Conformal renormalization of Einstein-AdS gravity in 8D

(Based on 2607.03679)

Ignacio J Araya
Universidad Andrés Bello

Physics Seminar - ICN-UNAM - Ciudad de México

August 5th, 2026

IN COLLABORATION WITH:

GIORGOS ANASTASIOU; NICOLAS BOULANGER; RODRIGO OLEA; DAVIDE ROVERE

AdS/CFT holography and FG expansion

- Partition function for AdS gravity ≈ generating functional of CFT correlators. [Gubser, Klebanov and Polyakov 1998 ; Witten 1998]
- In saddle point approximation: $I_{AdS}\left[\phi_i\right] \approx -\ln\left(\left\langle e^{\int \phi_{0,i} O^i}\right\rangle_{CFT}\right) = -W\left[\phi_{0,i}\right]$
- AlAdS metric near the boundary admits a FG expansion:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}\left(dz^2 + \bar{g}_{ij}(z,x)\,dx^i dx^j\right);$$

$$\bar{g}_{ij}(z,x) = g_{(0)ij}(x) + \frac{z}{\ell}g_{(1)ij}(x) + \frac{z^2}{\ell^2}g_{(2)ij}(x) + \frac{z^3}{\ell^3}g_{(3)ij}(x) + \ldots$$

- Line element in AlAdS spacetime diverges → AdS gravity action diverges.
- Finite $I_{AdS}\left[\phi_i\right]$ + well-defined Dirichlet variational principle → renormalization compatible with holography.

HR of Einstein-AdS gravity in 4D

- Consider the theory given by $I_{\rm EH} = \frac{1}{16\pi G}\int_{\mathscr{M}_4} d^4x\sqrt{|g|}\left(R+\frac{6}{\ell^2}\right)$.
- The odd power coefficients in the FG expansion are zero.
- $g_{(2)ij}$ is fixed in terms of $g_{(0)ij}$ by the EOM. $g_{(3)ij}$ is not fixed by the EOM.
- In holography, $g_{(0)ij}$ is the source and $g_{(3)ij}$ is the dual operator (CFT stress-tensor).
- Finiteness and variational principle achieved by Holographic Renormalization counterterms [Emparan, Johnson and Myers 1999]:

$$I^{ren}_{AdS} = I_{EH} + \frac{1}{8\pi G}\int_{\partial\mathscr{M}_4} d^3x\sqrt{|h|}\left(-K+\frac{2}{\ell}+\frac{\ell}{2}\mathscr{R}\right)$$

Topological Renormalization in 4D

- Alternatively, consider adding the topological Gauss-Bonnet term:

$$I_{\rm EH}^{\rm TR} = \frac{1}{16\pi G}\int_{\mathscr{M}_4} d^4x\sqrt{|g|}\left(R + \frac{6}{\ell^2} + \frac{\ell^2}{4}GB\right);\, GB = \frac{1}{4}\delta^{\nu_1\cdots\nu_4}_{\mu_1\cdots\mu_4}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4} = R^2 - 4R^\mu_\nu R^\nu_\mu + R^{\lambda\sigma}_{\mu\nu}R^{\mu\nu}_{\lambda\sigma}$$

- Resulting action has same Einstein EOM but is finite on AdS spaces. Noether-Wald charges reproduce conformal mass [Ashtekar and Magnon 1984; Ashtekar and Das 2000]. Topological Renormalization [Olea 2005].

- One can write identically

$$I_{EH}^{TR} = \frac{\ell^2}{256\pi G}\int d^4x\sqrt{|g|}\,\delta^{\mu_1\ldots\mu_4}_{\nu_1\ldots\nu_4}\Omega^{\nu_1\nu_2}_{\mu_1\mu_2}\Omega^{\nu_3\nu_4}_{\mu_3\mu_4}\,;\ \text{with}\ \ \Omega^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\delta^{\alpha\beta}_{\mu\nu}$$

- For pure AdS, $\Omega^{\alpha\beta}_{\mu\nu} = 0$ and so $I_{EH}^{TR} = 0$.

Topological Renormalization in 4D

- TR can be compared to HR by considering the Chern form.
- Related to the Gauss-Bonnet density through the Euler theorem:

$$\int_{\mathscr{M}_4} d^4x\sqrt{|g|}\,GB = 32\pi^2\chi\left(\mathscr{M}_4\right) + \int_{\partial\mathscr{M}_4} B_3 \;\; ; \;\; B_3 = -2d^3x\sqrt{|h|}\,\delta^{j_1j_2j_3}_{i_1i_2i_3}\left(K^{i_1}_{\;j_1}\mathscr{R}^{i_2i_3}_{\;\;j_2j_3} - \frac{2}{3}K^{i_1}_{\;j_1}K^{i_2}_{\;j_2}K^{i_3}_{\;j_3}\right)$$

- Up to the Euler characteristic, the Topologically-renormalized Einstein-AdS action is given by

$$I^{TR}_{AdS} = \frac{1}{16\pi G}\left(\int_{\mathscr{M}_4} d^4x\sqrt{|g|}\left(R + \frac{6}{\ell^2}\right) + \frac{\ell^2}{4}\int_{\partial\mathscr{M}_4} B_3\right)$$

Topological Renormalization in 4D

- To show equivalence between HR and TR, consider FG expansion of extrinsic curvature:

$$K^i_j = \frac{1}{\ell}\delta^i_j + \ell\,\mathcal{S}^i_j + \dots \;\; ; \;\; \mathcal{S}^i_j = \frac{1}{2}\left(\mathcal{R}^i_j - \frac{1}{6}\mathcal{R}\delta^i_j\right)$$

