Nuclear Fano factor effect for Dark Matter and CEνNS detection

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Contents

Direct Dark Matter Detection

Dark Matter

Elastic Scattering Kinematics

For a WIMP with non relativistic velocity, v and mass M_χ the recoil energy transfer to nuclei of mass M_N is: $E_R = \frac{\mu^2 v^2}{M_N} \sin^2(\theta/2)$, with $\mu = \frac{m_\chi M_N}{m_\chi + M_N}$.

- ρ_0: Local dark matter density (~ 0.4 GeV/cm^3)
- $f(\vec{v})$: WIMP velocity distribution in the laboratory frame
- $\frac{d\sigma}{dE_R} = \frac{m_N \sigma_0 F^2(q)}{2\mu^2 v^2}$: Differential WIMP-nucleus cross section
- $v_{\text{min}} = \sqrt{\frac{M_N E_R}{2\mu^2}}$: Minimum WIMP velocity to produce a recoil energy E_R.

Nuclear-Recoil Spectrum

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_\chi M_N} \int_{v_{\text{min}}}^{v_{\text{escape}}} v \cdot f(\vec{v}) \cdot \frac{d\sigma}{dE_R} \, d^3 v$$

Nuclear Ionization Efficiency

Integral Equation for $< \nu^m >$

- For an ionization detector, only electron energy is visible.
- For nuclear recoils energy is divided in: $\varepsilon_R = \bar{\eta} + \bar{\nu}$, $f_n = \bar{\eta}/\varepsilon_R$.[1].
- According to Lindhard theory[2] the integral equation for $< \bar{\nu}^m >$:

$$\int d\sigma_{n,e} \left\{ -\langle \nu^m(E) \rangle + \left\langle \left[\nu \left(E - T_n - \sum_i T_{ei} \right) + \nu(T_n - U) + \sum_i \nu_e(T_{ei} - U_i) \right]^m \right\rangle \right\} = 0$$

- Energy conservation for atomic motion in all successive collisions.
- Independent, $< \nu(E - T_n - \sum_i T_{ei}) \nu(T_n - U) >= \bar{\nu}(E - T_n - \sum_i T_{ei}) \bar{\nu}(T_n - U)$.
- This have to be true for classical collisions ($E > 1$ eV).

[1] In units: $\varepsilon = (8.13\,\mathrm{keV}^{-1}/\mathrm{Z}^{2.23})\mathrm{E}$

[2] Mat.Fys.Medd.Dan.Vid.Selsk.33,no. 10 (1963)

Approximations

Lindhard's modify approximations

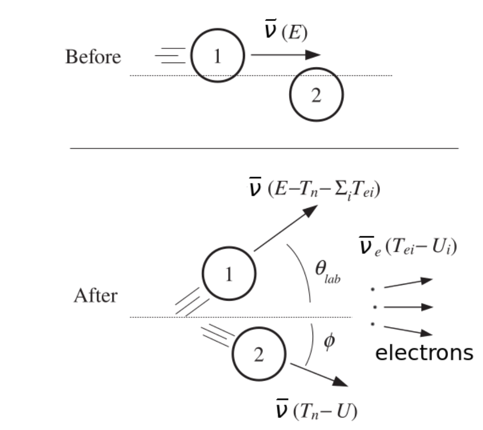
I Neglect contribution to atomic motion coming from electrons.

II **Consider binding energy U as a function of recoil energy**

III Effects of electronic and atomic collisions can be treated separately.

IV $\sum T_{ei}$ small compared to the energy E.

V T_n small compared to the energy E.



