Propagator of a massive charged vector boson in a magnetic field: Ritus eigenfunction method

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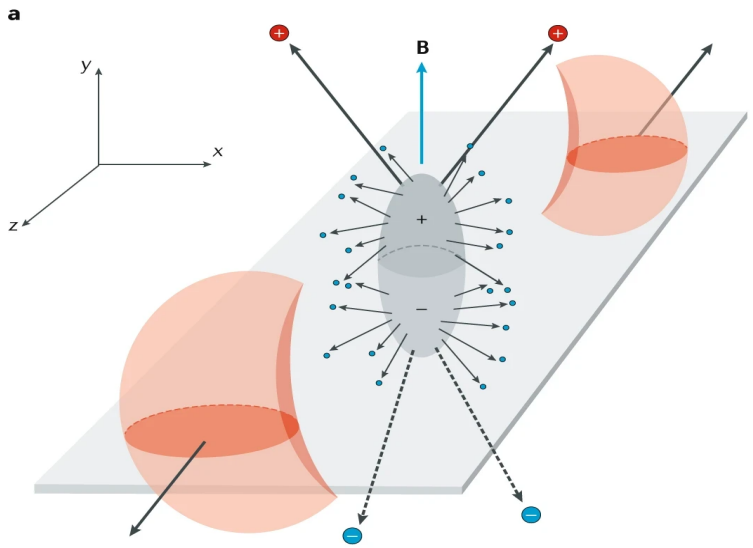
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4. LSZ Reduction Formula for W Bosons in a Magnetic Field
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Intense Magnetic Fields

- Presence of intense magnetic fields in high energy physics.
  Magnetic field intensity in magnetars between $10^{-2} m_\pi^2$ on the surface[1] and up to $10^0 m_\pi^2$ in its core[2]. In heavy ion colisions $\sim 1 - 10 m_\pi^2$.[3]



E. Kharzeev, & J. Liao, (2021) **3**, Nat. Rev. Phys., 55-63.

[1] R. Turolla, S. Zane, & A. L. Watts, (2015), Rep. Prog. Phys. **78**, 116901.
[2] G. Chanmugam, (1992), Annu. Rev. Astron. Astrophys. , **30**, 143-184.
[3] Y. Zhong, et. al, (2014), Adv. High Energy Phys., **193039**.

W Boson Propagator

A conventional way of building the propagator is

- Standard Model Lagrangian.
- Equation of Motion and Gauge Condition.
- Wave-like ansatz.
- Choose a set of polarization vectors.
- Canonical quantization of classical field.
- Propagator definition.

W Boson Propagator in a Magnetic Field

Mostly analogous methodology

- SM Lagrangian (including A^{μ}_{clas}).
- Equation of Motion (Proca Equation in a Magnetic Field) and Gauge Condition.
- (Mostly) Wave-like ansatz.
- Deriving sets of polarization vectors.
- Canonical quantization of the classical field.
- Propagator definition.

SM Lagrangian, EoM and Gauge Condition in a Magnetic Field

The Lagrangian that describes the $W^{\pm}$ bosons dynamics and interactions with the photon field A^{μ} is[4]

$$\mathcal{L} = -\frac{1}{2}\left(\partial^{\mu}W^{(+)\nu} - \partial^{\nu}W^{(+)\mu}\right)\left(\partial_{\mu}W^{(-)}_{\nu} - \partial_{\nu}W^{(-)}_{\mu}\right) + m_W^2 W^{(+)\mu}W^{(-)}_{\mu}$$
$$+ ie\left[F^{\mu\nu}W^{(+)}_{\mu}W^{(-)}_{\nu} - \left(\partial_{\mu}W^{(+)}_{\nu} - \partial_{\nu}W^{(+)}_{\mu}\right)A^{\mu}W^{(-)\nu} + \left(\partial_{\mu}W^{(-)}_{\nu} - \partial_{\nu}W^{(-)}_{\mu}\right)A^{\mu}W^{(+)\nu}\right]$$
$$+ e^2\left(A^{\mu}W^{(+)}_{\mu}A^{\nu}W^{(-)}_{\nu} - A^{\mu}A_{\mu}W^{(+)\nu}W^{(-)}_{\nu}\right).$$

The EoM will be

$$\left[\left(D_{\alpha}D^{\alpha} + m_W^2\right)g^{\mu\nu} + 2ieF^{\mu\nu}\right]W^{(-)}_{\nu} = 0.$$

where the condition

$$D^{\mu}W^{(-)}_{\mu}(x) = 0$$

has been used, and $D^{\alpha} \equiv \partial^{\alpha} + ieA^{\alpha}$.