- Extrinsic curvature has expansion in terms of intrinsic curvature near AdS boundary.
- In TR there is no addition of GHY term. Add and subtract in order to compare with HR:

$$I^{TR}_{EH} = I_{EH} + \frac{1}{8\pi G}\int_{\partial\mathcal{M}_4} d^3x\sqrt{|h|}K + \int_{\partial\mathcal{M}_4} d^3x\mathcal{L}_{KT},$$

$$\mathcal{L}_{KT} = -\frac{\ell^2}{32\pi G}\sqrt{|h|}\,\delta^{i_1 i_2 i_3}_{j_1 j_2 j_3}\left(K^{j_1}_{i_1}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - \frac{2}{3}K^{j_1}_{i_1}K^{j_2}_{i_2}K^{j_3}_{i_3} + \frac{2}{\ell^2}\,\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}K^{j_1}_{i_1}\right).$$

- Replace FG expansion of K^i_j, expand and keep up to normalizable order $\mathcal{O}\left(z^3\right)$.

Topological Renormalization in 4D

- The extrinsic "Kounterterm" becomes purely intrinsic near the AdS boundary:

$$\mathscr{L}_{KT} = \frac{\ell^2}{32\pi G}\sqrt{h}\,\delta^{i_1 i_2 i_3}_{j_1 j_2 j_3}\left(\frac{1}{\ell}\delta^{j_1}_{i_1} - \ell\,\mathcal{S}^{j_1}_{i_1}\right)\left(\mathscr{R}^{j_2 j_3}_{i_2 i_3} - \frac{2}{3}\left(\frac{1}{\ell}\delta^{j_1}_{i_1} - \ell\,\mathcal{S}^{j_1}_{i_1}\right)\left(\frac{1}{\ell}\delta^{j_1}_{i_1} - \ell\,\mathcal{S}^{j_1}_{i_1}\right) + \frac{2}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

- After some algebra…

$$\mathscr{L}_{KT} = \frac{1}{8\pi G}\sqrt{h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathscr{R}\right) + \ldots. \simeq \mathscr{L}_{HR},$$

- Equivalent up to terms that vanish as z→0. [Olea and Mišković 2009]
- Resulting action has a well defined Dirichlet variational principle with respect to $\delta g_{(0)ij}$ at the AdS boundary. However, NOT Dirichlet with respect to δh_{ij}.
- Resulting action is also finite. Good for holography.

Conformal Renormalization in 4D

- Topological Renormalization can be obtained from Conformal Gravity [Anastasiou and Olea 2016]:

$$I_{\rm CG} = C_{\rm CG}\int d^4x\sqrt{|g|}\,W^{\alpha\beta}_{\mu\nu}W^{\mu\nu}_{\alpha\beta} \quad ; \quad W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4S^{[\alpha}_{[\mu}\delta^{\beta]}_{\nu]}$$

- In 4D there is only one local conformal invariant.

- The EOM of $I_{\rm CG}$ gives $B_{\mu\nu} = 0$, where

$$B_{\mu\nu} = -4\left(\nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma}W_{\mu\lambda\nu\sigma}\right) \quad ; \quad C_{\mu\nu\lambda} = \nabla_\lambda S_{\mu\nu} - \nabla_\nu S_{\mu\lambda}\,.$$

- Einstein spacetimes have $B_{\mu\nu} = 0 \rightarrow$ Solutions of CG. In this case

$$S^\mu_\nu = -\frac{1}{2\ell^2}\delta^\mu_\nu \;;\; W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\delta^{\alpha\beta}_{\mu\nu} = \Omega^{\alpha\beta}_{\mu\nu} \;;\; I_{\rm CG}[E] = \frac{C_{\rm CG}}{4}\int d^4x\sqrt{|g|}\,\delta^{\mu_1\ldots\mu_4}_{\nu_1\ldots\nu_4}\Omega^{\nu_1\nu_2}_{\mu_1\mu_2}\Omega^{\nu_3\nu_4}_{\mu_3\mu_4}\,.$$

Einstein-AdS from Conformal Gravity

- For $C_{\rm CG} = \dfrac{\ell^2}{64\pi G}$, one has $I_{\rm CG}[E] = I_{EH}^{TR}$.
- Action of CG for Einstein-AdS spacetimes = Topological renormalization of Einstein-AdS action.
- Einstein-AdS spacetimes are solutions of CG and CG action becomes Einstein-AdS action → Einstein-AdS gravity can be embedded into CG.
- Renormalization of Einstein-AdS gravity by embedding it into CG is called Conformal renormalization.
- Imposing Einstein condition on CG can be understood holographically.
- Define $H_{\mu\nu} = R_{\mu\nu} - \dfrac{1}{4} R g_{\mu\nu}$, such that the Einstein-AdS condition corresponds to $H_{\mu\nu} = 0$.
- For Conformal Gravity $g_{(1)ij}$ and $g_{(2)ij}$ are not fixed. The theory is higher (4th) order. Expand $H_{\mu\nu}$.