Nuclear Ionization Efficiency

- We set an electronic stopping power, S_e, with Coulomb repulsion effects.
- We integrate a semi-classical nuclear stopping power, S_n, that incorporate U.
- And electronic straggling for each collision W.
- Electronic energy is given by a cascade of nuclear recoils for $m = 1$:

$$-\frac{1}{2}\varepsilon S_e(\varepsilon)\left(1+\frac{W(\varepsilon)}{S_e(\varepsilon)\varepsilon}\right)\bar{v}''(\varepsilon)+S_e(\varepsilon)\bar{v}'(\varepsilon)= \int_{\varepsilon u(\varepsilon)}^{\varepsilon^2} dt\frac{f(t^{1/2})}{2t^{3/2}}\times\{\bar{v}(\varepsilon-t/\varepsilon)+\bar{v}[t/\varepsilon-u(\varepsilon)]-\bar{v}(\varepsilon)\}$$

- Where $\varepsilon_R = \varepsilon + u = \bar{\eta} + \bar{\nu}$, then $f_n = (\varepsilon_R - \bar{\nu})/\varepsilon_R$.
- The cascade stops when $\varepsilon = u$.
- Visible energy, $\bar{\eta}$, is **Quenched and is subject to statistical fluctuations.**

f_n for Silicon [Y.Sarkis et al,PRA107,062811 (2023)]

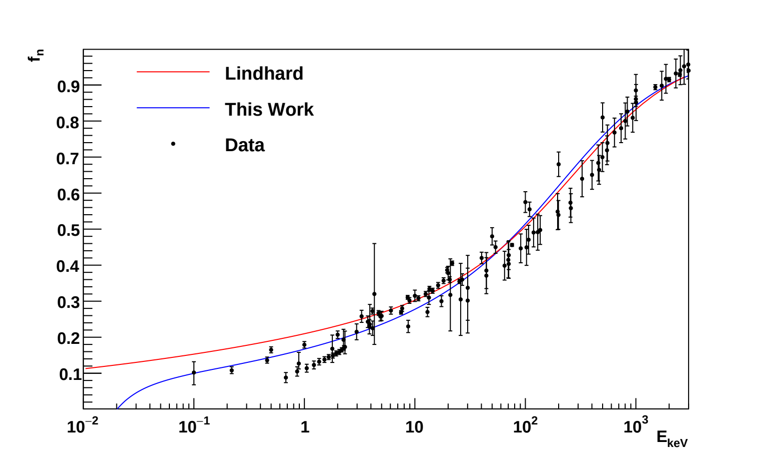


Figure 1: We have set an effective Frenkel energy $\bar{U}_F = U_F/2$ to account for residual atomic movement,[PRD112,L101303 (2025)], with $U_F = 20 \pm 2.4$ eV.

Nuclear Fano Factor

Electronic and Nuclear energy Fluctuations

- By definition η and ν fluctuate, $\varepsilon + u = \eta + \nu = \bar{\eta} + \bar{\nu}$.
- We define the variance: $\Omega_\eta^2 =< \eta^2 > - \bar{\eta}^2 =< (\eta - \bar{\eta})^2 >$.
- If we assume (at low energies) that u and η are uncorrelated,

$$\Omega_\nu^2 = \Omega_\eta^2 + \Omega_u^2.$$

- Ω_u^2 near threshold is, $\delta \bar{U}_F = 1.2$ eV.
- At high energies, Ω_u^2 can be defined by the TF theory and $\Omega_\eta^2 \gg \Omega_u^2$.
- We can define the *Nuclear Fano Factor*:

$$\boxed{F_n^\eta = \frac{\Omega_\eta^2}{\bar{\eta} W_{eh}} \text{ and } F_n^\nu = \frac{\Omega_\nu^2}{\bar{\nu} U_F}}$$

