RADPYC2026

NUMERICAL ANALYSIS OF INVERSE SEESAW MODELS AND LEPTON FLAVOR VIOLATION PROCESSES

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OVERVIEW

HOW DO NEUTRINOS OBTAIN MASS?

- This question arises because neutrino oscillation experiments have shown that neutrinos change flavor as they propagate, which is only possible if they have mass.
- However, in its original formulation, the Standard Model (SM) describes neutrinos as strictly massless particles.

$$-\mathcal{L}_Y = (Y_u)_{ij}\,\overline{Q}_{L_i}\,u_{R_j}\,\Phi + (\tilde{Y}_d)_{ij}\,\overline{Q}_{L_i}\,d_{R_j}\,\Phi + (Y_e)_{ij}\,\overline{L}_{L_i}\,e_{R_j}\,\Phi + \text{h.c.}$$

This is because no right fields are incorporated for neutrinos.

- Therefore, it is necesary to extend the SM introducing new fields or a new mechanism to generate small neutrino masses.

LEPTON FLAVOR VIOLATION (LFV)

- An important point to highlight is that **if neutrinos are massive, then LFV processes are allowed.**
  How significant are these effects?

- In the SM, leptons come in three generations, each associated with a conserved lepton flavor number +1, while their antiparticles have -1.

GENERATION	CHARGED LEPTON	NEUTRINO
1st	e	ν_e
2nd	μ	ν_μ
3rd	τ	ν_τ

Allowed Process

$$\mu^- \to e^- \, \bar{\nu}_e \, \nu_\mu$$

conserves number and leptonic flavor

Forbidden Process

$$\mu^+ \to e^+ \, \gamma$$

conserves total lepton number but violates lepton flavor

THE SEESAW MECHANISM

- If right hand fields for neutrinos are allowed then the Seesaw mechanism provide a natural explanation for the smallness of neutrino masses.

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

where M_D is the Dirac mass matrix, and M_R is the Majorana mass matrix for the sterile neutrinos.

- The light neutrino masses arise from their mixing with new heavy states.
- The observed light neutrino masses m_ν are naturally suppressed by the presence of a high mass scale M associated with sterile neutrinos.

$$m_\nu \sim \frac{m_D^2}{M}$$

The heavier the sterile neutrinos are, the lighter the active neutrinos we observe.

THE SEESAW MECHANISM (HIGH SCALE)

- Main drawback: **extremely high mass scale**

  In order to explain light neutrino masses of

  $$m_\nu \sim 0.1 \text{ eV}$$

  with a Dirac mass term of the electroweak scale,

  $$m_D \sim 100 \text{ GeV}$$

  the Type-I seesaw mechanism predicts a heavy mass scale

  $$M_R \sim \frac{m_D^2}{m_\nu} \sim 10^{23} \text{ eV}$$

- Far beyond experimental reach:
  current particle accelerators operate at energies of the order of the TeV scale.

- Decoupling of heavy states:
  effects of new heavy particles are strongly suppressed at low energies. For example, LFV processes.

- **Is there a similar small-scale mechanism?**

LOW-SCALE SEESAW MODELS (LSSM)

- LSSM can generate potentially observable signals:

$$\mu \to e\gamma,\ \mu \to 3e,\ \mu - eC$$

- These processes are currently being explored by:
  - MEG II, Mu3e, Mu2e, COMET, and PRISM

