation where $\hat{n}$ is a unit vector pointing from the source to the observer, R is the distance all evaluated at retarded time. Assuming that all particles move with the velocity of the shower front, the denominator in Eq. (9) can be rewritten to give

$$\begin{aligned} \mathcal{D} &= R(1 - \vec{\beta}_s \cdot \hat{n})|_{\text{ret}} \\ &= \sqrt{(-c\beta_s t + h)^2 + (1 - \beta_s^2 n^2) d^2} \end{aligned} \quad (10)$$

where n is the index of refraction. The distance between the observer and the point of impact of the core of the air shower is denoted by d.

In the present case the current density has only an $\hat{x}$-component, giving

$$A^x(t,d) = J \int dh \frac{\rho_p(h) f_t(t_r)}{\mathcal{D}} \ , \quad (11)$$

where the current density is assumed to be parameterized according to Eq. (5) with $J = \langle v_d \rangle N_e e/\{4\pi\varepsilon_0\}$. We use MKSA units where $\frac{e}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ [Vm] to get E in [V/m].

The upper limit of the integral over h extends only up to $c\beta_s t$ giving for the electric field

$$E_x(t,d) = J \frac{d}{dt} \int_0^{c\beta_s t} dh \frac{\rho_p(h) f_t(t_r)}{\mathcal{D}} \ . \quad (12)$$

After performing a partial integration and using the fact that $\rho_p(h) = 0$ for $h = 0$ we obtain

$$\begin{aligned} E_x(t,d) &= J \int_0^{c\beta_s t} dh \, \frac{f_t(t_r)}{\mathcal{D}} \beta \frac{d\,\rho_p(h)}{dh} \\ &\quad + J \int_0^{c\beta_s t} dh \, \frac{\rho_p(h)}{\mathcal{D}} \frac{df_t(t_r)}{dt_r} \end{aligned} \quad (13)$$

where one should be careful in evaluating the integral because of the $1/\sqrt{c\beta_s t - h}$ pole in $\frac{1}{\mathcal{D}}$.

To obtain a simple estimate for the emitted radiation one may at this point take the limit $\beta_s = 1$ $\beta_s^2 n^2 = 1$, resulting in

$$\mathcal{D} = c\beta_s t + \mathcal{O}(1 - \beta_s^2) \approx ct \quad (14)$$

and

$$ct_r = \frac{ct}{1+\beta_s} - \frac{d^2}{2c\beta_s t} + \mathcal{O}(1-\beta_s^2) \approx -\frac{d^2}{2ct} . \tag{15}$$

The retarded time is large and negative since $d^2 \gg ct^2$ for the cases of interest. A simple analytic expression for the electric field can now be obtained using $\int dh\, \rho_p(h)h \approx 0$ and $\int dh\, \rho_p(h) = 1$,

$$E_x(t,d) \approx -J\frac{c^2 t_r^2 4}{d^4}\left[t_r \frac{df_t(t_r)}{dt_r} + f_t(t_r)\right] . \tag{16}$$

This equation is valid for large distances to the shower core, d. For small distances the approximations made in deriving the expression for the retarded time, Eq. (15), no longer apply. From Eq. (16) it can be seen that the time structure of the pulse is given by a rather simple function depending on the retarded time, independent of the distance. If at distance d the peak of the pulse occurs at time t_0, at twice the distance, the signal peak occurs at a time $4 \times t_0$. The signal is four times as broad while the peak value of the pulse, occurring at the same retarded time, has decreased by a factor 2^4. Interesting to note here is that the earlier part of the signal contains the information of the earlier parts (at the higher altitude) in the air-shower development.

It should be noted that the emitted radiation does not contain a relativistic Lorentz γ factor and therefore does not depend strongly on the exact velocity of the shower front, as long as it is close to c.

Results

The effect of a finite pancake thickness can also be seen in the frequency response of the pulse as shown in Fig. 1. At shorter distances from the point of impact of the shower the effect of finite thickness is to suppress the higher frequency components since the signal is only coherent for wave lengths larger than the typical size of the emitting system. The signal, in the limit where the thickness is ignored, does depend strongly on the distance away from the shower core since the projected longitudinal extend enters, which equals to zero when viewing the shower head-on. Including a finite thickness reduces the dependence of the pulse shape on the distance from the core.