Integro-Differential-Non Homogeneous Equation for Ω_ν^2

- Using the approach described in [PRA107,062811 (2023)] we can deduced,

$$
\boxed{
\begin{aligned}
&-\frac{1}{2}\varepsilon S_e(\varepsilon)\left(1+\frac{W(\varepsilon)}{S_e(\varepsilon)\varepsilon}\right)\Omega_\nu^{2\,\prime\prime}(\varepsilon)+S_e(\varepsilon)\Omega_\nu^{2\,\prime}(\varepsilon)\\
&\quad=\int_{\varepsilon u(\varepsilon)}^{\varepsilon^2} dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}\times\{\Omega_\nu^2(\varepsilon-t/\varepsilon)+\Omega_\nu^2[t/\varepsilon-u(\varepsilon)]-\Omega_\nu^2(\varepsilon)\}+\\
&\quad+\int_{\varepsilon u(\varepsilon)}^{\varepsilon^2} dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}\times\left(\bar{\nu}(\varepsilon-t/\varepsilon-\Sigma t_{ei})+\bar{\nu}[t/\varepsilon-u(\varepsilon)]-\bar{\nu}(\varepsilon)\right)^2
\end{aligned}
}
$$

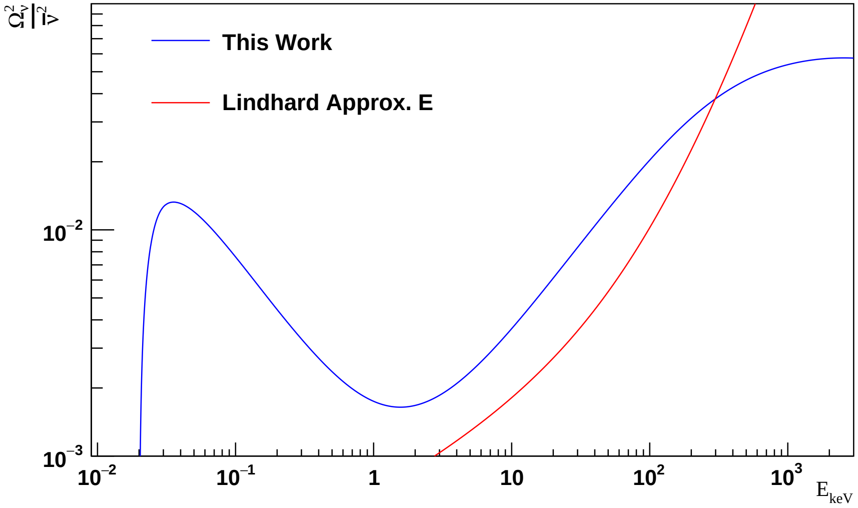
- Here the term Σt_{ei} is defined by the variable TF binding energy model.
- This equation can be solved by the shooting method ($u(\varepsilon)=\varepsilon$).

General Behavior

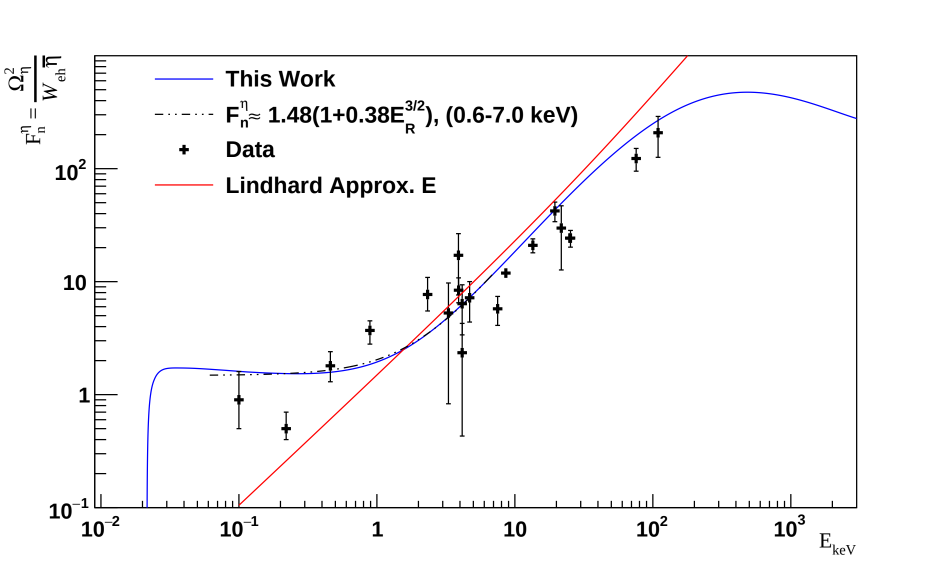
- For $\varepsilon_R < 2\bar{U}_F$, we have $\eta = \bar{\eta} = 0$ and $\bar{\nu} = \nu = \varepsilon_R$.
- Then $\Omega_\nu^2 = 0$ or equal to thermal fluctuations $\Omega_T^2 \ll \Omega_\nu^2$.
- The hing energy boundary condition is $\Omega_\nu''^2 \to 0_-$.
- This is because the nuclear stopping diminish with energy and the electronic stopping increases.
- Lindhard gave an approximate solution for energies among $10 - 100$ keV,
- He get using power law potentials, $\Omega_\eta^2 = 0.0562\,\bar{\eta}^2$ (s=2).
- For lower energies this approximation underestimate Ω_η^2.