[4] Schwartz, M., *Quantum Field Theory and the Standard Model*, United Kingdom, 2014, Ed. Cambridge University Press, Fifth Edition.

EoM Diagonalization

After defining the rotation matrices

$$N_\alpha{}^\mu \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Longrightarrow \left(N^{-1}\right)_\alpha{}^\mu \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

one gets

$$\left[\left(\hat{\tilde{\Pi}}^2 - m_W^2\right) g_\alpha{}^\beta - 2eB\tilde{\varphi}_\alpha{}^\beta\right] \tilde{W}_\beta^{(-)} = 0,$$

where $\hat{\tilde{\Pi}}_\pm^\alpha \equiv iD_\pm^\alpha$, with $\tilde{D}_-^\mu \equiv D^\alpha \left(N^{-1}\right)_\alpha{}^\mu$ and $\tilde{D}_+^\mu \equiv N^\mu{}_\alpha D^\alpha$.

Furthermore, $\tilde{\varphi}_\alpha{}^\beta \equiv iN_\alpha{}^\sigma \frac{F_\sigma{}^\rho}{B} \left(N^{-1}\right)_\rho{}^\beta$ is the diagonalized and normalized field strength tensor and $\tilde{W}_\alpha^{(-)} \equiv N_\alpha{}^\mu W_\mu^{(-)}$ is the rotated W^- field.

Projection Matrix

Defining

$$\tilde{\Delta}^{\mu}{}_{\nu}(s_3) \equiv \begin{pmatrix} 1-s_3^2 & 0 & 0 & 0 \\ 0 & s_3\left(\frac{1+s_3}{2}\right) & 0 & 0 \\ 0 & 0 & -s_3\left(\frac{1-s_3}{2}\right) & 0 \\ 0 & 0 & 0 & 1-s_3^2 \end{pmatrix},$$

that can be identified as a spin projection matrix with eigenvalues $s_3 = \pm 1, 0$. Then, the EoM can be rewritten as

$$\sum_{s_3} \left[\left(\hat{\tilde{\Pi}}^2 - m_W^2 \right) + 2eBs_3 \right] \tilde{W}_{\alpha}^{(-)s_3} = 0,$$

where $\tilde{W}_{\alpha}^{(-)s_3} \equiv \tilde{\Delta}_{\alpha}{}^{\beta}(s_3)\tilde{W}_{\beta}^{(-)}$.

EoM Solution in a Magnetic Field

A choice of gauge is made $A^\mu = \big(0, 0, Bx^1, 0\big)$ so that the resulting magnetic field is homogeneous and goes in the z direction. The proposed ansatz is

$$\tilde{W}_\alpha^{(-)s_3}(x) = e^{-ir\left(Ex_0 + p^2x_2 + p^3x_3\right)} X_n^r(x^1) \tilde{\bar{\varepsilon}}_\alpha^{(-)s_3},$$

Plugging the ansatz in the EoM one gets *Weber's differential equation*

$$\left[\frac{\partial^2}{\partial\rho_r^2} - \frac{1}{4}\rho_r^2 + \frac{1}{2} + n\right] X_n^r(\rho_r) = 0,$$

where

$$n = \frac{E^2 - (p^3)^2 - m_W^2 + 2eBs_3}{2|eB|} - \frac{1}{2} = \ell - 1 + \hat{s}_3,$$

$$\ell \equiv \frac{E^2 - (p^3)^2 - m_W^2}{2|eB|} + \frac{1}{2}, \qquad \text{and} \qquad \hat{s}_3 \equiv s_{eB} s_3,$$

and which solutions are parabolic cylinder functions: $X_n^r(\rho) = N_n D_n(\rho)$.