Holographic Einstein Condition

- Expanding $H_{\mu\nu}$ we have:

$$H^i_j = \frac{z}{\ell^3}\left(g^i_{(1)j} - \frac{1}{4}g_{(1)}\delta^i_j\right) + \frac{z^2}{\ell^2}\left(\frac{1}{16\ell^2}g_{(1)ms}g^{ms}_{(1)}\delta^i_j + \frac{1}{16\ell^2}g^2_{(1)}\delta^i_j - \frac{1}{4\ell^2}g^i_{(1)j}g_{(1)}\right.$$
$$\left. -\frac{1}{2\ell^2}g^m_{(1)j}g^i_{(1)m} + \frac{1}{\ell^2}g^i_{(2)j} + \mathscr{R}^i_{(0)j} - \frac{1}{4}\mathscr{R}_{(0)}\delta^i_j\right) + \mathscr{O}\left(z^3\right).$$

$$H^z_j = \frac{z^2}{2\ell^2}\left(D_{(0)j}g_{(1)} - D_{(0)m}g^m_{(1)j}\right) + \mathscr{O}\left(z^3\right);\ H^z_z = -\frac{z}{4\ell^3}g_{(1)} + \frac{z^2}{\ell^2}\left(\frac{5}{16\ell^2}g_{(1)ij}g^{ij}_{(1)} + \frac{1}{16\ell^2}g^2_{(1)} - \frac{1}{\ell^2}g_{(2)} - \frac{1}{4}\mathscr{R}_{(0)}\right) + \mathscr{O}\left(z^3\right)$$

- Einstein condition $H^\mu_\nu = 0$ understood holographically in terms of Neumann boundary condition for the induced metric (i.e. $\partial_z g_{ij}(z,x)\big|_{z=0} = g_{(1)ij} = 0$) [Maldacena 2011 ; Anastasiou, IJA and Olea 2020].

- Also $g_{(2)ij} = -\ell^2\mathscr{S}_{(0)ij}$ universally, consistent with PBH transformations [Imbimbo, Schwimmer, Theisen and Yankielowicz 1999].

Holographic data of CG

- The Weyl-squared action has three pieces: $W^{\alpha\beta}_{\mu\nu}W^{\mu\nu}_{\alpha\beta} = W^{ij}_{km}W^{km}_{ij} + 4W^{iz}_{jz}W^{jz}_{iz} + 4W^{iz}_{km}W^{km}_{iz}$.
- Each piece is $\mathcal{O}\left(z^4\right)$. Then, asymptotically the action behaves as

$$\int d^4x\sqrt{|g|}\,W^{\alpha\beta}_{\mu\nu}W^{\mu\nu}_{\alpha\beta} \sim \int d^3x \int dz \frac{\sqrt{|g_{(0)}|}}{z^4}\mathcal{O}\left(z^4\right) \sim \mathcal{O}\left(z\right) + \text{finite}$$

- The on-shell variation is given by [Grumiller et al. 2014]

$$\delta I_{\text{CG}}\Big|_{\text{EOM}} = \int_{\partial\mathcal{M}_4} d^3x\sqrt{|g_0|}\left(\tau^{ij}\delta g_{(0)ij} + P^{ij}\delta g_{(1)ij}\right)$$

- τ^{ij} and P^{ij} are the holographic stress tensor and the partially massless response. $g_{(0)ij}$ and $g_{(1)ij}$ are their respective holographic sources. Dirichlet variational principle for two independent sources in the CFT.
- For $\{g_{(1)ij} = 0\,, g_{(2)ij} = -\,\ell^2\mathcal{S}_{(0)ij}\,, tr[g_{(3)ij}] = 0\}$, $\tau_{ij} \propto g_{(3)ij} \rightarrow$ holographic stress-tensor for EH.

Renormalized EH from CG in 6D

- In 6D there are three linearly-independent local conformal invariants:

$$I_1 = W_{\alpha\beta\mu\nu}W^{\alpha\rho\lambda\nu}W_{\rho}{}^{\beta\mu}{}_{\lambda}\,,\; I_2 = W_{\mu\nu\alpha\beta}W^{\alpha\beta\rho\lambda}W_{\rho\lambda}{}^{\mu\nu}\,,\; I_3 = W_{\mu\rho\sigma\lambda}\left(\delta^{\mu}_{\nu}\Box + 4R^{\mu}_{\nu} - \frac{6}{5}R\delta^{\mu}_{\nu}\right)W^{\nu\rho\sigma\lambda} + \nabla_{\mu}J^{\mu},$$

where $J_{\mu} = 4R_{\mu}{}^{\lambda\rho}\nabla^{\nu}R_{\nu\lambda\rho} + 3R^{\nu\lambda\rho}\nabla_{\mu}R_{\nu\lambda\rho} - 5R^{\nu\lambda}\nabla_{\mu}R_{\nu\lambda} + \frac{1}{2}R\nabla_{\mu}R - R^{\nu}{}_{\mu}\nabla_{\nu}R + 2R^{\nu\lambda}\nabla_{\nu}R_{\lambda\mu}\,.$

- There is a unique combination (up to normalization) which admits all Einstein spacetimes as solution [Lu, Pang and Pope 2013]:

$$I_{CG} = \beta\int_{M_6} d^6x\sqrt{-G}\left(4I_1 + I_2 - \frac{1}{3}I_3\right).$$

- The action can be re-written as [Anastasiou, IJA and Olea 2020]:

$$I_{\rm LPP} = \int_{\mathscr{M}_6} {\rm d}^6x\sqrt{|g|}\,\mathscr{L}_{\rm LPP} + \int_{\partial\mathscr{M}_6} {\rm d}^5x\sqrt{|h|}\,n_{\mu}\hat{J}^{\mu}\,.$$