Results for Si

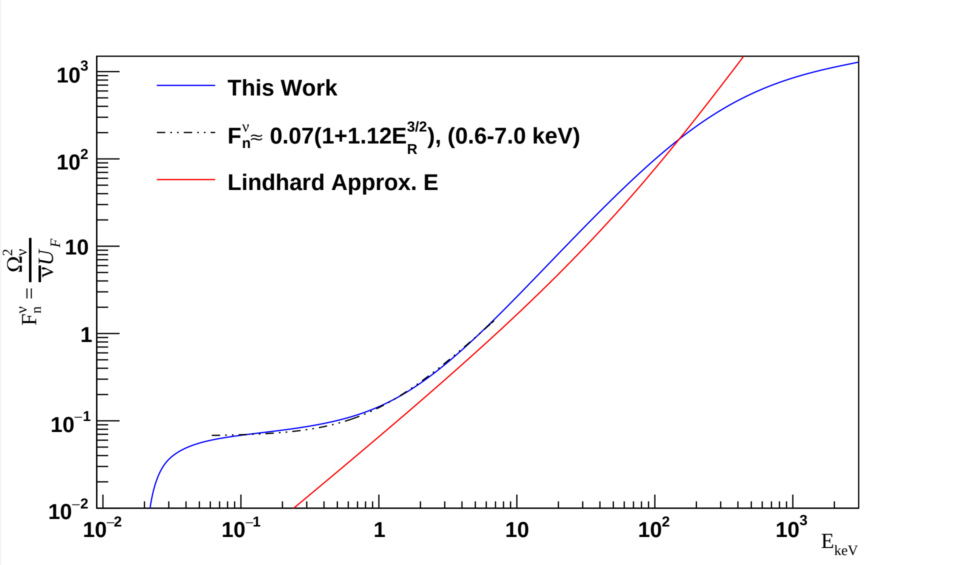
Solution for Si



Contrast With Measurements.