Gauge condition solution

Rewriting the gauge condition using the rotation matrices

$$\sum_{s_3} D^{\mu} W_{\mu}^{(-)s_3} = \sum_{s_3} D^{\alpha} \left(N^{-1}\right)_{\alpha}{}^{\beta} N_{\beta}{}^{\mu} W_{\mu}^{s_3-} = \sum_{s_3} \tilde{D}_{-}^{\mu} \tilde{W}_{\mu}^{(-)s_3} = 0.$$

Then, plugging the ansatz here, one gets

$$\left(\tilde{\mathbb{E}}^{r}_{\ell,0}\right)_{\mu}^{\ \nu}(p,x)\, \tilde{\tilde{p}}^{*\mu}\, \tilde{\tilde{\varepsilon}}^{(-)}_{\nu} = 0,$$

where

$$\tilde{\tilde{p}}^{\mu *} \equiv \left(E, i s_{eB}\sqrt{|eB|}\sqrt{\ell_+}, -i s_{eB}\sqrt{|eB|}\sqrt{\ell_-}, p^3\right),$$

$$\left(\tilde{\mathbb{E}}^{r}_{\ell,c}\right)_{\mu}^{\ \nu}(p,x) \equiv \sum_{s_3 \equiv \pm 1,0} e^{-ir\left(Ex_0 + p^2 x_2 + p^3 x_3\right)} X^{r}_{\ell-1+c\hat{s}_3}(\rho_r)\tilde{\Delta}_{\mu}{}^{\nu}(s_3),$$

$$\ell_{\pm} \equiv \ell - 1 + s_{\pm}, \qquad s_{\pm} \equiv (1 - s_{eB})/2.$$

Polarization Vectors

To completely know every factor in the ansatz, it is necessary to calculate a set of polarization vectors from the previous equations and conditions $\tilde{\tilde{\varepsilon}}^{(-)\mu}$. Additionally, the orthonormalization condition

$$\int dx^1 J_0 \left(W_\lambda^{(-)}(x), W_{\lambda'}^{(+)}(x) \right) = 4\pi E \delta_{\lambda,\lambda'},$$

will help us define each entry of the polarization vectors. These vectors satisfy the polarization sum relation

$$\sum_{\lambda=1}^{3} \tilde{\tilde{\varepsilon}}_\lambda^{(-)\alpha} \left(\tilde{\tilde{\varepsilon}}_\lambda^{(-)\beta} \right)^* = -g^{\alpha\beta} + \frac{\tilde{\tilde{p}}^\alpha \tilde{\tilde{p}}^{\beta *}}{m_W^2}.$$

Canonical Quantization

Following conventional canonical quantization

$$\hat{\tilde{W}}^{(-)\mu}(x) = \sum\!\!\!\!\!\!\int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\tilde{\tilde{p}}}}} \sum_{\lambda=1}^{3} \Bigg[\hat{a}_{\tilde{\tilde{p}},\lambda} \left(\tilde{\tilde{\mathbb{E}}}^{+}_{\ell,1}\right)^{\mu}_{\ \ \nu}(p,x)\, \tilde{\tilde{\varepsilon}}^{(-)\nu}_{\lambda}(\tilde{\tilde{p}},\ell) + \hat{b}^{\dagger}_{\tilde{\tilde{p}},\lambda} \left(\tilde{\tilde{\mathbb{E}}}^{-}_{\ell,1}\right)^{\mu}_{\ \ \nu}(p,x)\, \tilde{\tilde{\varepsilon}}^{(+)\nu *}_{\lambda}(\tilde{\tilde{p}},\ell) \Bigg],$$

where

$$\sum\!\!\!\!\!\!\int \frac{d^3p}{(2\pi)^3} \equiv \frac{1}{2\pi} \sum_{\ell=0}^{\infty} \int \frac{dp^2}{2\pi} \int \frac{dp^3}{2\pi}.$$