Renormalized EH from CG in 6D

- Where

$$\mathscr{L}_{\rm LPP} = \alpha\left(\frac{1}{4!}\delta^{\nu_1\cdots\nu_6}_{\mu_1\cdots\mu_6}W^{\mu_1\mu_2}{}_{\nu_1\nu_2}W^{\mu_3\mu_4}{}_{\nu_3\nu_4}W^{\mu_5\mu_6}{}_{\nu_5\nu_6} + \frac{1}{2}\delta^{\nu_1\cdots\nu_5}_{\mu_1\cdots\mu_5}W^{\mu_1\mu_2}{}_{\nu_1\nu_2}W^{\mu_3\mu_4}{}_{\nu_3\nu_4}S^{\mu_5}{}_{\nu_5} + 8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right),$$

$$\hat{J}^\mu = \alpha\left(8W^{\mu\kappa\lambda\nu}C_{\kappa\lambda\nu} - W^{\kappa\lambda}{}_{\nu\sigma}\nabla^\mu W_{\kappa\lambda}{}^{\nu\sigma}\right).$$

- For Einstein spaces, one has that

$$I_{CG}[E] = -4!\alpha\int_M d^6x\sqrt{-G}\,P_6(\Omega) - \alpha\int_{\partial M} d^5x\sqrt{-h}\,n^\mu\left(\Omega^{\kappa\lambda}{}_{\nu\sigma}\nabla_\mu\Omega^{\nu\sigma}{}_{\kappa\lambda}\right),$$

$$P_6(\Omega) = -\frac{1}{(4!)^2}\delta^{\nu_1\cdots\nu_6}_{\mu_1\cdots\mu_6}\Omega^{\mu_1\mu_2}{}_{\nu_1\nu_2}\Omega^{\mu_3\mu_4}{}_{\nu_3\nu_4}\Omega^{\mu_5\mu_6}{}_{\nu_5\nu_6} + \frac{1}{2(4!)\ell^2}\delta^{\nu_1\cdots\nu_4}_{\mu_1\cdots\mu_4}\Omega^{\mu_1\mu_2}{}_{\nu_1\nu_2}\Omega^{\mu_3\mu_4}{}_{\nu_3\nu_4}.$$

- Because $\ell^4 P_6(\Omega) = R + \frac{20}{\ell^2} - \frac{\ell^4}{72}\mathscr{E}_6$, where $\mathscr{E}_6 = \frac{1}{8}\delta^{\nu_1\cdots\nu_6}_{\mu_1\cdots\mu_6}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}$, one has that for $\alpha_{\rm EH} = -\frac{\ell^4}{384\pi G}$:

Renormalized EH from CG in 6D

$$I_{\rm CG}[E]\Big|_{\alpha_{\rm EH}} = \frac{1}{16\pi G}\int_M d^6x\sqrt{-g}\left(R+\frac{20}{\ell^2}-\frac{\ell^4}{72}\mathscr{E}_6+\frac{\ell^4}{48}\Box\left(\Omega^{\rho\lambda}_{\mu\nu}\Omega^{\mu\nu}_{\rho\lambda}\right)\right).$$

- Matches the Topological Renormalization of Einstein-AdS in 6D [Olea 2005], with extra term which cancels remaining divergence when boundary is not conformally flat [Anastasiou, IJA and Olea 2020; Anastasiou, IJA, Corral and Olea 2023].

- Action obtained from LPP CG and its asymptotic charges are finite. Holographic stress tensor matches the Holographic Renormalization result: $\tau_{ij} \propto g_{(5)ij}$.

- One can compare this action to what is obtained from Holographic renormalization:

$$\mathscr{L}_{ct} = -\frac{4}{\ell}-\frac{4\ell}{3}\mathcal{S}+\frac{\ell^3}{2}\delta^{ik}_{jl}\mathcal{S}^j_i\mathcal{S}^l_k,$$

$$I_{\rm ren} = \frac{1}{16\pi G}\int_{\mathscr{M}_6} d^6x\sqrt{-g}\left(R+\frac{20}{\ell^2}\right)+\frac{1}{8\pi G}\int_{\partial\mathscr{M}_6} d^5x\sqrt{-h}\,\left(K+\mathscr{L}_{ct}\right).$$

Renormalized EH from CG in 6D

- From the Euler theorem one has

$$\int_{M_6} d^6x\sqrt{-g}\,\mathscr{E}_6 = (4\pi)^3\, 3\,!\chi\left[M_6\right] + \int_{\partial M_6} d^5x B_5 \text{ where}$$

$$B_5 = -6\sqrt{-h}\,\delta^{i_1\cdots i_5}_{j_1\cdots j_5} K^{j_1}{}_{i_1}\int_0^1 dt\left(\frac{1}{2}\mathscr{R}^{j_2j_3}{}_{i_2i_3}(h) - t^2K^{j_2}{}_{i_2}K^{j_3}{}_{i_3}\right)\left(\frac{1}{2}\mathscr{R}^{j_4j_5}{}_{i_4i_5}(h) - t^2K^{j_4}{}_{i_5}K^{j_4}{}_{i_5}\right).$$

- Adding and subtracting the GHY term one defines

$$\mathscr{L}_{KT} = -\frac{\ell^4}{144}\frac{B_5}{\sqrt{-h}} - K\text{, such that}$$

$$I^{TR}_{\text{EH}} = \frac{1}{16\pi G}\int_{\mathscr{M}_6} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2}\right) + \frac{1}{8\pi G}\int_{\partial\mathscr{M}_6} d^5x\sqrt{-h}\left(K + \mathscr{L}_{KT}\right).$$

Renormalized EH from CG in 6D

- Using the expansion of K^i_j as before, one has

$$\mathscr{L}_{KT} = -\frac{4}{\ell} - \frac{4\ell}{3}\mathcal{S} + \frac{\ell^3}{2}\left[\delta^{i_1 i_2}_{j_1 j_2}\mathcal{S}^{j_1}{}_{i_1}\mathcal{S}^{j_2}{}_{i_2} + \frac{1}{12}\mathscr{W}^{mn}{}_{ij}\mathscr{W}^{ij}{}_{mn}\right],$$ such that

$$\mathscr{L}_{ct} - \mathscr{L}_{KT} = -\frac{\ell^3}{24}\mathscr{W}^{mn}{}_{ij}\mathscr{W}^{ij}{}_{mn}\,.$$