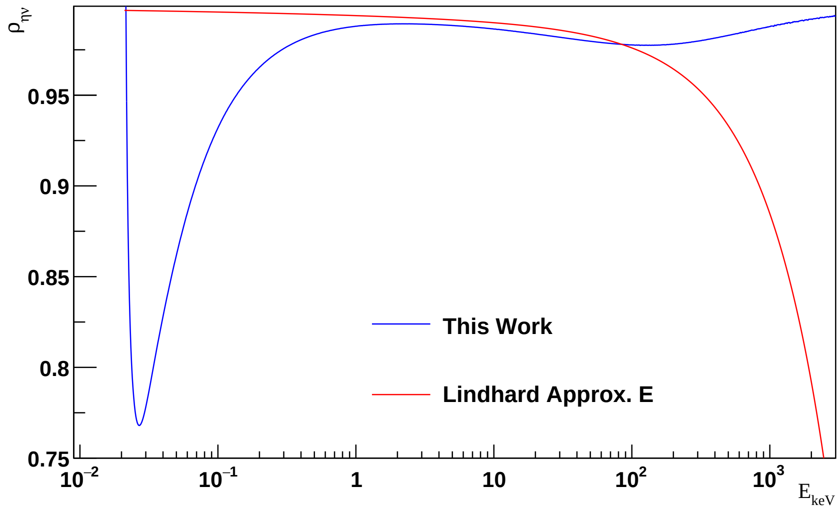


F_n^ν for Si

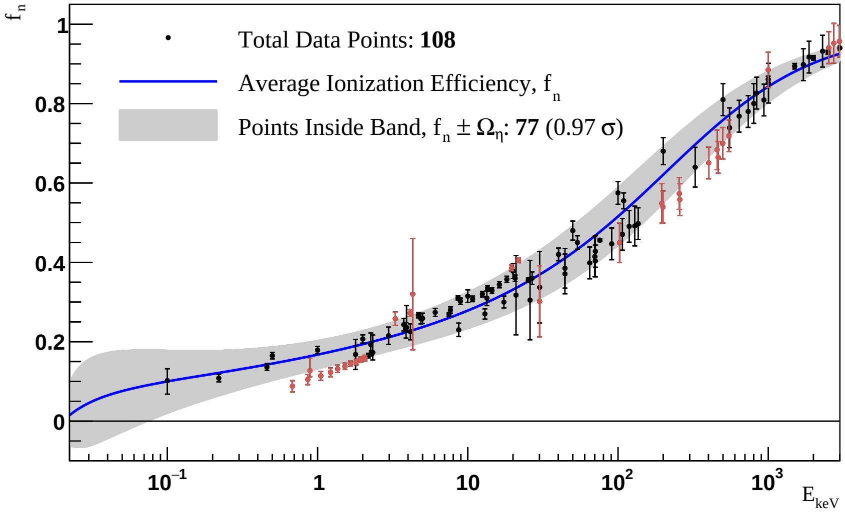


This Work
$F_n^\nu \approx 0.07(1+1.12E_R^{3/2})$, (0.6-7.0 keV)
Lindhard Approx. E
$F_n^\nu = \frac{\Omega_\nu^2}{\overline{\nu} U_F}$
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-2}
10^{-1}
1
10
10^2
10^3
E_{keV}

Correlation Energy



Statistical Error Band



Estimation of Visible Energy Spectrum

- We apply the Ionization efficiency, $\frac{dR}{d\bar{\eta}} = \frac{dR}{d\varepsilon_R}\left(f_n + \varepsilon_R f_n'\right)^{-1}$.
- Since $\bar{\eta}$ fluctuate, we a have to introduce a convolution.
- For low energies we can use, Beta distribution,
- Such that $\int_0^{\varepsilon_R} d\eta P_B(\eta, \varepsilon_R)\eta \approx \bar{\eta}$ and variance, $\Omega_0^2 + F_n^\eta W_{eh}\eta$.

$$\boxed{\frac{dR}{d\bar{\eta}} = \int_0^{\varepsilon_R} d\eta P_B(\eta - \bar{\eta}, \varepsilon_R)\frac{dR}{d\eta}}$$

- In this process only account for electronic energy fluctuation in the cascade process.

- Then the electron energy is converted to electron-hole pair in the detector.
- In this scenario we can consider an ideal solid detector.
- The atoms are just subject to ionization and thermal vibrations (phonons).
- We apply a convolution with a Gaussian that account electronics fluctuations for this case,

$$\boxed{\frac{dR}{d\bar{\eta}_M} = \int_0^{\infty} d\bar{\eta}\, G(\bar{\eta} - \bar{\eta}_M, \sigma_d) \frac{dR}{d\bar{\eta}}}$$