The commutation relations are defined as

$$\left[\hat{a}^{\lambda}_{\bar{p}}, \hat{a}^{\lambda'\dagger}_{\bar{p}'}\right] = (2\pi)^3\, \delta_{\ell\ell'} \delta_{\lambda\lambda'} \delta\left(p_2 - p_2'\right) \delta\left(p_3 - p_3'\right) = \left[\hat{b}^{\lambda}_{\bar{p}}, \hat{b}^{\lambda'\dagger}_{\bar{p}'}\right].$$

W Boson Propagator in a Magnetic Field

Using the propagator definition

$$\tilde{D}_B^{\mu\nu}(x,y) \equiv \langle 0|\, \mathcal{T}\left(\hat{\tilde{W}}^{(-)\mu}(x)\hat{\tilde{W}}^{(+)\nu}(y)\right)|0\rangle\,,$$

one arrives to the result

$$\tilde{D}_B^{\mu\nu}(x,y) = i\sum\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4}\,\frac{\left(\tilde{\mathbb{E}}^{+}_{\ell,1}\right)^{\mu}_{\ \alpha}(p,x)\left(-g^{\alpha\beta}+(1-\delta_{\ell,0})\,\frac{\tilde{\tilde{p}}^{\alpha}\tilde{\tilde{p}}^{\beta *}}{m_W^2}\right)\left(\tilde{\mathbb{E}}^{+*}_{\ell,1}\right)_{\beta}^{\ \nu}(p,y)}{\tilde{\tilde{p}}^2-m_W^2+i\varepsilon}$$

where the "magnetic 4-momentum" $\tilde{\tilde{p}}^{\mu}$ is now off-shell, therefore $\tilde{\tilde{p}}^2 \neq m_W^2$.

Comparison: With and without magnetic field

$$D^{\mu\nu}(x,y) = i\int \frac{d^4p}{(2\pi)^4}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_W^2}\right)\frac{e^{-ip\cdot(x-y)}}{p^2 - m_W^2 + i\varepsilon}$$

$$\downarrow$$

$$\tilde{D}_B^{\mu\nu}(x,y) = i\sum\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4}\frac{\left(\tilde{\mathbb{E}}^{+}_{\ell,1}\right)^{\mu}_{\ \alpha}(p,x)\left(-g^{\alpha\beta} + (1-\delta_{\ell,0})\,\frac{\tilde{\bar{p}}^{\alpha}\tilde{\bar{p}}^{\beta *}}{m_W^2}\right)\left(\tilde{\mathbb{E}}^{+*}_{\ell,1}\right)_{\beta}^{\ \nu}(p,y)}{\tilde{\bar{p}}^2 - m_W^2 + i\varepsilon}.$$

Comparison: With and without magnetic field

$$D^{\mu\nu}(x,y) = i\left(-g^{\mu\nu} + \frac{\hat{p}_x^{\mu}\hat{p}_y^{\nu *}}{m_W^2}\right)\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{p^2 - m_W^2 + i\varepsilon}$$

$$\downarrow$$

$$\tilde{D}_B^{\mu\nu}(x,y) = i\sum\!\!\!\!\!\!\int \frac{d^4p}{(2\pi)^4}\Bigg[(-g^{\alpha\beta})\,\frac{\left(\tilde{\mathbb{E}}^{+}_{\ell,1}\right)^{\mu}{}_{\alpha}(p,x)\left(\tilde{\mathbb{E}}^{+*}_{\ell,1}\right)^{\nu}{}_{\beta}(p,y)}{\tilde{\bar{p}}^2 - m_W^2 + i\varepsilon}$$

$$+ (1-\delta_{\ell,0})\frac{\hat{\tilde{\Pi}}_{+}^{\alpha}(x)\hat{\tilde{\Pi}}_{+}^{\beta *}(y)}{m_W^2}\,\frac{\left(\tilde{\mathbb{E}}^{+}_{\ell,0}\right)^{\mu}{}_{\alpha}(p,x)\left(\tilde{\mathbb{E}}^{+*}_{\ell,0}\right)^{\nu}{}_{\beta}(p,y)}{\tilde{\bar{p}}^2 - m_W^2 + i\varepsilon}\Bigg].$$

Schwinger Parameterization

As a final remark on the representations of the propagator, a novel methodology has been developed to write the $W^{\pm}$ boson propagator in the presence of a magnetic field using the Schwinger parameter

$$\frac{1}{x} = -i \int_0^{\infty} ds\ e^{isx},$$

and parting from the presented Ritus Eigenfunction representation of the propagator.