- Considering that $\int_{M_6} d^6x\sqrt{-g}\,\Box\left(\Omega^{\rho\lambda}_{\mu\nu}\Omega^{\mu\nu}_{\rho\lambda}\right) = -\frac{4}{\ell}\int_{\partial M_6} d^5x\sqrt{-h}\mathscr{W}^{mn}{}_{ij}\mathscr{W}^{ij}{}_{mn}$ one gets $I_{\rm CG}[E]\Big|_{\alpha_{\rm EH}} = I_{ren}\,.$

- Taking the Einstein condition can be interpreted asymptotically as extended Neumann boundary conditions: $\partial_z\bar{g}_{ij} = 0$, $\partial_z^3\bar{g}_{ij} = 0$ so that $g_{(1)ij} = 0$ and $g_{(3)ij} = 0$.

- More holographic sources at the boundary. [Anastasiou, IJA and Olea 2020]

CG in 8D with Einstein sector

- In 8D there are 12 linearly independent local conformal invariants [Boulanger and Erdmenger 2004].
- 7 of them are inequivalent contractions of four Weyl tensors.
- 3 of them are non-trivial invariants involving explicit covariant derivatives of curvatures.
- 2 of them are conformal invariants which can be written as divergences of vectors, such that they correspond to pure boundary terms.
- There is a unique combination, up to an arbitrary combination of the two pure-boundary CIs, which admits all Einstein spaces as solutions. The BR CG given in [Boulanger and Rovere 2025]:

$$I_{\mathrm{BR}}[g_{\mu\nu}] = \int_{\mathscr{M}_8} \mathrm{d}^8x\sqrt{-g}\left(\alpha\mathscr{L}_8 + \beta\nabla_\mu J^\mu_{(1)} + \gamma\nabla_\mu J^\mu_{(2)}\right),$$

$$\mathscr{L}_8 = -\frac{1}{4}\left(I_1 - \frac{23}{25}I_2 - \frac{21}{25}I_3 - 39I_6 + \frac{2}{5}I_7 - \frac{7}{2}I_8 + \frac{124}{5}I_9 + \frac{9}{20}I_{10} - 4I_{11} - \frac{568}{5}I_{12}\right).$$

Conformal Renormalization in 8D

- Consider the action of 8D Einstein-AdS gravity supplemented by the Euler term:

$$I_{EH}^{TR} = \frac{1}{16\pi G}\int_{M_8}\sqrt{-g}d^8x\left(R+\frac{42}{\ell^2}+\frac{\ell^6}{2880}\mathscr{E}_8\right), \text{ where } \mathscr{E}_8 = \frac{1}{16}\delta^{\nu_1\cdots\nu_8}_{\mu_1\cdots\mu_8}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}R^{\mu_7\mu_8}_{\nu_7\nu_8}\,.$$

- Action is finite for Einstein-AdS spaces with conformally flat boundary. Divergent in general. [Anastasiou, Mišković, Olea and Papadimitriou 2020]

- To find the divergence structure, consider the identity:

$$\left(R+\frac{42}{\ell^2}+\frac{\ell^6}{2880}\mathscr{E}_8\right) = \frac{\ell^6}{80}\left(\frac{1}{\ell^4}\Phi_{(2)}-\frac{1}{36\ell^2}\Phi_{(3)}+\frac{1}{576}\Phi_{(4)}\right), \text{ where}$$

$$\Phi_{(k)} := \delta^{\mu_1\cdots\mu_{2k}}_{\nu_1\cdots\nu_{2k}}\Omega_{\mu_1\mu_2}{}^{\nu_1\nu_2}\ldots\Omega_{\mu_{2k-1}\mu_{2k}}{}^{\nu_{2k-1}\nu_{2k}}\,.$$

- Also consider that for Einstein-AdS spaces $\Omega^{\mu\nu}_{\rho\lambda} = W^{\mu\nu}_{\rho\lambda}$.

Conformal Renormalization in 8D

- Bulk Weyl tensor for Einstein manifolds has radial decomposition:

$$W_{\mu\nu}{}^{\alpha\beta}W_{\alpha\beta}{}^{\mu\nu} = W_{mn}{}^{ij}W_{ij}{}^{mn} + 4W_{jz}{}^{iz}W_{iz}{}^{jz} + 4W_{mn}{}^{iz}W_{iz}{}^{mn}\,.$$

- Considering the FG expansion of the terms:

$$W_{jz}{}^{iz} = \frac{z^4}{3\ell^2}\mathscr{B}_{(0)j}{}^{i} + O(z^6),$$

$$W_{jk}{}^{iz} = -\frac{z^3}{\ell^2}\mathscr{C}_{(0)jk}{}^{i} + O(z^5),$$

$$W_{km}{}^{ij} = \frac{z^2}{\ell^2}\mathscr{W}_{(0)km}{}^{ij} - \frac{z^4}{\ell^2}\left[\frac{2}{3}\mathscr{B}_{(0)[k}{}^{[i}\delta^{j]}_{m]} + 2\mathcal{S}_{(0)n}{}^{[i}\mathscr{W}_{(0)km}{}^{j]n} - \mathcal{S}_{(0)[k}{}^{n}\mathscr{W}_{(0)m]n}{}^{ij} + D^{[i}_{(0)}\mathscr{C}_{(0)km}{}^{j]}\right] + O(z^6)\,.$$