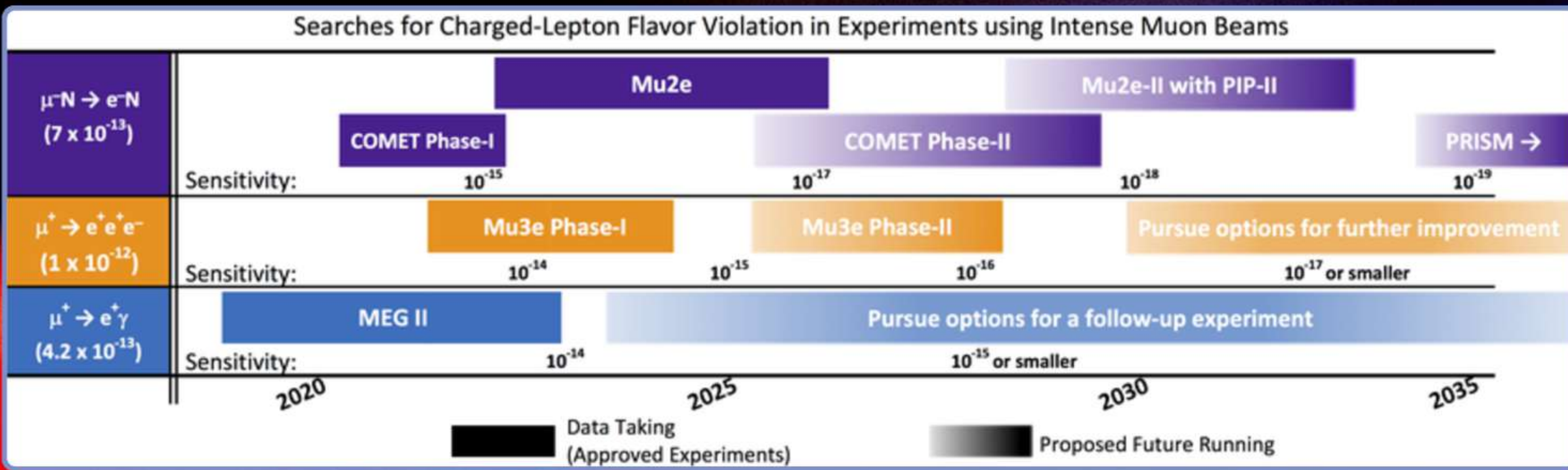


Figure 1: Current and projected sensitivities for muon-based LFV experiments [1].

[1] Mu2e Collaboration, FERMILAB-FN-1064 (2019).

LOW-SCALE SEESAW MODELS (LSSM)

- The full neutral lepton mass matrix in the LSSM takes the form

$$\mathcal{M}_N = \begin{pmatrix} 0 & M_D^T & M_L^T \\ M_D & 0 & M \\ M_L & M^T & \mu \end{pmatrix},$$

where M_D is a Dirac mass matrix, M is the heavy neutrino mass matrix, M_L and μ are lepton-number violating terms.

LINEAR [2]

$$\mu = 0$$

Small neutrino masses generated by a tiny M_L parameter.

Framework applicable

INVERSE [3]

$$M_L = 0$$

Small neutrino masses generated by a tiny μ parameter.

Focus of this work

[2] Forero, Morisi, Tórtola et al., JHEP 09 (2011).
[3] Gonzalez-Garcia & Valle, Phys. Lett. B 216 (1989).

INVERSE SEESAW MODEL

- We begin our analysis with the *inverse seesaw model*, where the smallness of the lepton number violating parameter μ naturally explains the lightness of active neutrino masses
- The mass matrix takes the 9×9 block form shown here, which describes a total of 6 heavy neutrino states

$$M_{9\times9}^{\mathrm{ISS}} = \begin{pmatrix} \mathbf{0}_{3\times3} & M_{D_{3\times3}} & \mathbf{0}_{3\times3} \\ M_{D_{3\times3}}^T & \mathbf{0}_{3\times3} & M_{3\times3} \\ \mathbf{0}_{3\times3} & M_{3\times3}^T & \mu_{3\times3} \end{pmatrix}.$$

the hierarchy among the mass scales is assumed to be

$$|\mu| \ll |M_D| \ll |M|.$$

NEUTRINO MASS MATRIX DIAGONALIZATION

- The neutrino mass matrix can be diagonalized using different methods. In this work, we consider two approaches. As a first step, we adopt the ***block matrix diagonalization method (BMDM)*** [4].
- This consists of constructing a unitary matrix U_ν such that

$$(U_\nu)^T M U_\nu = M_{\text{diag}}, \qquad M_{\text{diag}} = \begin{pmatrix} m_\nu^{\text{diag}} & 0 \\ 0 & M_N^{\text{diag}} \end{pmatrix},$$

where the light and heavy neutrino states are separated perturbatively.