From Ritus Eigenfunctions to Schwinger Parameter

After summing over all Landau Levels and integrating over the p^2 parameter

$$\tilde{D}_B^{\mu\nu}(x,y) = \int \frac{dp^0}{2\pi} \int \frac{dp^3}{2\pi} \int_0^\infty \frac{ds}{2\pi} e^{-is\left[-p_\parallel^2 + m_W^2\right]} \frac{-ieB}{2\sin(eBs)}$$
$$\times \left[\left(-g^{\alpha\beta}\right) \sum_{s_3} \tilde{\Delta}^\mu{}_\alpha(s_3)\tilde{\Delta}^\nu{}_\beta(s_3) e^{2ieBs_3 s} + \frac{\hat{\tilde{\Pi}}_+^\alpha(x)\hat{\tilde{\Pi}}_+^{\beta *}(y)}{m_W^2} \sum_{s_3, s_3'} \tilde{\Delta}^\mu{}_\alpha(s_3)\tilde{\Delta}^\nu{}_\beta(s_3') \right]$$
$$\times \Omega(x,y) e^{-ip\cdot(x-y)_\parallel} e^{\frac{ieB}{4}\cot(eBs)\left[(x-y)_\mu \varphi^{\mu\sigma}\varphi_\sigma{}^\nu (x-y)_\nu\right]},$$

where $\Omega(x,y) = e^{\frac{ieB}{2}(x_1+y_1)(x_2-y_2)}$ is the Schwinger Phase. This procedure has been done before by other authors[5]. However, this is where they stop.

[5] A. I. Nikishov. Vector boson in the constant electromagnetic field. J. Exp. Theor. Phys., 93(2):197–210, August 2001.

From Ritus Eigenfunctions to Schwinger Parameter

A straightforward calculation of the operators $\hat{\tilde{\Pi}}_{+}^{\alpha}(x)\hat{\tilde{\Pi}}_{+}^{\beta *}(y)$ acting on Schwinger's phase and the exponential factor allows us to rewrite the propagator as

$$\tilde{D}_B^{\mu\nu}(x,y) = \Omega(x,y)\tilde{D}_B^{\mu\nu}(x-y)$$

with

$$\tilde{D}_B^{\mu\nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{D}_B^{\mu\nu}(p),$$

where

$$\tilde{D}_B^{\mu\nu}(p) \equiv \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left[p_\parallel^2 + p_\perp^2 \frac{\tan(eBs)}{eBs} - m_W^2\right]}$$

$$\times \left[\left(-g^{\alpha\beta}\right) \sum_{s_3} \tilde{\Delta}^{\mu}{}_{\alpha}(s_3)\tilde{\Delta}^{\nu}{}_{\beta}(s_3) e^{2ieBss_3} + \frac{1}{m_W^2}\left(\tilde{p}_t^{\mu}\tilde{p}_t^{\nu\prime} - \frac{eB}{2}[\tilde{\varphi}^{\mu\nu} - i\tan(eBs)g_\perp^{\mu\nu}]\right)\right].$$

From Ritus Eigenfunctions to Schwinger Parameter

A straightforward calculation of the operators $\hat{\tilde{\Pi}}_{+}^{\alpha}(x)\hat{\tilde{\Pi}}_{+}^{\beta *}(y)$ acting on Schwinger's phase and the exponential factor allows us to rewrite the propagator as

$$\tilde{D}_B^{\mu\nu}(x,y) = \Omega(x,y)\tilde{D}_B^{\mu\nu}(x-y)$$

with

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where

$$\tilde{D}_B^{\mu\nu}(p) \equiv \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left[p_\parallel^2 + p_\perp^2 \frac{\tan(eBs)}{eBs} - m_W^2\right]}$$

$$\times \left[\left(-g^{\alpha\beta}\right) \sum_{s_3} \tilde{\Delta}^{\mu}{}_{\alpha}(s_3)\tilde{\Delta}^{\nu}{}_{\beta}(s_3) e^{2ieBss_3} + \frac{1}{m_W^2}\left(\tilde{p}_t^{\mu}\tilde{p}_t^{\nu\prime} - \frac{eB}{2}\left[\tilde{\varphi}^{\mu\nu} - i\tan(eBs) g_\perp^{\mu\nu}\right]\right)\right].$$