Conformal Renormalization in 8D

- Asymptotic behavior of terms in the action:

$$\Phi_{(2)}\Big|_E = \frac{4z^4}{\ell^4}\mathscr{W}^{ij}_{(0)mn}\mathscr{W}^{mn}_{(0)ij} + \frac{8z^6}{\ell^4}\left[\mathcal{S}^m_{(0)n}\mathscr{W}^{js}_{(0)mi}\mathscr{W}^{ni}_{(0)js} + 2\mathscr{C}^{ijm}_{(0)}\mathscr{C}_{(0)ijm} + \mathscr{W}_{(0)ijmn}D^n_{(0)}\mathscr{C}^{mij}_{(0)}\right] + O(z^8)\,.$$

$$\Phi_{(3)}\Big|_E = \frac{z^6}{\ell^6}\delta^{j_1\cdots j_6}_{i_1\cdots i_6}\mathscr{W}^{i_1i_2}_{(0)j_1j_2}\mathscr{W}^{i_3i_4}_{(0)j_3j_4}\mathscr{W}^{i_5i_6}_{(0)j_5j_6} + O(z^8)\,.$$

$$\sqrt{-g} = \frac{\ell^8}{z^8}\sqrt{-g_{(0)}}\left[1 - \frac{z^2}{2}\mathcal{S}^m_{(0)m} + O(z^4)\right]\,;\,\Phi_{(4)}\Big|_E \sim \mathcal{O}(z^8)$$

- Divergence structure of the action given by Topological Renormalization:

$$I_{\rm div}\left[g_{(0)ij}\right] = \frac{\ell^6}{1280\pi G}\int_{\partial\mathscr{M}_8} d^7x\sqrt{-g_{(0)}}\left[\frac{4}{3z^3}\mathscr{W}^{ij}_{(0)ms}\mathscr{W}^{ms}_{(0)ij} - \frac{1}{2z}\left(\delta^{i_1\cdots i_5}_{j_1\cdots j_5}\mathscr{W}^{j_1j_2}_{(0)i_1i_2}\mathscr{W}^{j_3j_4}_{(0)i_3i_4}\mathcal{S}^{j_5}_{(0)i_5}\right.\right.$$

$$\left.\left. + \frac{1}{18}\delta^{i_1\cdots i_6}_{j_1\cdots j_6}\mathscr{W}^{j_1j_2}_{(0)i_1i_2}\mathscr{W}^{j_3j_4}_{(0)i_3i_4}\mathscr{W}^{j_5j_6}_{(0)i_5i_6} + 32\mathscr{C}^{ijm}_{(0)}\mathscr{C}_{(0)ijm}\right)\right]\,.$$

Conformal Renormalization in 8D

- Analogy with 6D, look for terms to cancel divergences of the form $\Box^2\Phi_{(2)}$, $\Box\Phi_{(3)}$, $\frac{1}{\ell^2}\Box\Phi_{(2)}$.
- The asymptotic expansion of these terms is

$$\Box^2\Phi_{(2)}\Big|_E = \frac{576z^4}{\ell^8}\mathcal{I}_2 + \frac{z^6}{\ell^8}\left[288\mathcal{Q}_2 + 288\mathcal{S}^m_{(0)m}\mathcal{I}_2 - 72D^m_{(0)}D_{(0)m}\mathcal{I}_2\right] + O(z^8),$$

$$\Box\Phi_{(3)}\Big|_E = -\frac{6z^6}{\ell^8}\mathcal{I}_3 + O(z^8),$$

$$\Box\Phi_{(2)}\Big|_E = -\frac{48z^4}{\ell^6}\mathcal{I}_2 + \frac{z^6}{\ell^6}\left[4D^m_{(0)}D_{(0)m}\mathcal{I}_2 - 16\mathcal{S}^m_{(0)m}\mathcal{I}_2 - 48\mathcal{Q}_2\right] + O(z^8)$$

where

$$\mathcal{I}_2 := \mathcal{W}^{ij}_{(0)mn}\mathcal{W}^{mn}_{(0)ij}, \; ; \mathcal{I}_3 := \delta^{i_1\cdots i_6}_{j_1\cdots j_6}\mathcal{W}^{j_1j_2}_{(0)i_1i_2}\mathcal{W}^{j_3j_4}_{(0)i_3i_4}\mathcal{W}^{j_5j_6}_{(0)i_5i_6},$$

$$\mathcal{Q}_2 := \mathcal{S}^m_{(0)n}\mathcal{W}^{jl}_{(0)mi}\mathcal{W}^{ni}_{(0)jl} - 2\mathcal{C}^{ijm}_{(0)}\mathcal{C}_{(0)ijm} + D^n_{(0)}\left(\mathcal{W}_{(0)ijmn}\mathcal{C}^{mij}_{(0)}\right).$$

Conformal Renormalization in 8D

- Then, the following action is divergence free and has the same finite (universal) part as the one given by Topological Renormalization:

$$I_{\text{ren}}[g_{\mu\nu}]\Big|_E = \frac{\ell^6}{1280\pi G}\int_{\mathscr{M}_8} d^8x\sqrt{-g}\left[\frac{1}{\ell^4}\Phi_{(2)}\Big|_E - \frac{1}{36\ell^2}\Phi_{(3)}\Big|_E + \frac{1}{576}\Phi_{(4)}\Big|_E \right.$$
$$\left. + \frac{1}{72}\Box^2\Phi_{(2)}\Big|_E - \frac{1}{216}\Box\Phi_{(3)}\Big|_E + \frac{1}{4\ell^2}\Box\Phi_{(2)}\Big|_E\right].$$

- Action agrees with the result from Holographic Renormalization [Anastasiou, IJA, Boulanger, Olea and Rovere 2026].