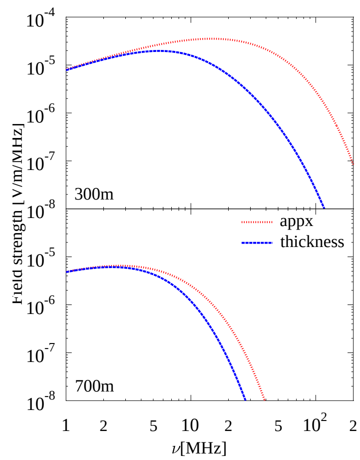

Figure 1: [color online] Frequency response of the pulse at 300 m and 700 m from the core for a 10^{17} eV shower. In the calculations labeled 'appx' and 'thickness' the thickness of the pancake is zero (Eq. (16)) and 10 m respectively.

In Fig. 2 the electric field is plotted as function of time for an observer at various distances d from the shower core. The primary energy is 10^{17} eV and the calculation includes only the effects of the pancake thickness. The shower core hits the Earths surface at $t = 0$. At large distances the pulse decreases in magnitude even faster than d^2, as predicted by Eq. (16). At small distances important deviations from the simple parametrization are observed. This is partially due to the fact that the approximations made to arrive at Eq. (16) are no longer valid, and partially since the finite thickness of the pancake has been taken into account.

Summary

In this work a relatively simple macroscopic picture is presented for the emission of electromagnetic radiation from the particle shower initiated by a high-energy cosmic ray. The radiation is emitted through the electromagnetic current which is induced by the Earths magnetic field in the plasma at the front end of the shower. Under some simplifying assumptions a simple algebraic equation can be derived which clearly shows that the time structure of the pulse directly reflects the longitudinal development of the number of electrons (and

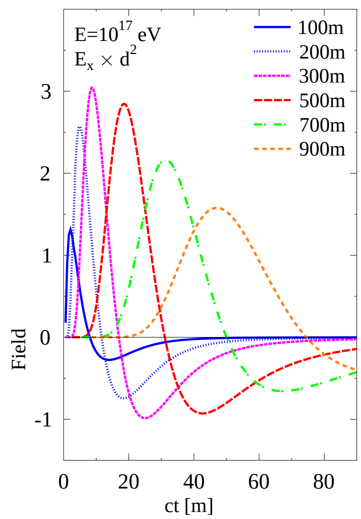

Figure 2: [color online] Electric field strength (multiplied by d^2) at different distances from the shower core as function of time.

positrons) in the shower. From Fig. 1 it is clear that the simple algebraic form, Eq. (16), is very appropriate to get insight into the gross structure of the pulse form. Due to the typical time scales involved the emitted radiation is in the regime of long wavelength radio waves, 1-30 MHz. If also the length scale of the pancake thickness is taken into account the collective response shifts to lower frequencies.

In this first paper we have restricted ourselves to a very simple geometry which allowed us to derive analytic formulas which show the direct relation between the emitted pulse and the development of the air shower. In a future publication emission from showers for a more general geometry will be investigated.

In our approach the collective aspect is strongly emphasized by treating the induced current as macroscopic. This is in contrast to the calculations of Refs. [6, 7] where the motion of individual particles is stressed (microscopic approach). However, in both the macroscopic and the microscopic approach the emission of the electromagnetic pulse is caused by moving charges in the Earths magnetic field. Therefore these two pictures should be regarded as presenting a complementary view of the same physical phenomenon. The basic features and magnitudes of the frequency responses are very similar in both approaches. A difference is seen in the time-structure of the peak. The pulse of Refs. [6, 7] has a simple unipolar structure. A simple explanation for the bi-polar structure of the pulse obtained in this work is the fact that the vector potential is positive definite and vanishes for both small and large times. The electric field is the time derivative of this vector potential and crosses zero at the time when the vector potential reaches a maximum. For the leading part of the pulse the two terms in Eq. (16) add constructively resulting it the large leading positive part.

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