In the last equation, we have introduced the notation and

$$p_\perp^2 \equiv p^1 p_1 + p^2 p_2, \quad \tilde{p}_t^{\mu} \equiv \left(p^0, p^- t_{eB}, p^+ t_{eB}^*, p^3\right), \quad \tilde{p}_t^{\mu\prime} \equiv \left(p^0, p^+ t_{eB}, p^- t_{eB}^*, p^3\right),$$

$$p^{\pm} \equiv \left(p^1 \pm ip^2\right)/\sqrt{2}, \qquad t_{eB} \equiv i\tan(eBs) + 1.$$

From Ritus Eigenfunctions to Schwinger Parameter

Finally, rotating back using the N and N^{-1} matrices, one arrives at

$$D_B^{\mu\nu}(x,y) = \Omega(x,y) D_B^{\mu\nu}(x-y) \implies D_B^{\mu\nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} D_B^{\mu\nu}(p)$$

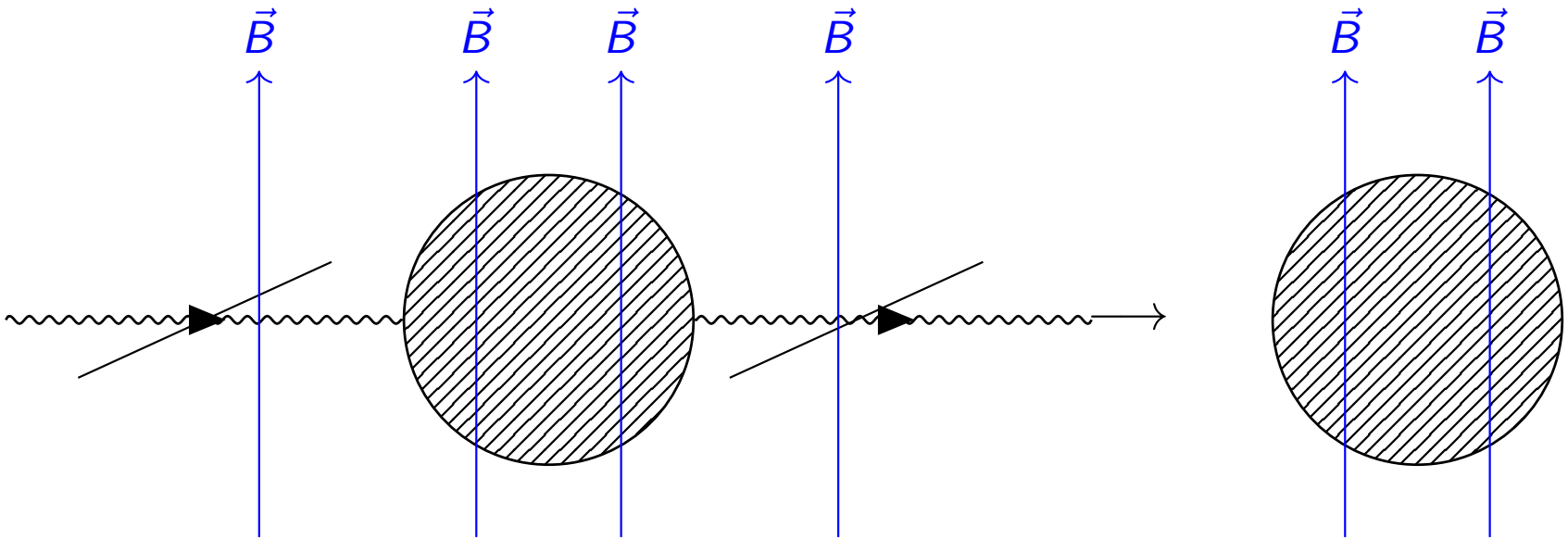
$$D_B^{\mu\nu}(p) = -\int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left[p_\parallel^2 + p_\perp^2 \frac{\tan(eBs)}{eBs} - m_W^2\right]} \left[g_\parallel^{\mu\nu} + g_\perp^{\mu\nu} \cos(2eBs) + \varphi^{\mu\nu} \sin(2eBs) \right.$$