- Now, compare the renormalized action with the BR conformal gravity for Einstein spaces:

- First, we note that $\displaystyle\int_{\mathscr{M}_8} \mathrm{d}^8x\sqrt{-g}\,\nabla_\mu J^\mu_{(i)}\Big|_E = 0, \qquad i = 1,2.$

Conformal Renormalization in 8D

- The integrated pure boundary conformal invariants vanish for Einstein spaces.
- They do not contribute to the action.
- Finally, $I_{\rm ren}[g_{\mu\nu}]\Big|_E = \alpha_{\rm EH}\int_{\mathscr{M}_8} {\rm d}^8x\sqrt{-g}\,\mathscr{L}_8\Big|_E$ for $\alpha_{EH} = -\dfrac{\ell^6}{504\pi G}$.
- Renormalized Einstein-AdS gravity action exactly reproduced by BR conformal gravity.
- Pure boundary conformal invariants do not contribute (therefore are not relevant) for Einstein.
- Einstein-AdS gravity embedded into conformal gravity.

Conclusions and outlook

- Conjecture:
  - There is a unique combination of conformal invariants in even dimensions, up to pure boundary conformal invariants, which admits all Einstein-AdS spacetimes as solutions.
  - When evaluated on Einstein manifolds, this combination gives the renormalized Einstein-AdS action
  - For Einstein-AdS spaces, the integrated pure boundary conformal invariants vanish such that they do not contribute to the action and therefore can be omitted.
- This would amount to Conformal Renormalization in all even dimensions.
- Evidence in 4, 6 and 8 dimensions. In 6 and 8 it is non-trivial evidence.
- Applications in codimension-2 for holographic entanglement measures and energy functionals in (A)dS/CFT [Anastasiou, IJA and Olea 2022; Anastasiou, IJA, Bueno, Moreno, Olea and Vilar Lopez 2024; Anastasiou, IJA, Das and Moreno 2026].

8D local conformal invariants

- For completeness, the invariants are [Rovere and Boulanger 2025]:

$$I_6 = W_{\alpha\beta}{}^{\nu\sigma} W^{\alpha\beta\gamma\delta} W_{\gamma\nu}{}^{\rho\mu} W_{\delta\sigma\rho\mu} \,. \;;\; I_7 = W_{\alpha}{}^{\nu}{}_{\gamma}{}^{\sigma} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\rho}{}_{\delta}{}^{\mu} W_{\nu\rho\sigma\mu} \,. \;;\; I_8 = W_{\alpha\beta}{}^{\nu\sigma} W^{\alpha\beta\gamma\delta} W_{\gamma\delta}{}^{\rho\mu} W_{\nu\rho\sigma\mu} \,.$$

$$I_9 = W_{\alpha\beta\gamma}{}^{\nu} W^{\alpha\beta\gamma\delta} W_{\delta}{}^{\sigma\rho\mu} W_{\nu\rho\sigma\mu} \,. \;;\; I_{10} = W_{\alpha\beta\gamma\delta} W^{\alpha\beta\gamma\delta} W_{\nu\rho\sigma\mu} W^{\nu\sigma\rho\mu} \,. \;;\; I_{11} = W_{\alpha}{}^{\nu}{}_{\gamma}{}^{\sigma} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\rho}{}_{\sigma}{}^{\mu} W_{\delta\mu\nu\rho} \,.$$

$$I_{12} = W_{\alpha\gamma}{}^{\nu\sigma} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\rho}{}_{\nu}{}^{\mu} W_{\delta\mu\sigma\rho} \,.$$

$$\begin{aligned} J^{\alpha}_{(1)} = & -\frac{1}{5} W_{\beta\gamma\delta}{}^{\sigma} W^{\beta\gamma\delta\epsilon} \nabla^{\rho} W^{\alpha}{}_{\epsilon\sigma\rho} + W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon\sigma\rho} \nabla_{\rho} W_{\gamma\delta\epsilon\sigma} \\ & -\frac{4}{15} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon}{}_{\gamma}{}^{\sigma} \nabla^{\rho} W_{\delta\epsilon\sigma\rho} + \frac{8}{15} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon}{}_{\gamma}{}^{\sigma} \nabla^{\rho} W_{\epsilon\sigma\delta\rho} \;; \\ & +\frac{4}{15} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon}{}_{\gamma}{}^{\sigma} \nabla^{\rho} W_{\delta\sigma\epsilon\rho} \,. \end{aligned}$$

$$\begin{aligned} J^{\alpha}_{(2)} = & W^{\alpha\beta\gamma\delta} W_{\gamma}{}^{\epsilon\sigma\rho} \nabla_{\rho} W_{\beta\epsilon\delta\sigma} - W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon\sigma\rho} \nabla_{\rho} W_{\gamma\delta\epsilon\sigma} \\ & -\frac{2}{5} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon}{}_{\gamma}{}^{\sigma} \nabla^{\rho} W_{\epsilon\sigma\delta\rho} - \frac{2}{5} W^{\alpha\beta\gamma\delta} W_{\beta}{}^{\epsilon}{}_{\gamma}{}^{\sigma} \nabla^{\rho} W_{\delta\sigma\epsilon\rho} \,. \end{aligned}$$