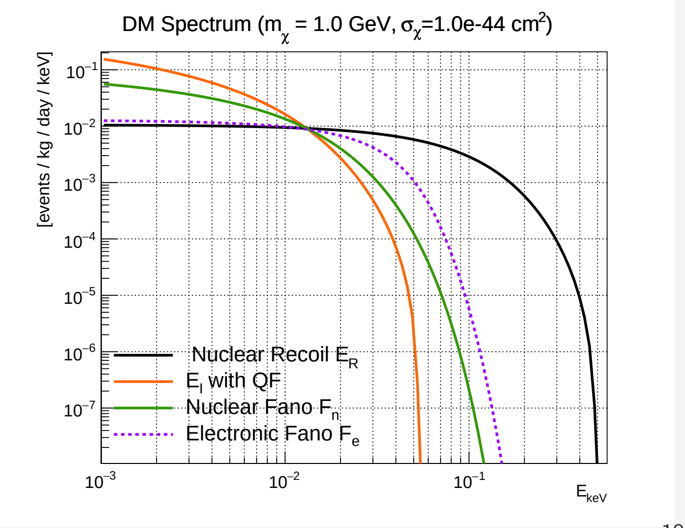
- Where $\sigma_d^2 = \sigma_0^2 + F_e W_{eh} \bar{\eta}$, with $F_e \cong 0.12$ is the electronic Fano factor,
- And σ_0 is the electronic noise defined by the detector.

Rates for DM and CEνNs

DM Rates for Si

Visible DM spectrum:

- Threshold of 10 eV, $\sigma_0 = 20$ eV and $\Omega_0 = 1$ eV.
- With $F_n^\eta = 1.5$, $R = 1.96 \times 10^{-4}$ Evt/kg/day.
- With $F_n^\eta = 0$, $R = 2.74 \times 10^{-4}$ Evt/kg/day.
- 40% **difference if we include nuclear Fano!**



Neutral-current Neutrino-Nucleus Scattering (CEνNS)

Coherent Elastic Neutrino-Nucleus Scattering

Standard Model Differential Cross Section

$$\frac{d\sigma_{SM}}{dE_R}(E_\nu) = \frac{G_F^2}{4\pi}\left[N - (1 - 4\sin^2\theta_W)Z\right]^2\left(1 - \frac{ME_R}{2E_\nu^2} - \frac{E_R}{E_\nu} + \frac{E_R^2}{2E_\nu^2}\right)MF^2(q)$$

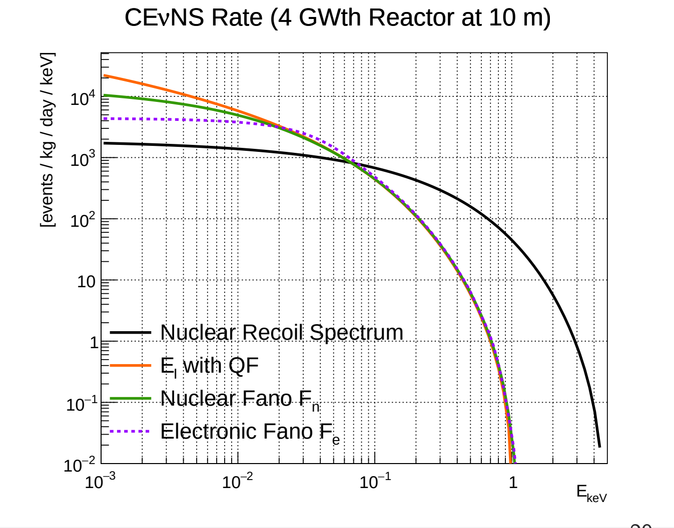
Weak nuclear charge $Q_W^2 \sim N^2$

- First observed by the **COHERENT Collaboration** with neutrinos of $E \sim$ 16–53 MeV:
  - **CsI Detector:** *Science 357, 1123 (2017)*
  - **Liquid Ar Detector:** *PRL 126, 012002 (2021)*
- Observation at lower energies (Reactors) may hint at **Physics Beyond the SM (BSM)**.