$$\left. - \frac{1}{m_W^2} \left[\left(p^\mu + (\varphi p)^\mu \tan(eBs)\right) \left(p^\nu + (p\varphi)^\nu \tan(eBs)\right) - \frac{ieB}{2} \left(\varphi^{\mu\nu} - g_\perp^{\mu\nu} \tan(eBs)\right) \right] \right]$$

where $(\varphi p)^\mu \equiv \varphi^{\mu\alpha} p_\alpha$ and $(p\varphi)^\mu \equiv p_\alpha \varphi^{\alpha\mu}$.

LSZ Reduction Formula for W Bosons in a Magnetic Field

As shown in the propagator under the presence of a constant magnetic field, the advantage of using Ritus Eigenfunctions is that the structure of the equations in the vacuum case are preserved.
In the magnetic case



Amputation process of a Feynman diagram in the presence of a magnetic field.

LSZ Reduction Formula for W Bosons in a Magnetic Field

As an application of the Ritus Eigenfunction representation we can write the scattering amplitude of a processes involving charged vector boson states in such a background magnetic field.

$$\begin{aligned}\langle f|\, S\,|i\rangle_B &= 2\sqrt{E_{\bar{p}_i} E_{\bar{p}_f}}\,\langle\Omega|\, a_{\bar{p}_f}(\infty) a^{\dagger}_{\bar{p}_i}(-\infty)\,|\Omega\rangle \\ &= 2\sqrt{E_{\bar{p}_i} E_{\bar{p}_f}}\,\langle\Omega|\,\mathcal{T}\Big\{\,[a_{\bar{p}_f}(\infty) - a_{\bar{p}_f}(-\infty)]\left[a^{\dagger}_{\bar{p}_i}(\infty) - a^{\dagger}_{\bar{p}_i}(-\infty)\right]\Big\}\,|\Omega\rangle\,,\end{aligned}$$

The core of the LSZ Reduction Formula can be simplified to the question:

How can we represent the creation and annihilation operators in terms of the fields they represent?

LSZ Reduction Formula for W Bosons in a Magnetic Field

In analogy to the methodology used for the non-magnetic field case, we can perform the following Fourier analysis

$$\hat{a}_{\bar{p},\lambda}(t) = -\frac{1}{\sqrt{2E_{\bar{p}}}\sqrt{2\pi}} \int d^3x \, \left(\mathbb{E}^{+}_{\ell,1}\right)^{\alpha}{}_{\mu}(p,x) \left(i\partial_t + E_{\bar{p}}\right) \hat{W}^{(-)\mu}(x).$$

Furthermore, to obtain the difference from the remote past and distant future of the creation and annihilation operators, we can perform the integral

$$\begin{aligned}\sqrt{2E_{\bar{p}}}\left[\hat{a}_{\bar{p},\lambda}(\infty) - \hat{a}_{\bar{p},\lambda}(-\infty)\right] &= \sqrt{2E_{\bar{p}}} \int_{-\infty}^{\infty} dt \; \partial_t \; \hat{a}_{\bar{p},\lambda} \\ &= \frac{-i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d^4x \, \left(\mathbb{E}^{+*}_{\ell,1}\right)^{\alpha}_{\mu}(p,x) \varepsilon^{(-)*}_{\alpha}\left(\vec{\bar{p}},\lambda\right) \left[\partial_t^2 + E_{\bar{p}}^2\right] \hat{W}^{(-)\mu}(x)\end{aligned}$$