- This method is used as a reference to validate the numerical diagonalization procedures discussed later.

[4] Schechter & Valle, Phys. Rev. D 25 (1982) 774.

NEUTRINO MASS MATRIX DIAGONALIZATION

- The second approach consists of diagonalizing the full matrix exactly via its eigenvalues and eigenvectors, constructing a unitary matrix U such that:

$$U^\dagger \, M \, U = \mathrm{diag}(m_1, m_2, \ldots, m_n).$$

- This procedure provides an exact diagonalization of the full square mass matrix, including all mixings between light and heavy states.

The numerical analysis is carried out using three complementary approaches to explore the parameter space

DIAGONALIZATION METHOD	EXPLORATION STRATEGY
Block diagonalization	Random scan
Full diagonalization	Random scan
Full diagonalization	Bio-inspired algorithms (BA)

PARAMETRIZATION

- Following the reference [5], the matrices M and μ are assumed to be diagonal:

$$M = v_M \, \text{diag}\left(1+\epsilon^m_{11},\, 1+\epsilon^m_{22},\, 1+\epsilon^m_{33}\right),$$
$$\mu = v_\mu \, \text{diag}\left(1+\epsilon^\mu_{11},\, 1+\epsilon^\mu_{22},\, 1+\epsilon^\mu_{33}\right).$$

- The key difference lies in the construction of the Dirac mass matrix:

Garnica et al. (2023)

M_D is constructed using the Casas-Ibarra parametrization [6]:

$$M_D = U_{\text{PMNS}} \sqrt{m_\nu^{\text{diag}}}\, R^T \sqrt{\mu^{-1}}\, M^T.$$

With U_{PMNS}: leptonic mixing matrix, m_ν^{diag}: light neutrino mass matrix, R: complex orthogonal matrix (θ, ϕ, ψ).

This work

M_D is a fully general complex matrix

$$(M_D)_{ij} = m_{ij} + i\,\tilde{m}_{ij},$$

with $i, j = 1, 2, 3.$

[5] Garnica, Peinado & Hernández-Tomé, Phys. Rev. D 108 (2023).
[6] Casas & Ibarra, Nucl. Phys. B 618 (2001) 171.

PARAMETER SPACE

For the first two numerical approaches, the analysis is performed using **17 free parameters**, which define the scanned parameter space:

PARAMETER	RANGE
$\epsilon_{11}^{\mu},\ \epsilon_{22}^{\mu},\ \epsilon_{33}^{\mu}$	$[-0.5,\ 0.5]$
$\epsilon_{11}^{M},\ \epsilon_{22}^{M},\ \epsilon_{33}^{M}$	$[-0.5,\ 0.5]$
$\theta,\ \phi,\ \psi$	$[0,\ 2\pi]$
δ_{CP}	$[0,\ 2\pi]$
$\sin^2\theta_{12}$	$[0.271,\ 0.369]$
$\sin^2\theta_{23}$	$[0.434,\ 0.610]$
$\sin^2\theta_{13}$	$[0.02000,\ 0.02405]$
v_μ	$[1, 1000]\mathrm{eV}$
m_1	$[0, 0.12/3]\mathrm{eV}$
m_2	$\left[\sqrt{6.94\times10^{-5}+m_1^2},\ \sqrt{8.14\times10^{-5}+m_1^2}\right]\ \mathrm{eV}$
m_3	$\left[\sqrt{2.47\times10^{-3}+m_1^2},\ \sqrt{2.63\times10^{-3}+m_1^2}\right]\ \mathrm{eV}$