CEνNS Rates for Si

Visible CEνNS spectrum:

- Threshold of 10 eV, $\sigma_0 = 20$ eV and $\Omega_0 = 1$ eV.
- With $F_n^\eta = 1.5$, $R = 1.80 \times 10^2$ Evt/kg/day.
- With $F_n^\eta = 0$, $R = 1.97 \times 10^2$ Evt/kg/day.
- 9% **difference if we include nuclear Fano!**
- Equivalent to change $\sin^2(\theta_W)$ by, $\Delta \sin^2(\theta_W) = 1.4 \times 10^{-2}$.



Conclusions

Conclusions

- We have set a formal framework to study cascade recoil process.
- Using integral equations for the first and second moment.
- Where we have also set an algorithm to solve for the second moment base in shooting method.
- Obtaining the nuclear ionization efficiency and nuclear Fano factor in a range from 50 eV to 3 MeV.
- We have shown that an additional convolution is needed to estimate correctly the visible spectrum for DM and CEνNs.
- In particular for DM the limits are overestimated.
- At low energies, we have evidence of an asymmetric behavior for f_n.

Thanks for your attention!

Backup

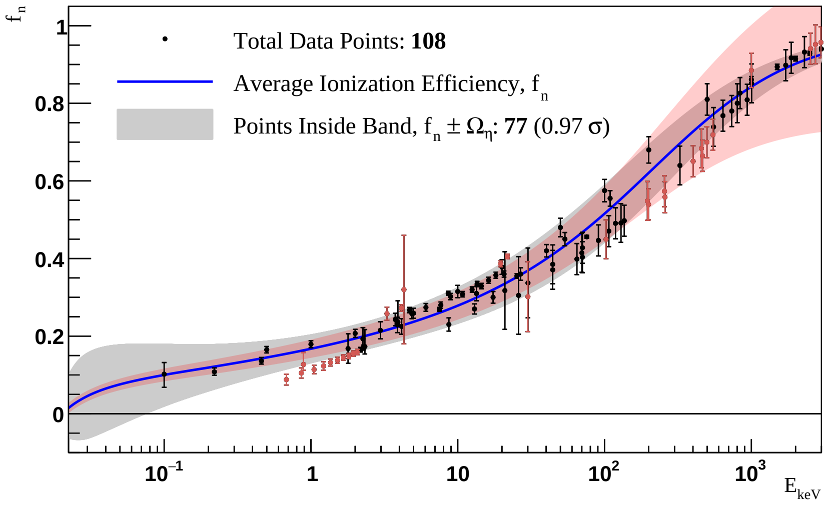


Figure 2: Red is Lindhard error band.

Migdal Effect (1940s attributed to A.B. Migdal)

- Electrons and the nucleus are coupled in atomic systems:
- Perturbation of the nucleus can induce electronic transitions.
- This only happen for sudden perturbations.

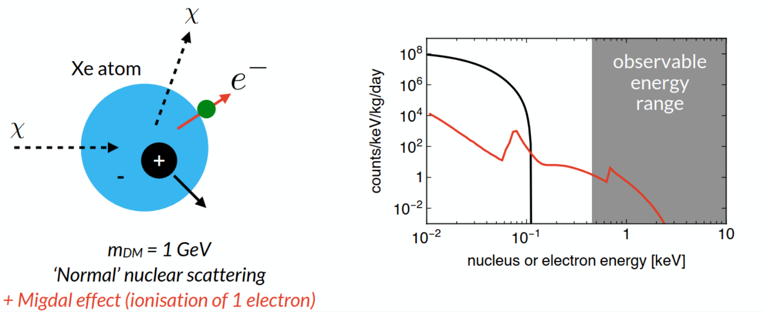


Image from:Christopher McCabe, KCL 2026

- For $E_R \ll 1$ keV atoms, ground state is the most probable condition.