8D local conformal invariants

$$
\begin{aligned}
I_1 = {} & W^{\rho\gamma\mu\sigma}\mathcal{D}^{\alpha}\mathcal{D}_{\alpha}\mathcal{D}^{\beta}\mathcal{D}_{\beta}W_{\rho\gamma\mu\sigma} + \frac{48}{25}\mathcal{D}_{\beta}W^{\beta}{}_{\gamma\mu\alpha}\mathcal{D}^{\alpha}\mathcal{D}_{\rho}W^{\rho\gamma\mu\alpha} \\
& + 2\mathcal{D}_{\alpha}W_{\mu\beta\gamma\nu}\mathcal{D}_{\rho}\mathcal{D}^{\rho}\mathcal{D}^{\alpha}W^{\mu\beta\gamma\nu} + \frac{42}{125}\mathcal{D}^{\alpha}\mathcal{D}^{\beta}W_{\gamma\alpha\beta\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho}W^{\gamma\nu\rho\mu} \\
& + \frac{9}{10}\mathcal{D}^{\gamma}\mathcal{D}_{\gamma}W_{\alpha\mu\nu\beta}\mathcal{D}^{\rho}\mathcal{D}_{\rho}W^{\alpha\mu\nu\beta} + \frac{3}{5}\mathcal{D}_{\alpha}\mathcal{D}_{\beta}W_{\nu\gamma\mu\rho}\mathcal{D}^{\alpha}\mathcal{D}^{\beta}W^{\nu\gamma\mu\rho} \\
& + \frac{96}{125}\mathcal{D}_{\alpha}\mathcal{D}_{\gamma}W^{\gamma}{}_{\mu\nu\beta}\mathcal{D}^{\alpha}\mathcal{D}_{\rho}W^{\rho\mu\nu\beta} + \frac{74}{25}W_{\beta}{}^{\alpha\gamma\mu}W_{\nu\alpha\gamma\mu}\mathcal{D}^{\rho}\mathcal{D}^{\sigma}W^{\beta}{}_{\rho\sigma}{}^{\nu} \\
& + \frac{208}{5}W_{\mu\beta\gamma\alpha}W_{\sigma}{}^{\nu\rho\alpha}\mathcal{D}^{\beta}\mathcal{D}^{\gamma}W^{\mu}{}_{\nu\rho}{}^{\sigma} - 8W_{\alpha}{}^{\gamma}{}_{\beta}{}^{\mu}W^{\alpha}{}_{\nu}{}^{\beta}{}_{\rho}\mathcal{D}^{\sigma}\mathcal{D}_{\sigma}W^{\nu}{}_{\gamma}{}^{\rho}{}_{\mu} \\
& + \frac{16}{5}W_{\alpha\gamma\mu\rho}W_{\beta\nu}{}^{\alpha\gamma}\mathcal{D}_{\sigma}\mathcal{D}^{\sigma}W^{\beta\nu\mu\rho} - \frac{144}{25}W^{\gamma}{}_{\alpha}{}^{\mu}{}_{\beta}\mathcal{D}^{\rho}W_{\rho\gamma}{}^{\nu}{}_{\mu}\mathcal{D}^{\sigma}W_{\sigma}{}^{\alpha}{}_{\nu}{}^{\beta} \\
& + \frac{104}{5}W_{\alpha}{}^{\gamma}{}_{\beta}{}^{\mu}\mathcal{D}^{\alpha}W^{\beta}{}_{\mu}{}^{\sigma\nu}\mathcal{D}^{\rho}W_{\rho\gamma\sigma\nu} - \frac{88}{25}W_{\alpha\beta\gamma\mu}\mathcal{D}^{\rho}W_{\rho\nu}{}^{\alpha\beta}\mathcal{D}^{\sigma}W_{\sigma}{}^{\nu\gamma\mu}\,.
\end{aligned}
$$

8D local conformal invariants

$$I_2 = W_\beta{}^{\alpha\gamma\mu} W_{\nu\alpha\gamma\mu} \mathscr{D}^\rho \mathscr{D}^\sigma W^\beta{}_{\rho\sigma}{}^\nu + 5 W_{\alpha\gamma\mu\rho} W_{\beta\nu}{}^{\alpha\gamma} \mathscr{D}^\sigma \mathscr{D}_\sigma W^{\beta\nu\mu\rho}$$
$$+5 W_{\alpha\beta\gamma\mu} \mathscr{D}_\nu W^{\alpha\beta\rho\sigma} \mathscr{D}^\nu W^{\gamma\mu}{}_{\rho\sigma} + \frac{12}{5} W_{\alpha\beta\gamma\mu} \mathscr{D}^\rho W_{\rho\nu}{}^{\alpha\beta} \mathscr{D}^\sigma W_\sigma{}^{\nu\gamma\mu} .$$

$$I_3 = W_\beta{}^{\alpha\gamma\mu} W_{\nu\alpha\gamma\mu} \mathscr{D}^\rho \mathscr{D}^\sigma W^\beta{}_{\rho\sigma}{}^\nu - 20 W_\alpha{}^\gamma{}_\beta{}^\mu W^\alpha{}_\nu{}^\beta{}_\rho \mathscr{D}^\sigma \mathscr{D}_\sigma W^\nu{}_\gamma{}^\rho{}_\mu$$
$$-\frac{48}{5} W^\gamma{}_\alpha{}^\mu{}_\beta \mathscr{D}^\rho W_{\rho\gamma}{}^\nu{}_\mu \mathscr{D}^\sigma W_\sigma{}^\alpha{}_\nu{}^\beta - 20 W^\alpha{}_\mu{}^\gamma{}_\beta \mathscr{D}_\nu W^\mu{}_\rho{}^\beta{}_\sigma \mathscr{D}^\nu W^\rho{}_\alpha{}^\sigma{}_\gamma .$$

$\mathscr{D}_\mu$ defined in [Boulanger 2005].