LSZ Reduction Formula for W Bosons in a Magnetic Field

After performing the same type of Fourier analysis over the EoM we obtain

$$\int d^4x \left(\mathbb{E}^{+\dagger}_{\ell,1}\right)^{\mu}{}_{\alpha}(p,x)\bar{\varepsilon}^{(-)*}_{\mu}(\vec{p},\lambda)\sum_{s_3}\left(D^{\sigma}D_{\sigma}+m_W^2-2eBs_3\right)\Delta^{\alpha}{}_{\beta}(s_3)W^{(-)\beta}(x)$$

$$= \int d^4x \left(\mathbb{E}^{+\dagger}_{\ell,1}\right)^{\mu}{}_{\alpha}(p,x)\bar{\varepsilon}^{(-)*}_{\mu}(\vec{p},\lambda)\left[\partial_t^2+E_{\bar{p}}^2\right]W^{(-)\alpha}(x)$$

$$= i\sqrt{2\pi}\sqrt{2E_{\bar{p}}}\left[\hat{a}_{\bar{p},\lambda}(\infty)-\hat{a}_{\bar{p},\lambda}(-\infty)\right].$$

LSZ Reduction Formula for W Bosons in a Magnetic Field

Finally, the scattering amplitude can be rewritten as

$$\begin{aligned}\langle f|\, S\,|i\rangle_B = \frac{1}{2\pi}\int d^4x_f\, \left(\mathbb{E}^{+\dagger}_{\ell_f,1}\right)^{\mu}{}_{\alpha}(p_f,x_f)\bar{\varepsilon}^{(-)*}_{\mu}\left(\vec{\bar{p}}_f,\lambda_f\right)\\ \times\int d^4x_i\, \left(\mathbb{E}^{+}_{\ell_i,1}\right)^{\nu}{}_{\beta}(p_i,x_i)\bar{\varepsilon}^{(-)}_{\nu}\left(\vec{\bar{p}}_i,\lambda_i\right)\\ \times\left(\bar{p}_f^2-m_W^2\right)\left(\bar{p}_i^2-m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{(-)\alpha}(x_f)W^{(+)\beta}(x_i)\right\}|\Omega\rangle\,.\end{aligned}$$

LSZ Formula: Comparison

$$\langle f|\, S\,|i\rangle = \int d^4x_f \int d^4x_i\; e^{ip_f\cdot x_f} e^{-ip_i\cdot x_i} \varepsilon^*_\mu(\vec{p}_f,\lambda_f)\varepsilon_\nu(\vec{p}_i,\lambda_i)$$
$$\times \left(p_f^2 - m_W^2\right)\left(p_i^2 - m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{\mu-}(x_f)W^{\nu+}(x_i)\right\}|\Omega\rangle$$

$$\downarrow$$

$$\langle f|\, S\,|i\rangle_B = \frac{1}{2\pi}\int d^4x_f\; \left(\mathbb{E}^{+\dagger}_{\ell_f,1}\right)^{\mu}{}_{\alpha}(p_f,x_f)\bar{\varepsilon}^{(-)*}_{\mu}\left(\vec{\bar{p}}_f,\lambda_f\right)\int d^4x_i\; \left(\mathbb{E}^{+}_{\ell_i,1}\right)^{\nu}{}_{\beta}(p_i,x_i)\bar{\varepsilon}^{(-)}_{\nu}\left(\vec{\bar{p}}_i,\lambda_i\right)$$
$$\times \left(\bar{p}_f^2 - m_W^2\right)\left(\bar{p}_i^2 - m_W^2\right)\langle\Omega|\,\mathcal{T}\left\{W^{(-)\alpha}(x_f)W^{(+)\beta}(x_i)\right\}|\Omega\rangle$$

Summary and Results

- The W boson propagator was explicitly calculated in the presence of a magnetic field using the Ritus Eigenfunction methodology.
- This methodology and calculation is heavily inspired on a very well known way of arriving at the W boson propagator in a vacuum.
- A novel way of calculating the propagator in a magnetic field in the Schwinger parameter representation is presented.
- A small discrepancy with the literature is found and reported.
- An analogous derivation of the LSZ reduction formula for the case of an external constant magnetic field is presented. The resulting formula, and the derivation in between, maintains the same structure as in the $B = 0$ case.

Thank you for your attention!