PARAMETERS WITH BA

For this approach, the analysis is performed using **25 free parameters:**

PARAMETER	RANGE
$\epsilon_{11}^{\mu}, \epsilon_{22}^{\mu}, \epsilon_{33}^{\mu}$	$[-0.5, 0.5]$
$\epsilon_{11}^{M}, \epsilon_{22}^{M}, \epsilon_{33}^{M}$	$[-0.5, 0.5]$
v_{μ}	$[100, 1000]\mathrm{eV}$
m_{ij}^{R}, m_{ij}^{I}	$[5\,\mathrm{Mev}, 5\,\mathrm{GeV}]$

PRELIMINARY RESULTS

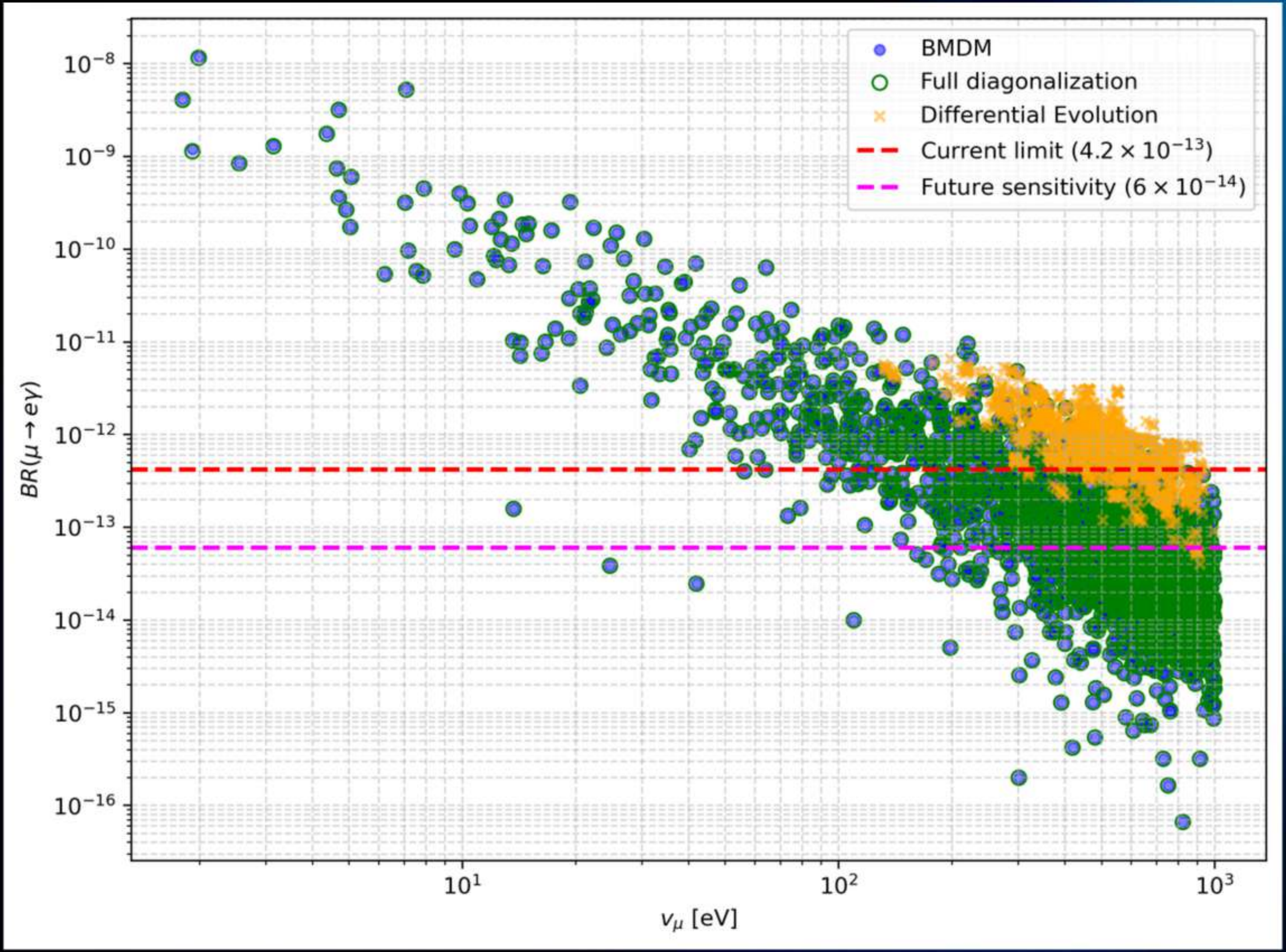


Figure 2: $BR(\mu \to e\gamma)$ vs v_μ for three complementary approaches: block diagonalization (random scan), full diagonalization (random scan), and full diagonalization (BA).

PERSPECTIVES

- Implement constraints from $\mu \to 3e$ and μ - eC.
- Apply the framework to other low-scale seesaw parametrizations.
- Extend the bio-inspired analysis to the Linear Seesaw Model.
- Include τ lepton flavor violating channels ($\tau \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to 3\ell$)

Thank you!

$\mathbf{BR}(\mu \to e\gamma)$

The branching ratio is given by [7]:

$$\mathrm{BR}(\mu \to e\gamma) = \frac{12\pi^2\alpha}{G_F^2 m_\mu^4} \left|F_M^\gamma(0)\right|^2$$

where the dipole form factor is

$$F_M^\gamma(0) = \frac{\alpha_W}{8\pi M_W^2} \frac{m_\mu}{2} \sum_i B_{\mu i}^* \, B_{ei} \, f_M^\gamma(x_i)$$

with $x_i = m_{\chi_i}^2 / M_W^2$, $f_M^\gamma(x)$ the loop function, and B the mixing matrix.

[7] Hernández-Tomé, Illana, Masip, López Castro & Roig, Phys. Rev. D 101, 075020 (2020).

BMDM

The full mass matrix is diagonalized perturbatively by constructing a block unitary matrix [4]:

$$U_\nu = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}$$

where $V_1 = U_{\text{PMNS}}$ and

$$V_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}.$$

The resulting block-diagonal mass matrix is:

$$M_{\text{diag}} = \begin{pmatrix} m_\nu^{\text{diag}} & 0 \\ 0 & M_N^{\text{diag}} \end{pmatrix},$$

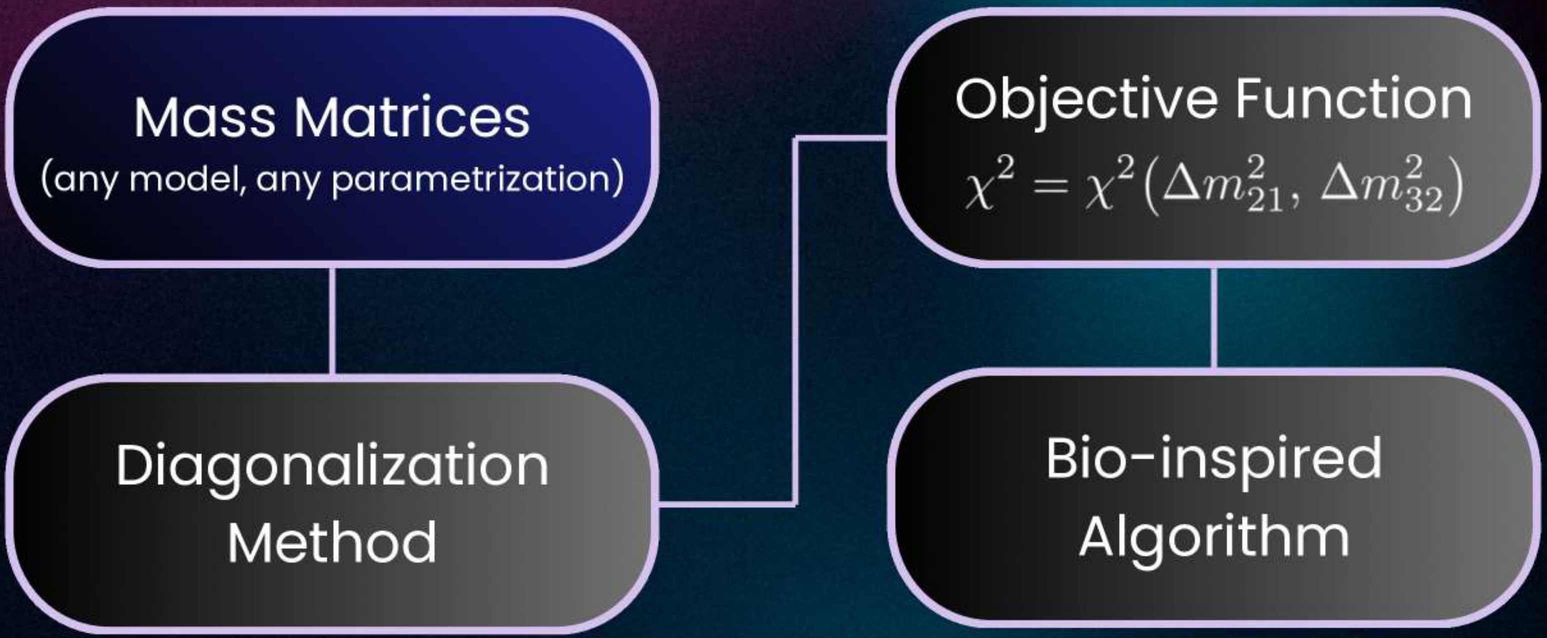
The mixing matrix is constructed as $B = \begin{pmatrix} B_L & B_H \end{pmatrix}$, where:

$$\begin{aligned} B_L &= (1-\eta)\, U_{\text{PMNS}}, \qquad \eta = \frac{1}{2} M_D^* (M^*)^{-T} M^{-1} M_D^\dagger \\ B_H &= M_D^* (M^\dagger)^{-1} V_2 \end{aligned}$$

[4] Schechter & Valle, Phys. Rev. D 25 (1982) 774.

FRAMEWORK ARCHITECTURE

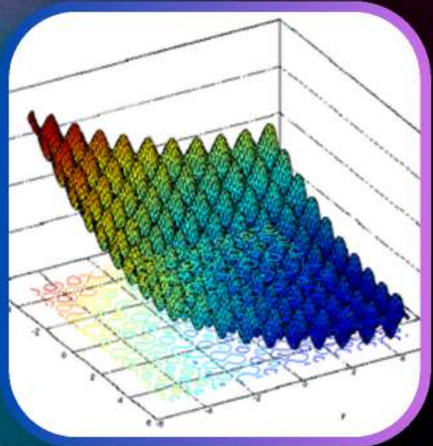
Our code is built following a modular library philosophy, allowing the same methodology to be applied to any seesaw model and any parametrization without rewriting from scratch.



BIO-INSPIRED OPTIMIZATION ALGORITHMS

Why use them?

- Well-suited for non-linear optimization problems.
- Efficient exploration of high-dimensional parameter spaces.
- Robust against noisy functions and multiple local minima.



How do they work? – Differential Evolution

- This algorithm evolves a population of candidate solutions through mutation, crossover, and selection, mimicking natural evolutionary processes. DE iterates until the population converges to the optimal region.
- The fitness function is a χ2 built from the neutrino mass squared differences:

$$\chi^2 = \frac{1}{2}\left[\left(\frac{\Delta m^2_{21} - \Delta m^2_{21,\,\mathrm{exp}}}{\sigma_{21}}\right)^2 + \left(\frac{\Delta m^2_{32} - \Delta m^2_{32,\,\mathrm{exp}}}{\sigma_{32}}\right)^2\right]$$

DIFFERENTIAL EVOLUTION